Design Example for Beams with Web Openings

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DUE TO the increasing cost of energy and the difficulty of obtaining raw materials, economy has a high priority in all aspects of design. In the design of multistory steel buildings, savings can be realized by passing ductwork for heating, ventilation, and air conditioning systems *through* steel floor beams, rather than under them. Not only does this practice save in the overall height of the structure, along with all the related benefits in material savings, but it also saves in the cost of heating and air conditioning by enclosing less volume to be heated or cooled. Eccentric openings are of special interest because not all floor beams and girders are of the same depth at any given level. It is desirable to keep the ductwork on a relatively level plane to cut down on the cost of fabricating bends. It is doubtful that it costs any more to fabricate an eccentric opening in a steel beam than it does to fabricate a concentric opening. Thus, even more savings can be realized by using the eccentric opening. The addition of reinforcement is also sometimes desirable so that a heavier section is not required because of the opening. Design formulas have been developed for beams with concentric and eccentric web openings, both unreinforced and reinforced;[1] the purpose of this paper is to illustrate the application of these formulas in a typical design problem.

PROBLEM STATEMENT

A portion of the floor system supporting a concourse in a multistory building is shown in Fig. 1. The floor system consists of girders spanning from column to column, with floor beams supported by the girders. The floor beams and girders support a 4-in. concrete slab, which in turn provides continuous lateral support to the top flanges of the floor members. Moment connections are provided between the columns and girders; therefore, the girder ends are assumed to be fixed. It is further assumed that the columns are W14 sections and that the girder span is taken from column face to column face. The floor beams are attached to the girders with shear connections; simple supports are therefore assumed. For architectural reasons the floor beams are limited to a 21-in. depth.

The heating, ventilation, and air conditioning (HVAC) ducts run parallel to the girders, with service ducts branching out at right angles. For both architectural and aesthetic reasons it is undesirable to run the HVAC system below the floor members, so they must penetrate them. It is also desirable to keep the HVAC system on a level plane, thereby reducing installation costs by decreasing the number of bends in the duct material. It is necessary to provide a duct area of 144 sq. in. with 1-in.-thick insulation on all sides. Vertical positioning of the ducts used in this example are shown in Fig. 2. The corner radius was determined from recommendations for members subjected to fatigue loadings.[2] While structures designed by plastic methods are not subject to fatigue situations, these guidelines were used to provide a reasonable basis for determining the corner radii of web openings. It is also possible that a slightly smaller opening could have been used to accommodate the duct. The use of a smaller opening might, however, cause problems in the installation of the duct insulation and thereby increase costs. A liberal clearance was therefore provided in this example.

The floor system is to be designed to carry a live load of 100 psf[3] and a dead load of 80 psf (50 psf for the concrete slab and 30 psf for other dead loads). Using A36 steel and the AISC Specification[4] for plastic design, along with current research results, the locations where openings can be placed along the length of the floor beams and girders will be explored with the opening reinforcement varying as follows: (1) no opening reinforcement is provided, (2)

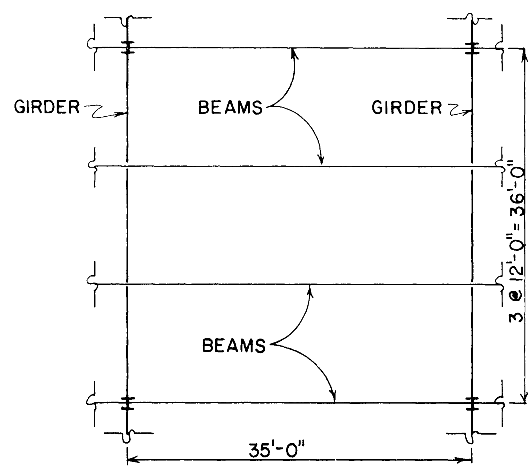

Fig. 1. Plan view of floor system

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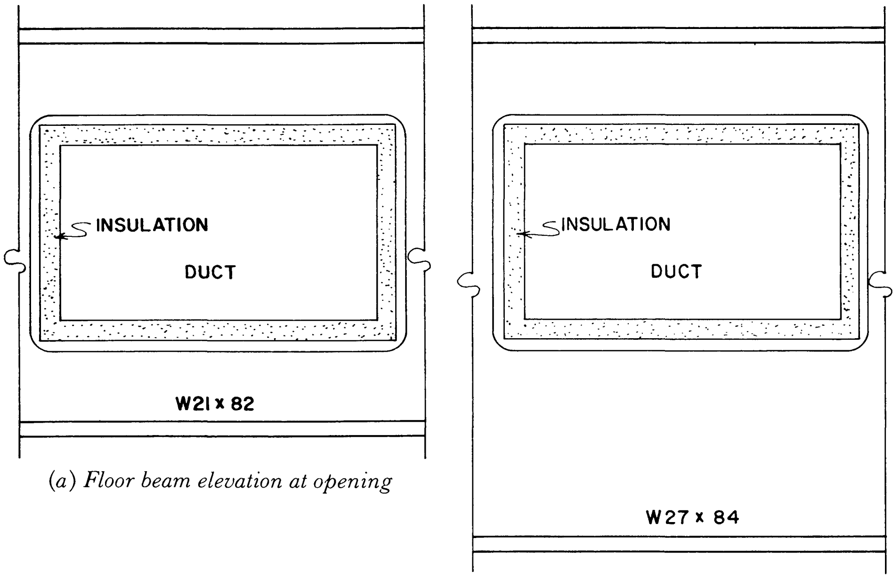


(*a*) *Floor beam elevation at opening*

(*b*) *Girder elevation at opening*

Figure 2

enough reinforcement is supplied to resist the maximum shear force in the beam, and (3) the minimum reinforcement required to develop the shear strength of the section is furnished.

