Directional Moment Connections— A Proposed Design Method for Unbraced Steel Frames

Paper presented by ROBERT O. DISQUE
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Discussion by **Kenneth B. Wiesner**

In the author's equations for Elastic G_T on pgs. 16 and 17, the result is correct, but the factor 0.5 should be in the denominator rather than the numerator of the equations.

The writer considers it important to calculate the story drift, based upon the column and girder sizes initially selected, and assuming plastic hinges at leeward ends of girders. If the story drift is excessive, another cycle of design may be necessary. This should be done prior to designing the girder end connections.

For many cases in which this design method is used, it is advisable to consider the leeward ends of girders to have plastic hinges, as shown in the author's Fig. 3. Figure 7 shows a statically determinate subassembly, with column wind shear Q_i and story drift Δ_i. Using virtual work or other approach, it can be shown that the story drift for this subassembly, neglecting $P\Delta$ effect, is given by Eq. (1).

$$\Delta_i = \frac{Q_i h^2}{12E}\left(\frac{h}{I_c} + \frac{L}{0.25 I_g}\right) \quad (1)$$

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Since all three such subassemblies must have the same drift, their relative stiffnesses, and thus the column wind shear distribution, can readily be calculated. For the author's example and sizes, the following column shears are found:

Exterior column:
Q_{ext} = 8.80 kips (vs. 6.0 assumed)

Two interior columns:
Q_{int} = 10.60 kips (vs. 12.0 assumed)

The column wind shear distribution at factored load condition will change, due to development of further plastic yielding of girder connections.

The story drift is 0.575 in., for a drift ratio of 0.00399. This drift ratio is in excess of normal practice in many offices. A recheck of the interior and exterior column design calculations shows that the author's sizes are still satisfactory for strength, however.

In the author's design example, he incorrectly distributed the interior column moment of 12 kips × 12 ft = 144 kip-ft into both connecting girders. Because the girder at the left side of the column already has a gravity load connection plastic hinge, the entire moment must be taken by the right side girder. The plastic moment capacity of the author's example connection is calculated as follows:

$$M_p = A_n F_y d/12 = 1.50 \times 36 \times 24/12 = 108 \text{ kip-ft}$$

(For comparison, if the connection is reasonably flexible, the girder end moment due to the 3.0 kips/ft gravity load could be approximately $WL/20$, or 90 × 30/20 = 135 kip-ft.)

The right side girder connection will therefore "unload" from its initial gravity plastic moment of −108 kip-ft to a resultant moment of +36 kip-ft.

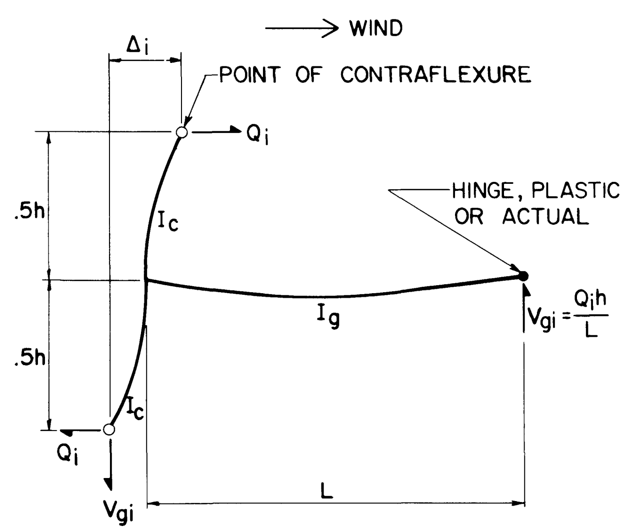


Fig. 7. Subassembly with wind load

ALTERNATE DESIGN APPROACH

The writer proposes an alternate design approach based upon ultimate yield strength and factored loads, an approach which includes $P\Delta$ amplification of all wind moments. The specified minimum load factor to be used in plastic design of steel frames is 1.3 × gravity load combined with 1.3 × wind load, unless the design is governed by 1.7 × gravity load alone. If the author's design approach is used, the true load factor or margin of safety may vary widely, because the approach does not account for the $P\Delta$ effect.

