A Simplified Method for the Flexural Design of Stiffened Hybrid Girders

KEN P. CHONG

HYBRID GIRDERS are useful when strength is the governing factor in design. Due to the availability of higher strength steels and their structural efficiency, hybrid beams and girders are becoming more and more popular. The behavior of hybrid beams has been under extensive research and reported in various publications.[1,2] However, there is still very little literature available on the design of hybrid beams.

In 1970, Milek[3] presented a comprehensive paper on hybrid girder design according to the 1969 AISC Specification. Recently, Schilling has published a paper which explores the optimal dimensions of both homogeneous and hybrid I-shaped beams due to bending moment only (web slenderness is assumed to be known).

The difficult part in the flexural design of hybrid beams is that the adjusted allowable bending stress F_b' depends on α and β as given by:[5]

$$F_b' \leq F_b \left[\frac{12 + \beta(3\alpha - \alpha^3)}{12 + 2\beta} \right] \tag{1}$$

where

α = F_{yw}/F_{yf}
β = A_w/A_f
F_b = allowable bending stress defined in AISC Spec. Sect. 1.5.1
F_{yw} = yield stress of web
F_{w} = yield stress of flanges
A_w = area of web
A_f = area of one flange

In design, generally α is known, F_b' is assumed[3] as a percentage of F_b, and then F_b' must be refined after β is found. In other words, an iteration procedure is required. The purpose of this paper is to eliminate this iteration so that a direct design is possible.

Ken P. Chong is Associate Professor of Civil Engineering, Dept. of Civil and Architectural Engineering, The University of Wyoming, Laramie, Wyoming.

THEORY

The moment resistance M of a hybrid girder is given by

$$M = F_b' S_x \tag{2}$$

in which S_x is the section modulus, given with sufficient accuracy by

$$S_x = A_f h + \frac{th^2}{6} \tag{3}$$

where

A_f = area of one flange
h = clear distance between flanges
t = thickness of web

To avoid any iterations in F_b', the following analysis has been conceived by the author:

Let

$$M = F_b S_f \tag{4}$$

where

$$S_f = A_f h + \frac{Kth^2}{6} \tag{5}$$

K is a dimensionless correction factor which is less than, or at best equal to, unity. The rationale is that since the web is of lower strength material, only part of the web is effective in resisting the moment if the full unreduced flange stress F_b is used. Equating Eqs. (2) and (4), utilizing Eqs. (1), (3) and (5), there results:

$$A_f h + \frac{Kth^2}{6} = \left[\frac{12 + \beta(3\alpha - \alpha^3)}{12 + 2\beta} \right] \left(a_f h + \frac{th^2}{6} \right) \tag{6}$$

Equation (6) can be rewritten as:

$$A_f h \left(1 + \frac{K\beta}{6} \right) = \left[\frac{12 + \beta(3\alpha - \alpha^3)}{12 + 2\beta} \right] A_f h \left(1 + \frac{\beta}{6} \right) \tag{7}$$

Expanding and solving for K:

$$K = \frac{\alpha(3 - \alpha^2)}{2} \leq 1.0 \tag{8}$$

Therefore, it can be seen that the correction factor K is independent of β and depends only on α. This independency has considerable advantage in design, since β is an unknown quantity at the beginning. Table 1 illustrates the dependency of K on α.

EXAMPLES

The following two examples are taken respectively from Milek's paper[3] and page 2-114 of the AISC Manual.[6] They illustrate the directness of the simplified method compared with conventional methods in flexural design.

Example 1

Given:

Design a simply supported beam, 72 ft long, carrying a uniform load of 3.6 kips/ft and concentrated loads of 180 and 300 kips at left and right third-points, respectively. Flanges are to be A514 steel and the web is to be A36 steel.

Solution:

Governing moment: $M = 8315$ kip-ft

Eq. (4): $S_f = \dfrac{8315 \times 12}{0.6 \times 100} = 1663 \text{ in.}^3$

$\alpha = F_{yw}/F_{yf} = 36/100 = 0.36$

From Table 1 or Eq. (8): $K = 0.52$

From Ref. 3: Let $h = 72$ in., $t = \frac{1}{2}$-in.

Eq. (5): $A_f = \dfrac{1663}{72} - \dfrac{0.52 \times 72 \times \frac{1}{2}}{6}$

$= 19.98 \text{ in.}^2$

Use: **20 × 1-in. A514 flanges**
72 × ½-in. A36 web
(Same as Ref. 3)

Table 1

α	K
0.1	0.1495
0.2	0.2960
0.3	0.4365
0.36	0.5167
0.4	0.5680
0.5	0.6875
0.6	0.7920
0.7	0.8785
0.72	0.8934
0.8	0.9440
0.9	0.9855
1.0	1.0000

Example 2

Given:

Design a hybrid girder 80 ft long, subject to a uniform load of 2 kips/ft, and three concentrated loads of 200 kips each at quarter-points. The depth of the girder is limited to five feet and the compression flange will be continuously braced. A572 Gr. 50 flanges and A36 web are to be used.

Solution:

$M = 9600$ kip-ft

Eq. (4): $S_f = \dfrac{9600 \times 12}{0.6 \times 50} = 3840 \text{ in.}^3$

$\alpha = 36/50 = 0.72$

From Table 1: $K = 0.893$

From Ref. 6: Let $h = 54$ in., $t = \frac{9}{16}$-in.

Eq. (5): $A_f = \dfrac{3840}{54} - \dfrac{0.893 \times 54 \times \frac{9}{16}}{6} = 66.6 \text{ in.}^2$

Use: **24 × ⅞-in. A572 Gr. 50 flanges**
54 × 9⁄16-in. A36 web
(Ref. 6 uses 24 × 1-in. 50 ksi flanges.)

CONCLUSION

The theory and simplified method presented herein consider full unadjusted allowable bending stress in conjunction with reduced web contribution to the section modulus. The advantage of this exact approach in flexural design of hybrid girders is that the dependency on β (ratio of web area to one flange area) is eliminated. Thus, no guesswork is required and no iterations are performed with respect to β. In other words a direct flexural design is possible by the simplified method. The method can also be applied for the analysis of a hybrid girder when the cross section is given.

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