FLOOR BEAMS

Selection of Beam Section

Uniformly Distributed Load:
$w = (1.7)(12)(0.10 + 0.08) = 3.67$ kips/ft

Design Moment:
$M = (3.67)(35)^2/8 = 562$ kip-ft

Required Plastic Section Modulus:
$Z = (562)(12)/36 = 187$ in.^{3}

Try W21 × 82 ($Z = 192$ in.$^3 > 187$ in.^{3}):

Design Shear:
$V = (3.67)(35)/2 = 64.2$ kips

Plastic Shear Force:
$V_p = (0.55)(36)(20.86)(0.499)$
$= 206$ kips > 64.2 kips

Use W21 × 82

Properties for Investigating Local Strength at the Opening

Opening Parameters:
$h = 6$ in.; $a = 9.5$ in.; $e = 0$

Cross Section Properties:
$A_f = (8.962)(0.795) = 7.12$ in.^{2}
$A_w = (20.86)(0.499) = 10.4$ in.^{2}

Reference Values:
$M_p = (192)(36)/12 = 576$ kip-ft
$V_p = 206$ kips (see above)

Calculation of Internal Beam Forces—The internal beam forces are described by the two expressions below, based on Fig. 3. These forces are plotted in Fig. 4 along with the interaction diagrams for the floor beams.

$$\left|\frac{V}{V_p}\right|_x = \frac{64.2 - 3.67x}{206}$$

$$\left|\frac{M}{M_p}\right|_x = \frac{64.2x - 1.84x^2}{576}$$

Opening Locations for $A_r = 0$—Calculate the interaction diagram coordinates, using formulas presented in the Appendix, with $A_r = e = 0$:

$$\alpha = \left(\frac{3}{16}\right)\left(\frac{20.86}{9.5}\right)^2\left(1 - \frac{12}{20.86}\right)^2 = 0.163$$

$$\beta = \frac{10.4}{(2)(7.12)}\left[\left(1 - \frac{12}{20.86}\right)^2 \frac{1}{1.163}\right]^{1/2} = 0.288$$

$$\left(\frac{M}{M_p}\right)_0 = \frac{1 + \frac{10.4}{7.12}\left[\frac{1}{4} - \left(\frac{1}{20.86}\right)^2\right]}{1 + \frac{10.4}{(4)(7.12)}} = 0.911$$

$$\left(\frac{M}{M_p}\right)_1 = \frac{1 - 0.288}{1 + \frac{10.4}{(4)(7.12)}} = 0.522$$

$$\left(\frac{V}{V_p}\right)_1 = 2\left[\left(\frac{1}{2} - \frac{6}{20.86}\right)^2 - \left(\frac{7.12}{10.4}\right)^2 (0.288)^2\right]^{1/2}$$
$$= 0.158$$