The alternate design approach is outlined below:

1. Establish preliminary girder sizes, neglecting connection end moment capacity.
2. Establish preliminary column sizes.
3. Determine drift (Δ) at service wind load (Q), assuming plastic hinge at leeward end of each girder.
4. Set up the equation for ultimate strength of the story subject to factored loads, then solve for the required connection M_p to provide the desired load factor.
5. Design the connections, using plastic design criteria to assure adequate rotation capacity without premature failure.
6. Check column sizes for two cases:
   a. Full gravity load plus wind and $P\Delta$.
   b. Checkerboard live load, no wind. (This case will produce single curvature bending in interior columns.)
7. If final sizes differ from trial sizes, recycle calculations as required.

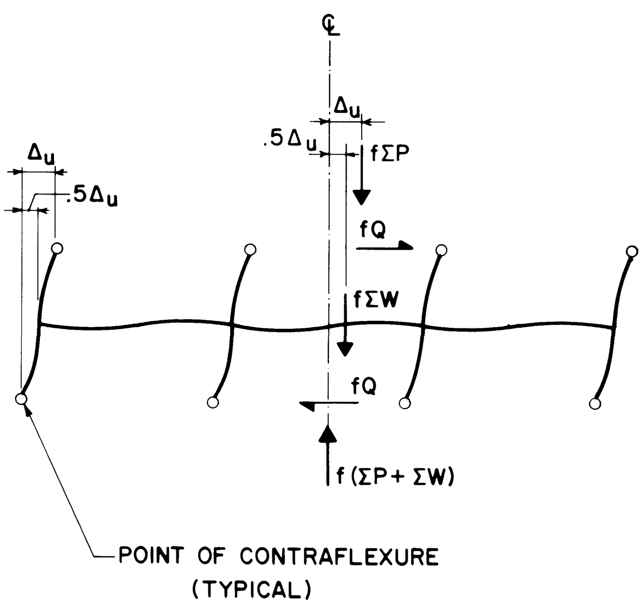


Fig. 8. Deflected frame at factored load (schematic)

ULTIMATE LOAD EQUATION FOR STORY

Referring to Fig. 8, one can write the equation of moment equilibrium of the deflected frame at ultimate, or factored, load:

$$f_w Qh + (f_g \Sigma P\Delta_u + f_g \Sigma W\Delta_u/2) - \Sigma M_{gu} = 0 \quad (2)$$

The load factor for wind is f_w and the gravity load factor is f_g; the term ΣM_{gu} represents the sum of all girder plastic end moments, and the term in brackets represents the total $P\Delta$ moment. Let $\Delta_u = f_w(\alpha\Delta)$, in which Δ = story elastic deflection due to force Q alone, and α = the amplification factor which accounts for $P\Delta$ effects. By logical extension of the approach used in Ref. 10, this factor can be shown to be:

$$\alpha = \frac{1}{1 - \dfrac{f_g\bar{P}\Delta}{Qh}} \quad (3)$$

The term $\bar{P}$ in Eq. (3) is defined as $\bar{P} = \Sigma P + \Sigma W/2$. Equation (2) may be rewritten:

$$f_g\bar{P}\alpha f_w\Delta + f_w Qh - \Sigma M_{gu} = 0$$

or in another form

$$\frac{1}{\dfrac{1}{f_g} - \dfrac{\bar{P}\Delta}{Qh}} + \frac{Qh}{\bar{P}\Delta} - \frac{\Sigma M_{gu}}{f_w \bar{P}\Delta} = 0 \quad (4)$$

Consider the author's example, with connection M_p = 108 kip-ft, Q = 30 kips, h = 12 ft, $\bar{P} = 600 + 270/2 = 735$ kips:

$\Sigma M_{gu} = 6 \times 108 = 648$ kip-ft

$\Delta = 0.575$ in. $= 0.0479$ ft, based on Eq. (1).

Substituting into Eq. (4), compute $f = f_w = f_g$:

$$\frac{1}{1/f - 0.0978} + 10.225 - \frac{14.40}{f} = 0$$

The resulting load factor $f = 1.530$. Also, from Eq. (3) the amplification factor is found to be $\alpha = 1.176$ at $f = 1.53$, and $\alpha = 1.146$ at $f = 1.3$.

The equilibrium shears and moments in the example frame at ultimate load ($f = 1.530$) are shown in Fig. 9.

