Stiffness Design of Unbraced Steel Frames

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IN CONSIDERING the serviceability of unbraced multistory steel buildings at working loads, the ability of the bents to limit lateral deflections is of prime importance. The parameter most commonly used in the measurement of their stiffnesses is the *frame drift index,* which is defined as the sway recorded at the uppermost level of a frame divided by its total height.

To control cracking in light partitions and in windows, the frame drift index is not an appropriate criteria to use. A more suitable indicator would be the *story drift index,* which is the ratio of the relative sway of a story to its height. This is so because story deflections are nonuniform and generally vary markedly throughout the frame's height. For example, computer analysis of a 40 story, 2 bay, regular unbraced steel frame (Frame 7, Design 1 of Ref. 2) shows that the drift indices of the top, middle, and lowest stories are 0.003, 0.005, and 0.0015, respectively, whereas the frame drift index is 0.0045.

For design purposes, there is a need for a simple method which can fulfill the following requirements:

1. It must be capable of predicting, with sufficient accuracy, story stiffnesses.
2. It must be capable of considering both first-order deflections and second-order deflections ($P\Delta$ effect included).
3. It must be capable of providing rapid means for modifying member sizes to meet pre-selected story deflection constraints.

One such method has been developed and reported elsewhere.[1] Although its accuracy as an analytical tool has been confirmed by comparison with a conventional second-order elastic method and by model tests, its application as a design tool on practical multistory frames has not yet been demonstrated.

This paper, therefore, aims at illustrating the design features of this method[1] by considering a typical frame proportioned by plastic theory. See Fig. 1 and Table 1(a). The frame chosen is appropriate, since it demonstrates that although experience suggests that increases in beam sizes are more effective than increases in column sizes in reducing frame deflections, there are cases when simultaneous improvements in both beam inertias and column inertias are desirable.

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NOMENCLATURE

h = story height
m = number of columns (subassemblages) in a story
n = factor by which member sizes are increased
s = subassemblage stiffness
E = modulus of elasticity
I_b = beam inertia
I_c = column inertia
L_b = beam length
P = column axial load
ΣP = total story load
Q = total wind load on a story
Δ = relative story sway
ψ = $(I_c/h)/\Sigma(I_b/L_b)$
= ratio of column stiffness to beam stiffness

WORKING FORMULAS

The concept upon which the method rests is that the stiffness of a story can be represented by the sum of the stiffnesses of its subassemblages, a subassemblage being a substructure consisting of one column and its restraining girders (Fig. 2). Obviously, a story with m columns will contain m subassemblages. Also, an exterior subassemblage will have only one restraining beam and an interior subassemblage will have two restraining beams.

The following subassemblage stiffness equations have been derived for rigidly-jointed unbraced steel frames with equal story heights:[1]

For a subassemblage in the uppermost story:

$$s = \frac{12EI_c}{h^3(1+\psi)} - \frac{P}{h} \quad (1)$$

For a subassemblage in an intermediate story:

$$s = \frac{12EI_c}{h^3(1+2\psi)} - \frac{P}{h} \quad (2)$$

For a subassemblage with fixed column base:

$$s = \frac{12EI_c}{h^3}\left(\frac{3+\psi}{3+4\psi}\right) - \frac{P}{h} \quad (3)$$

For a subassemblage with pinned column base:

$$s = \frac{12EI_c}{h^3(4+3\psi)} - \frac{P}{h} \quad (4)$$

In the above equations, s = subassemblage stiffness, E = modulus of elasticity, I_c = column moment of inertia, P = column axial load, and h = story height. The quantity ψ is a measure of the ratio of column stiffness, I_c/h, to beam stiffness, $\Sigma(I_b/L_b)$, in a subassemblage:

$$\psi = \frac{I_c}{h} \Big/ \Sigma \left(\frac{I_b}{L_b}\right) \qquad (5)$$

The parameter ψ is of significant importance in the decision to increase member sizes for best results in reducing sway and in design configurations. The following rules apply.[1]

1. Increase beam sizes if $\psi \gg 0.5$.
2. Increase column sizes if $\psi \ll 0.5$.
3. Increase beam and column sizes if ψ is approximately 0.5.

In applying the subassemblage stiffness equations, it is not necessary to calculate the axial forces on the individual columns when $P\Delta$ effects are included. Instead, the readily evaluated total gravity load on the story is required. For example, the story stiffness, S_T, at an intermediate level with m columns is:

$$S_T = \frac{Q}{\Delta} = \frac{12E}{h^2} \sum_1^m \left(\frac{I_c/h}{1 + 2\psi}\right) - \sum_1^m \frac{P}{h} \qquad (6)$$