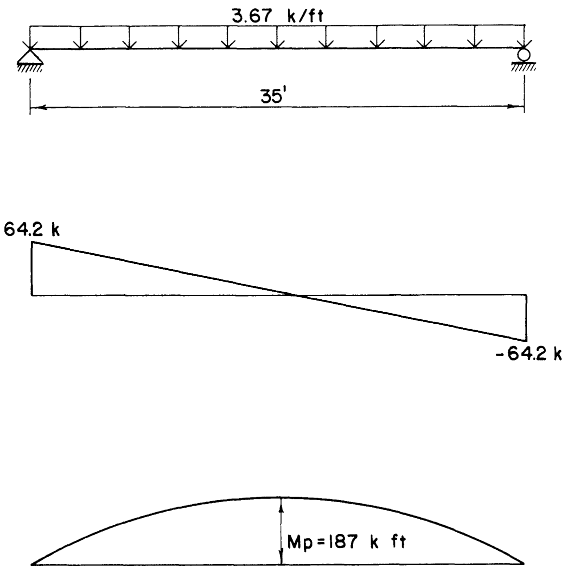


Fig. 3. Floor beam loading, shear, and moment diagrams

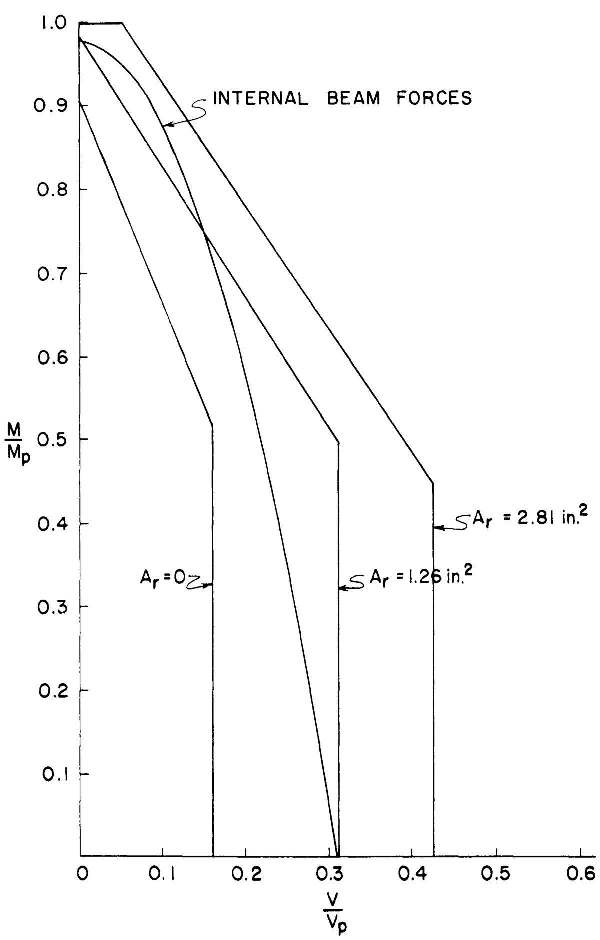


Fig. 4. Floor beam interaction diagrams

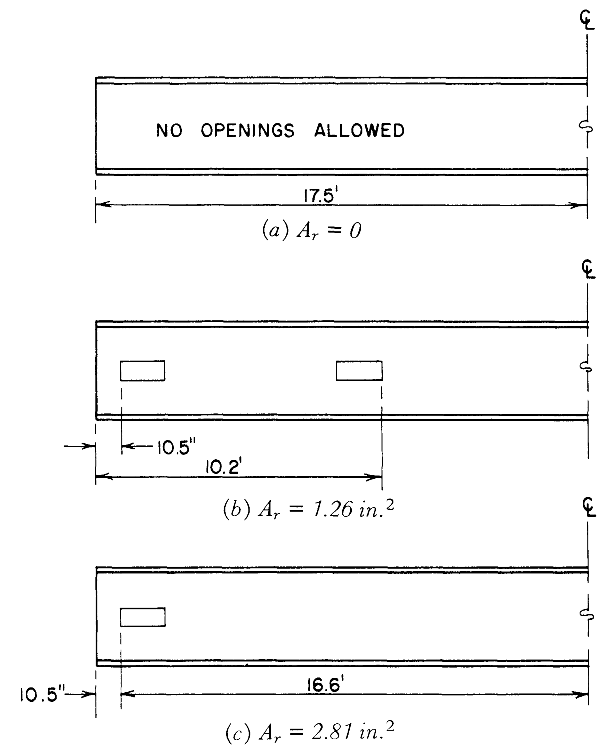


Fig. 5. Permissible opening locations for floor beams

The above coordinates are plotted to form the approximate interaction diagram shown in Fig. 4 for $A_r = 0$. As shown schematically in Fig. 5 and graphically in Fig. 4, there are no positions along the beam where the opening can be placed for $A_r = 0$. This is because all the points on the internal beam force curve lie outside the failure envelope for $A_r = 0$.

Opening Locations for A_r Large Enough to Accommodate Maximum Beam Shear—The following quadratic equation in A_r is obtained from the generalized equation for $(V/V_p)_1$ presented in the Appendix, with $A_r \leq (A_r)_{min}$:

$$AA_r^2 + BA_r + C = 0$$

where

$$A = \frac{16\alpha}{A_w^2(1 + \alpha)}$$

$$B = \frac{8\alpha}{(1 + \alpha)A_w}\left[\left(1 - \frac{2h}{d}\right)^2 - \left(\frac{V}{V_p}\right)^2\right]^{1/2}$$

$$C = \left(1 - \frac{2h}{d}\right)^2\left(\frac{\alpha}{1 + \alpha}\right) - \left(\frac{V}{V_p}\right)^2$$

Calculation of A_r:

$V_{max}/V_p = 0.312$

$$A = \frac{(16)(0.163)}{(10.4)^2(1.163)} = 0.0207$$

$$B = \frac{(8)(0.163)}{(1.163)(10.4)}\left[\left(1 - \frac{12}{20.86}\right)^2 - (0.312)^2\right]^{1/2}$$

$$= 0.0311$$

$$C = \left(1 - \frac{12}{20.86}\right)^2 \frac{(0.163)}{(1.163)} - (0.312)^2 = -0.0721$$

$$A_r = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$= \frac{-0.0311 \pm \sqrt{(0.0311)^2 - (4)(0.0207)(-0.0721)}}{(2)(0.0207)}$$

$$= 1.26 \text{ in.}^2$$

Use 1-Bar 3 × 7⁄16 above and below the opening on one side only.