Ultimate wind shear:
$fQ = 1.53 \times 30 = 45.9$ kips

Ultimate wind story moment:
$fQh = 45.9 \times 12 = 551$ kip-ft

Ultimate story drift:
$\Delta_u = f\alpha\Delta = 1.53 \times 1.176 \times 0.0479 = 0.0862$ ft

Ultimate $P\Delta$ moment:
$f\bar{P}\Delta_u = 1.53 \times 735 \times 0.0862 = 97$ kip-ft

Check: $97/551 = 0.176$ ($= \alpha - 1.0$) **o.k.**

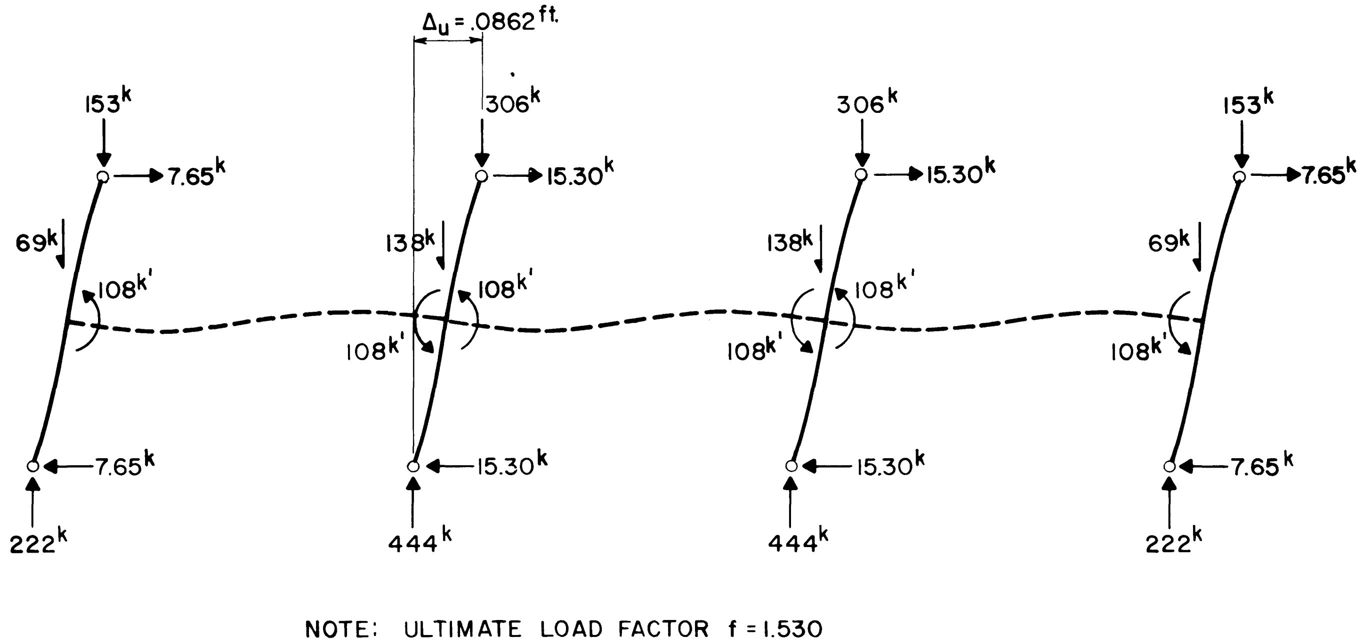
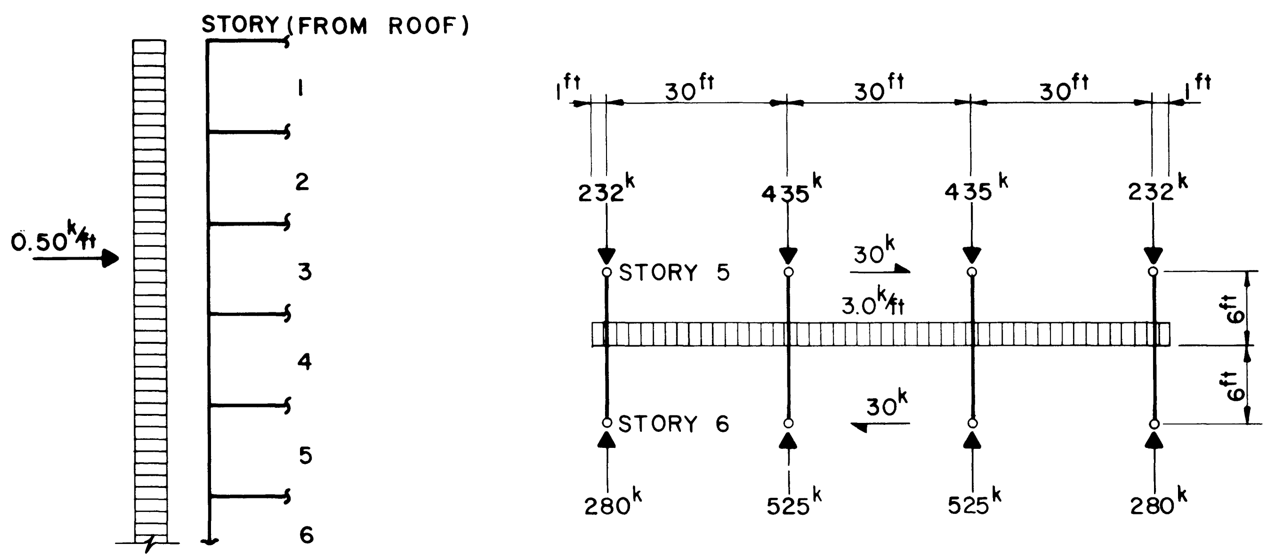