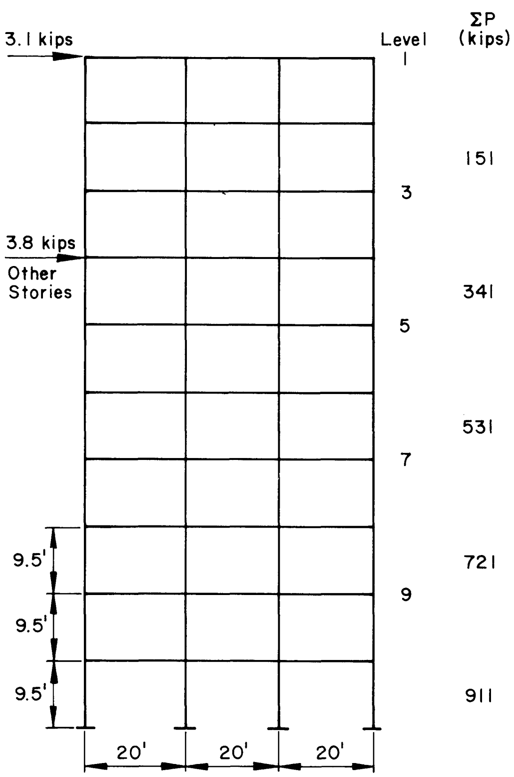

Fig. 1. Frame loads and dimensions

where

Q = total wind force on the story

$\sum_1^m P$ = total story gravity loads

Note that when first-order results only are to be found, $P = 0$ in the stiffness equations.

ILLUSTRATIVE EXAMPLE

The applied loads on and the dimensions of the chosen frame are shown in Fig. 1, and member sizes are given in Table 1(a). The frame has a failure load factor of 1.32 under proportional loading conditions and has a frame deflection index of 0.0044 at working loads. It is required to modify the frame so that story deflections do not exceed $0.003h$, $P\Delta$ effects being accounted for.

Calculations for the values of ψ for various subassemblages have been performed and are given in Table 2. Since ψ is approximately equal to 0.5, especially in the interior subassemblages, both beam and column inertias should be increased.

Computations for the story below level 8 are summarized below:

For $\Delta = 0.003h$, the required story stiffness is:

$$S_T = \frac{Q}{\Delta} = \frac{29.7}{0.003h} = 87 \text{ kips/in.}$$

Suppose all moments of inertia are increased by a factor n. Using Eq. (6),

$$87 = \frac{12E}{h^2} \sum_1^4 \left(\frac{nI_c/\mathrm{h}}{1 + 2\psi}\right) - \frac{721}{h}$$

so that n = 1.66. Hence all beam and column sizes should be increased by a factor of 1.66.

In choosing a new column section, the new area of cross section must be approximately equal to that of the old section to avoid premature hinge formation. The final beams and columns selected to meet the sway limit of $0.003h$ per story are given in Table 1(b).

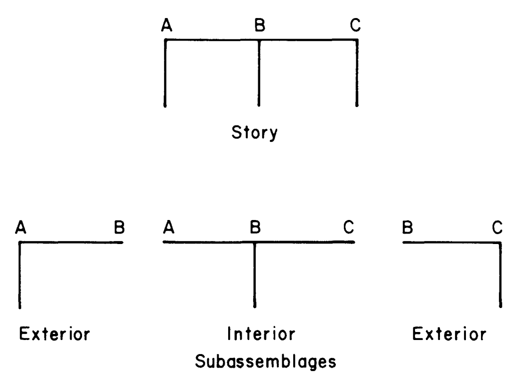

Fig. 2. Story components

It is of interest to investigate the type of design which would have resulted had beam sections only been changed. Using the information provided by Table 2 for level 8 and denoting the new beam inertias by nI_b, it can be shown from Eq. (6) that

$$87 = \frac{12E}{h^2} \times 2 \left(\frac{1.10}{1 + 2.16/n} + \frac{1.10}{1 + 1.08/n} \right) - 6.33$$

Table 1. Member Sizes for Frame in Fig. 1

Level	Beam	Level	Columns: Exterior	Columns: Interior
		(a) Plastic Design		
1 – 2	W12 × 22	1 – 3	W8 × 13	W8 × 13
3 – 4	W14 × 22	3 – 5	W8 × 20	W8 × 20
5 – 8	W14 × 26	5 – 7	W8 × 28	W8 × 28
9 – 10	W16 × 26	7 – 9	W8 × 35	W8 × 35
		9 – 11	W8 × 40	W8 × 40
		(b) Stiffness Design		
1 – 2	W12 × 22	1 – 3	W8 × 13	W8 × 13
3	W14 × 26	3 – 5	W8 × 31	W8 × 31
4	W16 × 26	5 – 7	W8 × 31	W8 × 31
5 – 7	W16 × 31	7 – 9	W14 × 38	W14 × 38
8 – 10	W16 × 36	9 – 11	W14 × 38	W14 × 38

Table 2. ψ-values for Subassemblages of the Frame in Table 1(a)

Level	Values of ψ: Exterior	Values of ψ: Interior
4	0.74	0.37
6	0.92	0.46
8	1.08	0.54

and $n = 3.1$. Consequently, the new beam inertias are $3.1 \times 244 = 756$ in.^{4}, and the section chosen would be W16 × 64..

Compare now the results of the two different actions taken. When both beams and columns are modified for the story below level 8, the beams are W16 × 36 and the columns W14 × 38 sections. When only beam sizes are changed, the beams are W16 × 64 and the columns W8 × 35. Comparison of weights show that the latter design is about 44% higher than the former. Obviously, in this example, increasing both beam sizes and column sizes gives a far better design.

CONCLUSION

In most conventional frames, improving beam inertias will efficiently reduce lateral sway; however, in some cases it is more desirable to increase column sizes and beam sizes simultaneously. This paper provides simple guidelines in deciding which course of action to take. The method is simple, fast, direct, and suitable for dealing with first-order as well as second-order deflections.

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