($A_r = 1.31$ in.$^2 > 1.26$ in.^{2}; $b_r/t_r = 6.9 < 8.5$)

Calculation of interaction diagram coordinates using formulas given in the Appendix with $A_r = 1.26$ in.^{2}, $e = 0$:

$$\beta = -\frac{(2)(0.163)}{1.163}\left(\frac{1.26}{7.12}\right) + \frac{10.4}{(2)(7.12)} \times \left[\left(1 - \frac{12}{20.86}\right)^2 \frac{1}{1.163} - \frac{(16)(0.163)}{(1.163)^2}\left(\frac{1.26}{10.4}\right)^2\right]^{1/2}$$

$$= 0.210$$

$$\left(\frac{M}{M_p}\right)_0 = \frac{1 + \frac{1.26}{7.12}\left(\frac{12}{20.86}\right) + \frac{10.4}{7.12}\left[\frac{1}{4} - \left(\frac{6}{20.86}\right)^2\right]}{1 + \frac{10.4}{(4)(7.12)}}$$

$$= 0.986$$

$$\left(\frac{M}{M_p}\right)_1 = \frac{1 - \frac{1.26}{7.12} - 0.210}{1 + \frac{10.4}{(4)(7.12)}} = 0.449$$

$$\left(\frac{V}{V_p}\right)_1 = 2\left[\left(\frac{1}{2} - \frac{6}{20.86}\right)^2 - \left(\frac{7.12}{10.4}\right)^2 (0.210)^2\right]^{1/2}$$

$$= 0.313$$

These coordinates are plotted in Fig. 4 to describe the center interaction diagram. It should be noted that the maximum shear in the beam is just within the failure envelope of the diagram, as shown by the point on the internal beam force curve corresponding to M/M_p equals zero. In the regions of higher moment, part of the internal beam force curve falls outside the envelope, indicating that the opening should not be placed in the areas shown in Fig. 5 with $A_r = 1.26$ in.^{2}

Opening Locations for $(A_r)_{min}$

$$(A_r)_{min} = \frac{(9.5)(0.499)}{\sqrt{3}} = 2.74 \text{ in.}^2$$

Use 1-Bar 3¾ × ¾ above and below the opening on one side only.

($A_r = 2.81$ in.$^2 > 2.74$ in.^{2}; $b_r/t_r = 5.0 < 8.5$)

Calculation of interaction diagram coordinates using formulas given in the Appendix with $e = 0$, $A_r = 2.81$ in.^{2}:

$$\left(\frac{M}{M_p}\right)_0 = \frac{1 + \frac{2.81}{7.12}\left(\frac{12}{20.86}\right) + \frac{10.4}{7.12}\left[\frac{1}{4} - \left(\frac{6}{20.86}\right)^2\right]}{1 + \frac{10.4}{(4)(7.12)}}$$

$$= 1.08$$

$$\left(\frac{M}{M_p}\right)_1 = \frac{1 - \frac{2.74}{7.12}}{1 + \frac{10.4}{(4)(7.12)}} = 0.451$$

Note that in the above calculation for $(M/M_p)_1$, the calculated value of $(A_r)_{min}$ was used in place of the area of the bar chosen. This is because A_r is subtracted from one in this equation, and logic tells us that more reinforcement must increase the load carrying capacity of a member, not decrease it. Therefore, the $(A_r)_{min}$ value calculated as above should be used to prevent overly conservative results.

$$\left(\frac{V}{V_p}\right)_1 = \left(1 - \frac{12}{20.86}\right) = 0.425$$

These coordinates are plotted on Fig. 4 to form the outside interaction diagram. With the reinforcing area set at $(A_r)_{min}$, the opening can be placed anywhere along the beam as shown in Fig. 5.

GIRDERS

Selection of Girder Section

Load:
$P = (3.67)(35) = 128$ kips

Design Moment:
$M = (128)(11.4)/2 = 730$ kip-ft

Required Plastic Section Modulus:
$Z = (730)(12)/36 = 243$ in.^{3}

Try W27 × 84 ($Z = 244$ in.$^3 > 243$ in.^{3})

Design Shear:
$V = 128$ kips

Plastic Shear Force:
$V_p = (0.55)(36)(0.463)(26.69) = 245 > 128$ kips

Use W27 × 84

Properties for Investigating Local Strength at the Opening

Opening Parameters:
h = 6 in.; a = 9.5 in.; e = 3 in.

Cross Section Properties:
$A_f = (9.963)(0.636) = 6.34 \text{ in.}^2$
$A_w = (0.463)(26.69) = 12.4 \text{ in.}^2$

Reference Values:
$M_p = (244)(36)/12 = 732$ kip-ft
$V_p = 245$ kips (see above)

Calculation of Internal Beam Forces—The internal beam forces form a discontinuous curve described by the expressions below, based on Fig. 6, with the appropriate limits. This curve is shown in Fig. 7.

$$\left|\frac{V}{V_p}\right| = \frac{128}{245} = 0.522, \quad \left|\frac{M}{M_p}\right|_x = \frac{-730 + 128x}{732} \qquad \text{Limits: } 0 \leq x \leq 11.4 \text{ ft}$$

$$\left|\frac{V}{V_p}\right| = 0, \quad \left|\frac{M}{M_p}\right| = 0.997 \qquad \text{Limits: } 11.4 \text{ ft} \leq x \leq 17.4 \text{ ft}$$

where x is taken to be positive from the column face toward midspan.