Fig. 9. Author's example frame at ultimate load



(*a*) *Wind loading*

(*b*) *Loading and geometry—Story 6*

Fig. 10. Second design example

ANOTHER DESIGN EXAMPLE

The writer has prepared another design example to show how the $P\Delta$ effect changes the load factor, and to show some features of his alternate design approach. In this example, the frame geometry and wind shear are the same as the author's example, but column gravity loads are higher. This represents the sixth story below the roof in a moderate height office building. The geometry and loading are shown in Fig. 10.

Gravity dead + live load:
Roof = 100 psf
Typical floor = 120 psf

Wind load: 20 psf for full height

Bay sizes:
25 ft × 30 ft, wind bents at 25 ft centers

Exterior col. trib. area:
25(1 + 15) = 400 sq ft/floor

Interior col. trib. area:
25 × 30 = 750 sq ft/floor

Girder load:
25 ft × 0.120 kips/sq ft = 3.0 kips/ft distributed load

Column loads at 5th story below roof:
P_{ext} = 400(0.10 + 4 × 0.12) = 232 kips
P_{int} = 750(0.10 + 4 × 0.12) = 435 kips
667 kips
Total (4 cols.) = 667 × 2 = 1334 kips

Total wind shear:
Q = 5 stories × 0.020 ksf × (12 ft × 25 ft) = 30 kips

This is a story average, based on 27 kips at the story above this floor and 33 kips at the story below.

The steps in the design process are summarized below:

Girders: W24 × 76; F_y = 36 ksi
Int. Columns: W14 × 111; F_y = 36 ksi
Ext. Columns: W14 × 74; F_y = 36 ksi

Service wind distribution:
Ext. Column: 9.47 kips
Int. Column: 10.27 kips

Service wind first order story drift:
Δ = 0.489 in. = 0.0408 ft

$\bar{P} = 1334 + 276/2 = 1472$ kips

$\bar{P}\Delta = 1472 \times 0.0408 = 60.06$

$\bar{P}\Delta/Qh = (60.06)/(30 \times 12) = 0.1668$

Design for $f \geq 1.30$, and set up Eq. (4):

$$\frac{1}{1/1.30 - 0.1668} + 5.994 - \frac{6M_p}{60.06 \times 1.3} = 0$$

$$1.660 + 5.994 - 0.07685\, M_p = 0$$

Required M_p = 99.6 kip-ft

Use 8-in. × ¼-in. plate and four ⅞-in. diameter bolts top and bottom:

$A_n = (8 - 2)(1/4) = 1.50$ sq in.

$M_p = 1.50 \times 36 \times 24/12$

= 108 kip-ft >99.6 kip-ft **o.k.**

The resulting load factor is computed, using Eq. (4):

$f = 1.384$, and the amplification factor is

$$\alpha = \frac{1}{(1 - 0.1668) \times 1.384} = 1.300$$

Therefore the ultimate drift is:

$\Delta_u = f\alpha\Delta = 1.384 \times 1.300 \times 0.0408$

= 0.0734 ft

Note that the load factor of 1.384 is approximately 10 percent lower than the 1.530 load factor for the author's example, although the only difference is in the column axial load. Similarly, this amplification factor is about 10 percent larger than that for the author's example.