Opening Locations for $A_r = 0$—Calculation of interaction diagram coordinates using formulas presented in the Appendix with $A_r = 0$, $e = 3$ in.:

$$\alpha_T = \frac{3}{16}\left(\frac{26.69}{9.5}\right)^2\left(1 - \frac{12}{26.69} - \frac{6}{26.69}\right)^2 = 0.157$$

$$\alpha_B = \frac{3}{16}\left(\frac{26.69}{9.5}\right)^2\left(1 - \frac{12}{26.69} + \frac{6}{26.69}\right)^2 = 0.889$$

$$\beta_T = \frac{12.4}{(2)(6.34)}\left[\left(1 - \frac{12+6}{26.69}\right)^2 \frac{1}{1.157}\right]^{1/2} = 0.296$$

$$\beta_B = \frac{12.4}{(2)(6.34)}\left[\left(1 - \frac{12-6}{26.69}\right)^2 \frac{1}{1.889}\right]^{1/2} = 0.552$$

$$\left(\frac{M}{M_p}\right)_0 = \frac{1 + \frac{12.4}{6.34}\left[\frac{1}{4} - \frac{(6)^2 + (2)(6)(3)}{(26.69)^2}\right]}{1 + \frac{12.4}{(4)(6.34)}} = 0.867$$

$$\left(\frac{M}{M_p}\right)_1 = \frac{1 - 0.552}{1 + \frac{12.4}{(4)(6.34)}} = 0.301$$

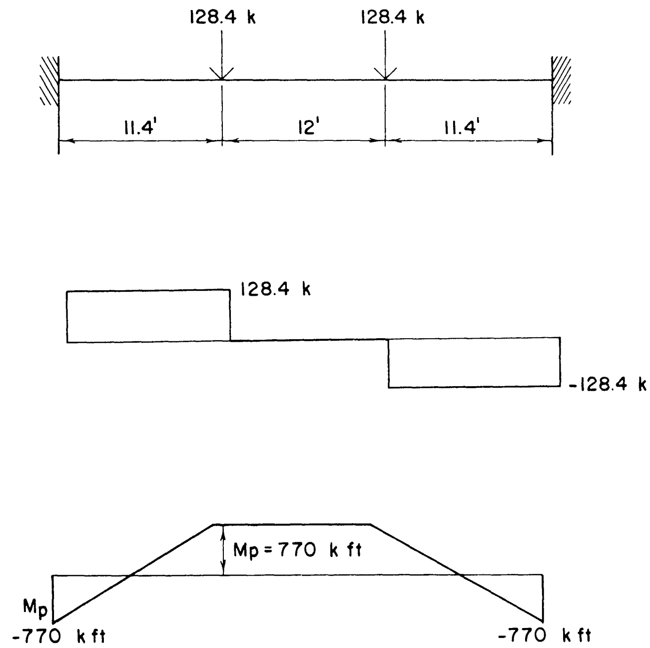


Fig. 6. Girder loading, shear, and moment diagrams

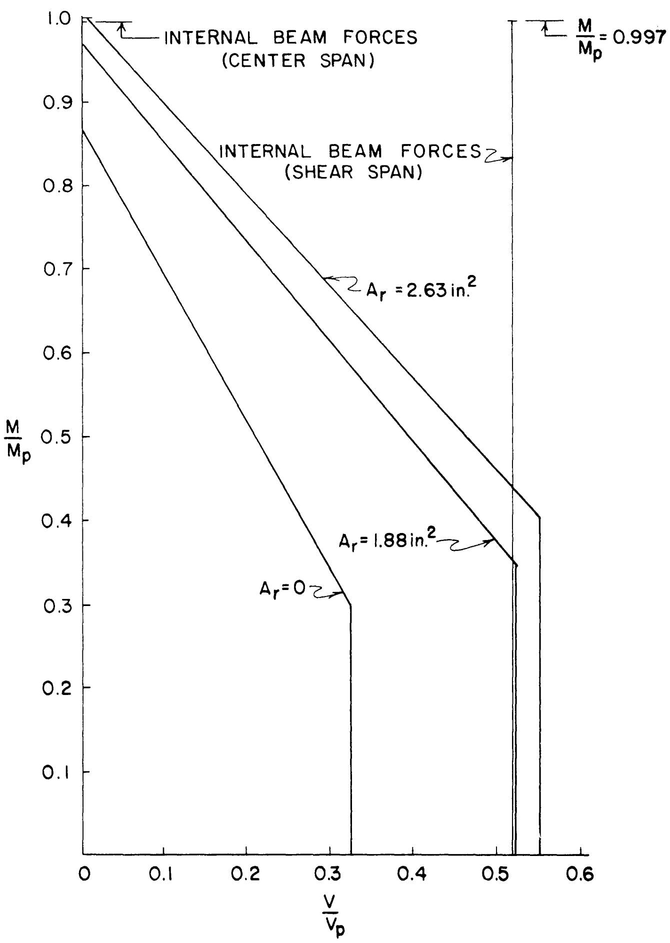


Fig. 7. Girder interaction diagrams

$$\left(\frac{V_T}{V_p}\right)_1 = \left[\left(\frac{1}{2} - \frac{6+3}{26.69}\right)^2 - \left(\frac{6.34}{12.4}\right)^2 (0.296)^2\right]^{1/2}$$
$$= 0.060$$

$$\left(\frac{V_B}{V_p}\right)_1 = \left[\left(\frac{1}{2} - \frac{6-3}{26.69}\right)^2 - \left(\frac{6.34}{12.4}\right)^2 (0.552)^2\right]^{1/2}$$
$$= 0.266$$

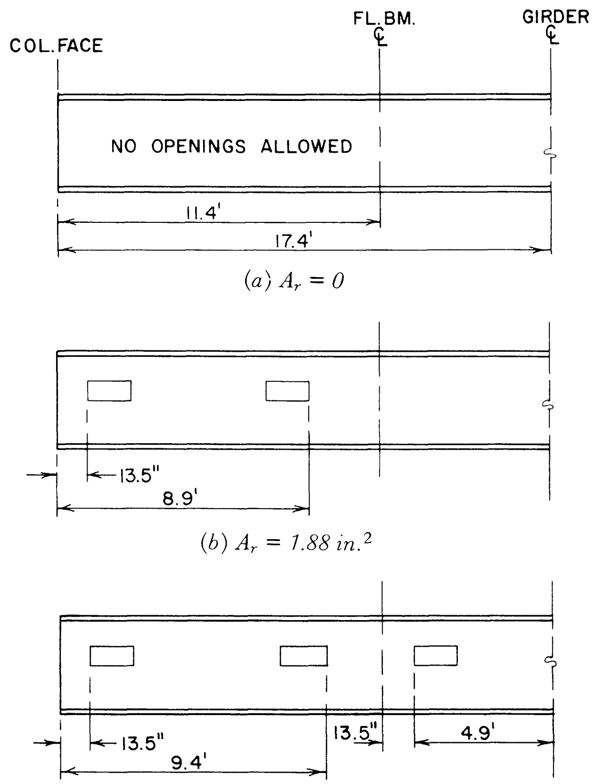
$$\left(\frac{V}{V_p}\right)_1 = 0.060 + 0.266 = 0.326$$

The above coordinates are plotted to form the approximate interaction diagram shown in Fig. 7 for $A_r = 0$.

There are no positions along the beam where the opening can be placed for the case where $A_r = 0$, since all points on the internal beam force curve lie outside the failure envelope, as shown in Fig. 5.

Opening Locations for A_r Large Enough to Accommodate Maximum Beam Shear—To determine the reinforcing area required to reach the maximum beam shear, a trial and error process is used, since the equations for eccentric web openings become quite unwieldy when the A_r term is isolated.

A brief examination of Fig. 7 will show that the V/V_p values for the shear spans are a fairly large distance from the vertical leg of the interaction diagram for $A_r = 0$. This indicates that a relatively large area of reinforcement is required to include these points in the failure envelope.



(a) $A_r = 0$

(b) $A_r = 1.88$ in.^{2}

(c) $A_r = 2.63$ in.^{2}

Fig. 8. Permissible opening locations for girders

After some preliminary trials, try $A_r = 1.88$ in.^{2}

Use 1-Bar 3 × ⅝ above and below the opening on one side only.

($A_r = 1.88$ in.^{2}; $b_r/t_r = 4.8 < 8.5$)

Calculation of interaction diagram coordinates using formulas presented in the Appendix with $A_r = 1.88$ in.^{2}, $e = 3$ in.:

$$\beta_T = -\frac{(2)(0.157)}{1.157}\left(\frac{1.88}{6.34}\right) + \frac{12.4}{(2)(6.34)} \times \left[\left(1 - \frac{12+6}{26.69}\right)^2 \frac{1}{1.157} - \frac{(16)(0.157)(1.88)^2}{(1.157)^2(12.4)^2}\right]^{1/2}$$
$$= 0.135$$

$$\beta_B = -\frac{(2)(0.889)}{1.889}\left(\frac{1.88}{6.34}\right) + \frac{12.4}{(2)(6.34)} \times \left[\left(1 - \frac{12-6}{26.69}\right)^2 \frac{1}{1.889} - \frac{16(0.889)}{(1.889)^2}\left(\frac{1.88}{12.4}\right)^2\right]^{1/2}$$
$$= 0.186$$

$$\left(\frac{M}{M_p}\right)_0 = \frac{1 + \frac{1.88}{6.34}\left(\frac{12}{26.69}\right)}{1 + \frac{12.4}{(4)(6.34)}} + \frac{\frac{12.4}{6.34}\left[\frac{1}{4} - \frac{(6)^2 + (2)(6)(3) - (3)^2}{(26.69)^2} - \frac{(0.463)(3)^2}{(3.463)(26.69)^2}\right]}{1 + \frac{12.4}{(4)(6.34)}}$$
$$= 0.971$$

$$\left(\frac{M}{M_p}\right)_1 = \frac{1 - \frac{1.88}{6.34} - 0.186}{1 + \frac{12.4}{(4)(6.34)}} = 0.348$$

$$\left(\frac{V_T}{V_p}\right)_1 = \left[\left(\frac{1}{2} - \frac{6+3}{26.69}\right)^2 - \left(\frac{6.34}{12.4}\right)^2 (0.135)^2\right]^{1/2}$$
$$= 0.147$$

$$\left(\frac{V_B}{V_p}\right)_1 = \left[\left(\frac{1}{2} - \frac{6-3}{26.69}\right)^2 - \left(\frac{6.34}{12.4}\right)^2 (0.186)^2\right]^{1/2}$$
$$= 0.376$$

$$\left(\frac{V}{V_p}\right)_1 = 0.147 + 0.376 = 0.523$$

$(V/V_p)_1$ is greater than the maximum V/V_p ratio in the girders (0.522); therefore, the trial A_r is o.k.

The above coordinates are plotted in Fig. 7 to describe the interaction diagram for $A_r = 1.88$ in.^{2} It should be noted that, while the maximum shear alone and shear along with low moment are within the failure envelope, as the moment increases some regions of the girder cannot accommodate the opening, as shown in Fig. 8.