LOADING HISTORY

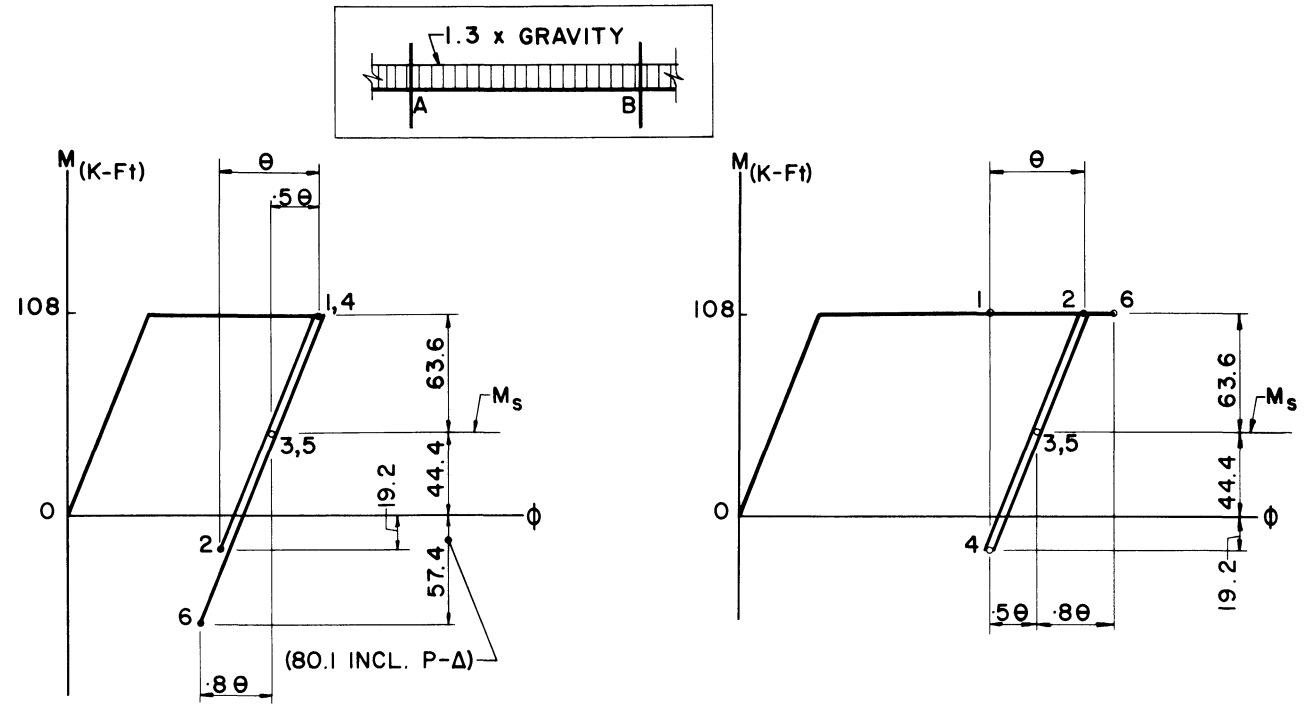
It is of interest to investigate how the wind loading history of the frame affects the assumption that the leeward girder connections can take no wind moment, and thus affects the frame lateral stiffness.

Figure 11 shows an example loading history, in the form of first-order M-ϕ diagrams for the two end connections of a girder. Figure 12 shows a different loading history. The loading history for these examples is summarized in Table 1. Note that both figures start from an initial point (1) at which the connection has yielded under factored gravity load prior to the application of wind moment. In Table 1, Q represents the design wind shear at service load.

In Figs. 11 and 12, note that the "shakedown" or stabilized moment M_s is dependent upon the magnitude of the