Opening locations for $(A_r)_{min}$

$$(A_r)_{min} = \frac{(9.5)(0.463)}{\sqrt{3}} = 2.54 \text{ in.}^2$$

Use 1-Bar 3½ × ¾ above and below the opening on one side only.

$(A_r = 2.63 \text{ in.}^2 > 2.54 \text{ in.}^2; \quad b_r/t_r = 4.7 < 8.5)$

Calculations of interaction diagram coordinates using Appendix formulas with $A_r = 2.63$ in.^{2}, $e = 3$ in.:

$$\left(\frac{M}{M_p}\right)_0 = \frac{1 + \frac{2.63}{6.34}\left(\frac{12}{26.69}\right) +}{1 + \frac{12.4}{(4)(6.34)}}$$

$$\frac{\frac{12.4}{6.34}\left[\frac{1}{4} - \frac{(6)^2 + (2)(6)(3) - (3)^2}{(26.69)^2} - \frac{(0.463)(3)^2}{(3.963)(26.69)^2}\right]}{1 + \frac{12.4}{(4)(6.34)}}$$

$$= 1.007$$

$$\left(\frac{M}{M_p}\right)_1 = \frac{1 - \frac{2.54}{6.34}}{1 + \frac{12.4}{(4)(6.34)}} = 0.403$$

Note that in the above calculation for $(M/M_p)_1$, the calculated value of $(A_r)_{min}$ was used in place of the reinforcing bar area. This is because A_r is subtracted from 1 in the equation, and logic tells us that more reinforcement must increase the load carrying capacity of a member, not decrease it. Therefore the $(A_r)_{min}$ value calculated as above should be used to prevent overly conservative results.

$$\left(\frac{V}{V_p}\right)_1 = 1 - \frac{12}{26.69} = 0.550$$

These coordinates are plotted in Fig. 7 to form the interaction diagram for $A_r = 2.63$ in.^{2} With this area of reinforcement, the center line of the opening can be up to 8.6 ft from the supports in the shear spans and anywhere in the center portion, as shown in Fig. 8.

If A_r were increased above $(A_r)_{min}$, point 1 on the interaction diagram would move upward to include more points on the internal beam force curve. However, it would be uneconomical to try to include additional positions along the beam, since little is gained for a significant increase in A_r.

SUMMARY AND DESIGN DETAILS

The possible opening locations for the beams and girders are summarized in Figs. 5 and 8, respectively. The positions where openings can be placed are obtained graphically from Figs. 4 and 7, noting that where the internal beam forces fall inside the failure envelope, an opening can be placed. On the interaction diagram for the floor beams, Fig. 4, it can be seen that if A_r were reduced from $(A_r)_{min}$, the sloping portion of the diagram could be brought closer to the internal beam forces. Conversely, in Fig. 7, the interaction diagram for the girders, a substantial increase in A_r over $(A_r)_{min}$ would provide a very minor expansion in the possible locations of the opening. In general, when concentrated loads are involved it would be difficult as well as uneconomical to provide sufficient reinforcement to allow positioning the opening at any desired location along the beam. Further restrictions are placed on opening locations in the girders, due to limited experimental data on web crippling in beams with web openings in the vicinity of concentrated loads and reactions. It is therefore recommended that the edge of the opening be a distance of at least $d/2$ from the points where concentrated loads or reactions are introduced unless bearing stiffeners are provided.[2]

The following suggestions for design details are provided to supplement the information in the example:

1. Bars were used on one side of the web only.
   a. This gives approximately the same results as bars placed on both sides of the web.[5]
   b. The reinforcement is cheaper to fabricate, since welding is required on one side only.
2. The reinforcing bars should be checked for:
   a. Weld compatibility with web.
   b. The b_r/t_r ratio.
3. The reinforcing bars should be extended beyond the edge of the opening:
   a. A sufficient length to develop the strength of the bar in the welds.
   b. A minimum of 3 in.
4. The welds should:
   a. Be in accordance with the AISC Specification.
   b. Have a weld stress of 1.7 times the allowable weld stress or less.

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NOMENCLATURE

A_f	Area of one flange ($b_f \times t_f$)
A_r	Area of reinforcement above opening, also area of reinforcement below opening
$(A_r)_{min}$	Minimum area of reinforcement required to reach shear capacity of section
A_w	Area of web ($d \times t_w$)
M	Moment at center line of opening
M_p	Plastic moment of section
$\|M/M_p\|_x$	Absolute value of moment to plastic moment ratio along beams and girders
$(M/M_p)_0$, $(M/M_p)_1$	Coordinates of points on approximate interaction diagram
P	Concentrated load
V	Shear force at center line of opening
V_p	Plastic shear force of section
$\|V/V_p\|_x$	Absolute value of shear force to plastic shear force along beams and girders
$(V/V_p)_1$	Coordinate of point on approximate interaction diagram
$(V_B/V_p)_1$	Ratio of shear force in bottom tee section (V_B) to plastic shear force for point 1 on approximate interaction diagram
$(V_T/V_p)_1$	Ratio of shear force in top tee section (V_T) to plastic shear force for point 1 on approximate interaction diagram
Z	Plastic section modulus
a	One-half opening length
b_f	Flange width
b_r	Width of reinforcement
c	Web thickness plus width of reinforcement ($b_r + t_w$)
d	Beam depth
e	Eccentricity (distance between mid-depth of section and mid-depth of opening)
h	One-half opening depth
t_f	Flange thickness
t_r	Thickness of reinforcement
t_w	Web thickness
u	Distance from edge of opening to face of reinforcement
w	Uniformly distributed load
x	Coordinate defining positions along beams and girders
α,β	Coefficients used in approximate design formulas with subscripts T and B to denote top and bottom tee sections, respectively

REFERENCES

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APPENDIX

APPROXIMATE INTERACTION DIAGRAM EQUATIONS

The equations presented below are used to construct an approximate interaction diagram similar to Fig. 9.[1] Point 0 on the diagram has the coordinates $[0,(M/M_p)_0]$, Point 1 is described by $[(V/V_p)_1,(M/M_p)_1]$, and the point on the V/V_p-axis is $(V/V_p)_1$ from the origin. These equations are for the general case of a reinforced eccentric web opening. To obtain equations for other, less general cases, A_r, e, or both A_r and e, are taken equal to zero as appropriate.

For $e \leq u$:

$$\left(\frac{M}{M_p}\right)_0 = \frac{1 + \frac{A_r}{A_f}\left(\frac{2h}{d}\right) + \frac{A_w}{A_f}\left(\frac{1}{4} - \frac{h^2 + 2he}{d^2}\right)}{1 + \frac{A_w}{4A_f}}$$

For $u \leq e \leq u + q + A_r/t_w$:

$$\left(\frac{M}{M_p}\right)_0 = \frac{1 + \frac{A_r}{A_f}\left(\frac{2h}{d}\right) + \frac{A_w}{A_f}\left(\frac{1}{4} - \frac{h^2 + 2he - e^2}{d^2} - \frac{t_w e^2}{cd^2}\right)}{1 + \frac{A_w}{4A_f}}$$

For $e \geq u + q + A_r/t_w$:

$$\left(\frac{M}{M_p}\right)_0 = \frac{1 + \frac{A_r}{A_f}\left(\frac{2h + 2e}{d} - \frac{A_r}{A_w}\right) + \frac{A_w}{A_f}\left(\frac{1}{4} - \frac{h^2 + 2he}{d^2}\right)}{1 + \frac{A_w}{4A_f}}$$

$$(A_r)_{min} = \frac{at_w}{\sqrt{3}}$$

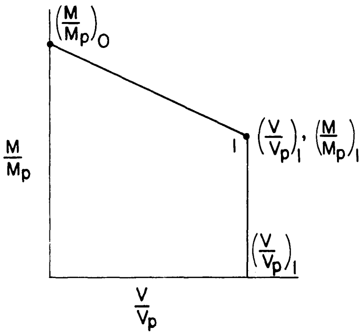

Figure 9

For $A_r < (A_r)_{min}$:

$$\left(\frac{M}{M_p}\right)_1 = \frac{1 - \dfrac{A_r}{A_f} - \beta_B}{1 + \dfrac{A_w}{4A_f}}$$

$$\left(\frac{V}{V_p}\right)_1 = \left(\frac{V_T}{V_p}\right)_1 + \left(\frac{V_B}{V_p}\right)_1$$

$$\left(\frac{V_T}{V_p}\right)_1^2 = \left(\frac{1}{2} - \frac{h+e}{d}\right)^2 - \left(\frac{A_f}{A_w}\beta_T\right)^2$$

$$\left(\frac{V_B}{V_p}\right)_1^2 = \left(\frac{1}{2} - \frac{h-e}{d}\right)^2 - \left(\frac{A_f}{A_w}\beta_B\right)^2$$

For $A_r \geq (A_r)_{min}$:

$$\left(\frac{M}{M_p}\right)_1 = \frac{1 - \dfrac{A_r}{A_f}}{1 + \dfrac{A_w}{4A_f}} \quad \text{where the } A_r \text{ value is } (A_r)_{min}$$

$$\left(\frac{V}{V_p}\right)_1 = 1 - \frac{2h}{d}$$

$$\beta_T = -\frac{2\alpha_T}{1+\alpha_T}\left(\frac{A_r}{A_f}\right) + \frac{A_w}{2A_f} \times \left[\left(1 - \frac{2h+2e}{d}\right)^2 \frac{1}{1+\alpha_T} - \frac{16\alpha_T}{(1+\alpha_T)^2}\left(\frac{A_r}{A_w}\right)^2\right]^{1/2}$$

$$\beta_B = -\frac{2\alpha_B}{1+\alpha_B}\left(\frac{A_r}{A_f}\right) + \frac{A_w}{2A_f} \times \left[\left(1 - \frac{2h-2e}{d}\right)^2 \frac{1}{1+\alpha_B} - \frac{16\alpha_B}{(1+\alpha_B)^2}\left(\frac{A_r}{A_w}\right)^2\right]^{1/2}$$

$$\alpha_T = \frac{3}{16}\left(\frac{d}{a}\right)^2\left(1 - \frac{2h}{d} - \frac{2e}{d}\right)^2$$

$$\alpha_B = \frac{3}{16}\left(\frac{d}{a}\right)^2\left(1 - \frac{2h}{d} + \frac{2e}{d}\right)^2$$