Table 1

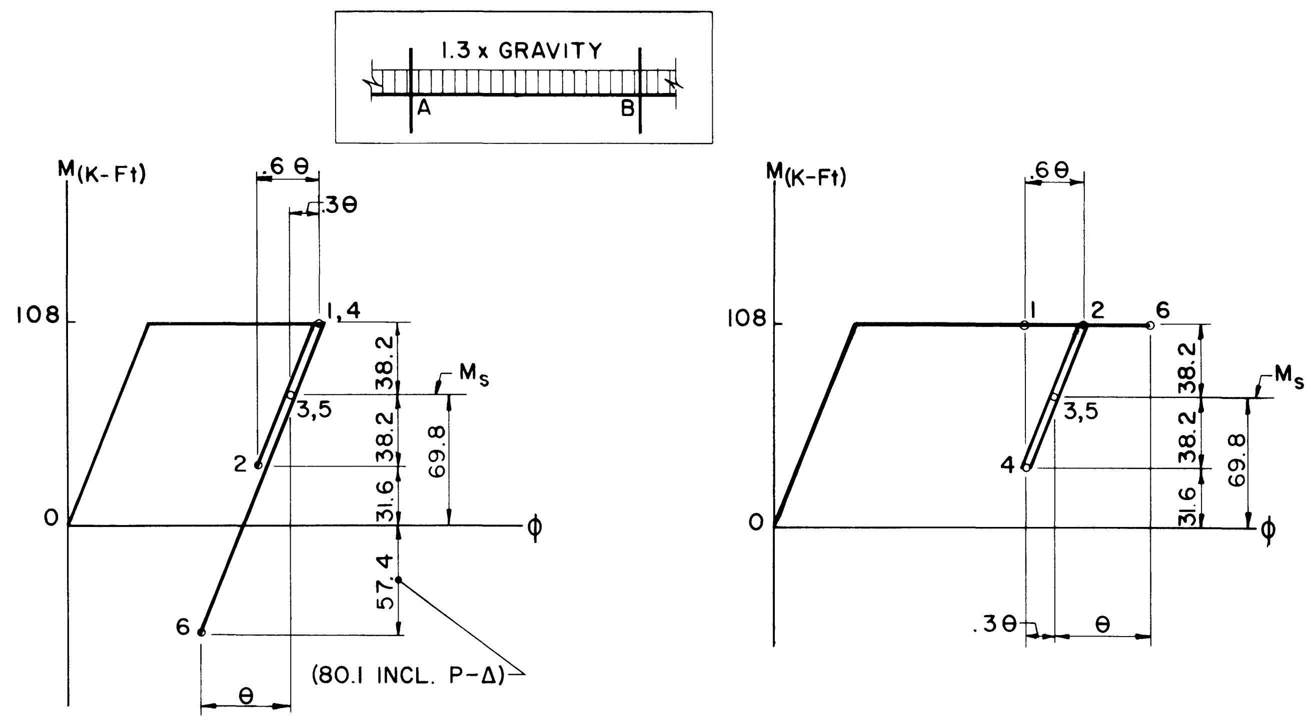
Figure No.	Wind History		M-ϕ Curve Points	Connection Rotation	
	Force	Direction		End A	End B
11	$+Q$	L-R	1-2	$-\theta$	$+\theta$
	0	—	2-3	$+0.5\theta$	-0.5θ
	$-Q$	R-L	3-4	$+0.5\theta$	-0.5θ
	0	—	4-5	-0.5θ	$+0.5\theta$
	$+1.3Q$	L-R	5-6	-0.8θ	$+0.8\theta$
12	$+0.6Q$	L-R	1-2	-0.6θ	$+0.6\theta$
	0	—	2-3	$+0.3\theta$	-0.3θ
	$-0.6Q$	R-L	3-4	$+0.3\theta$	-0.3θ
	0	—	4-5	-0.3θ	$+0.3\theta$
	$+1.3Q$	L-R	5-6	$-\theta$	$+\theta$



(*a*) *Girder end A*

(*b*) *Girder end B*

Fig. 11. Girder M-φ loading history #1



(*a*) *Girder end A*

(*b*) *Girder end B*

Fig. 12. Girder M-φ loading history #2

first cycle of wind moment, also that end A develops more than end B of the total girder moment at factored wind load. In Fig. 11, 62 percent is developed, and in Fig. 12, 77 percent is developed at end A. The exact ratio is dependent upon the magnitude of previously applied wind moment. $M_s = M_p - 0.5\, M_{w1}$, in which M_{w1} = the first cycle wind moment. Therefore, since it is quite possible that wind moments applied prior to an extreme (i.e., factored) wind loading may be small, almost all the wind moment may have to be taken at end A, the windward connection. Column stability, story drift, and actual load factor should be calculated accordingly.

In conclusion, the writer agrees that frame design using Type 2 Directional Moment Connections is a useful and simple design method. Engineers using this method should be aware of its limitations, including the likelihood of connection yielding under gravity service loading, the need for connection plastic rotation capacity, and the effect of plastic hinges on the lateral story stiffness. Also the frame should be checked by a method properly accounting for $P\Delta$ effects, to determine whether it has an adequate load factor for combined wind plus gravity load.

REFERENCES

10. *Wiesner, K. B.* Discussion of "Applied Plastic Design of Unbraced Multistory Frames" *by R. O. Disque, AISC Engineering Journal, July 1972.*