The Effective Length of Unbraced Single Story Columns

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THE AISC Manual[1] provides the engineer with a simplified method for the design of columns in unbraced frames for stability in the form of an "Alignment Chart for Effective Length of Columns in Continuous Frames, Sidesway Uninhibited." In addition, there have been many significant papers written recently which enable engineers to achieve a greater understanding and accuracy in the design of steel columns.

The effective length concept is based on the relative stiffness of the column to its bracing members. As long as these bracing members are steel, the connections are fairly standard, and the AISC Manual can be used directly with reasonably accurate results. But when the bracing member is a concrete footing, there is a choice given to the designer. The Manual states that "for column ends supported by but not rigidly connected to a footing or foundation, G . . . may be taken as 10 . . . . If the column end is rigidly attached to a properly designed footing, G may be taken as 1.0. Smaller values may be used if justified by analysis." For multistory buildings this does not usually represent a very wide range when making a column selection. However, for single story structures, the designer's choice of 1.0 or 10 will frequently result in a significant difference in total column weight. In such cases it therefore seems warranted to make a further investigation.

Galambos[2] has developed formulas for the stiffness of spread footings and base plates by relating their bracing effects to a fictitious base restraint beam. For a footing:

$$\frac{I}{L} = \left(\frac{q g f^3}{12}\right)\left(\frac{1}{6E_s}\right)$$

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where

q = modulus of subgrade reaction*
g = width of footing
f = length of footing
E_s = modulus of elasticity for steel

When $E_s = 29 \times 10^3$ ksi, the preceding stiffness formula becomes:

$$\frac{I}{L} = \frac{q g f^3}{2088 \times 10^6} \qquad (1)$$

For a base plate:

$$\frac{I}{L} = \left(\frac{bd^2E_c}{12}\right)\left(\frac{1}{6E_s}\right)$$
$$= \frac{bd^2}{72n} \qquad (2)$$

where

b = width of base plate
d = length of base plate
n = modular ratio

By using the smaller of these stiffness values, the most conservative result will be obtained. This stiffness, together with the I/L of the column, can be used to determine the value of G_{bottom} for use with the AISC Alignment Chart. This procedure is illustrated in the following example.

* *Various methods for determining q, including testing, are available. Table 1 shows values based on Fig. 11-8 in Ref. 3, "Immediate Settlement of Isolated Footings in Coarse Grained Soils."*

DESIGN EXAMPLE NO. 1 (PART 1)

Given:

Using a pipe section, design a column for a large one story industrial building, located in an interior area, isolated by expansion joints. Bracing at the top is provided by 24H8 open web steel joists ($I_{effective}$ = 281.4 in.^{4}) spanning 40 ft in one direction, and 46-in. deep truss girders ($I_{effective}$ = 3,180 in.^{4}) spanning 40 ft in the other. Column gravity load is 81 kips; the unbraced height is 20 ft to the underside of the joists and 17.96 ft to the underside of the truss girders.

Solution:

Try a Pipe 8 Std (I = 72.5 in.^{4}):

In joist direction:

$$G_{top} = \frac{(72.5/20) \times 12}{2 \times (281.4/40) \times 12}$$
$$= 0.26 \text{ (controls)}$$

In truss girder direction:

$$G_{top} = \frac{(72.5/17.96) \times 12}{2 \times (3{,}180/40) \times 12} = 0.03$$

Assume a base plate 16 in. square, and concrete footing 5 ft square, bearing on coarse grained soil having an allowable compressive strength of 2 tons/ft^2 (f'_c = 3,000 psi; $E_c = 3.2 \times 10^6$ psi; n = 9):

For footing:

From Table 1, q = 150 lbs/in.^{3}

Using Eq. (1):

$$\frac{I}{L} = \frac{150 \times 60^4}{2{,}088 \times 10^6} = 0.931 \text{ (controls)}$$

For base plate:

Using Eq. (2)

$$\frac{I}{L} = \frac{16^3}{72 \times 9} = 6.32$$

$$G_{bottom} = \frac{72.5/20 \times 12}{931} = 0.33$$

This value of G_{bottom} is below the 1.0 recommended by the AISC for a "rotation fixed" end. To be conservative, a value of G_{bottom} = 1.0 will be used.

Column Selection:

From AISC Manual, pg. 3-5:
K = 1.20 (for G_{top} = 0.26 and G_{bottom} = 1.0)

$KL = 1.20 \times 20 = 24$ ft.

From AISC Manual, pg. 3-38:
$P_{allowable} = 111 \text{ kips} < P = 81 \text{ kips}$

Use: Pipe 8 Std Column

Note that if the designer had selected G_{bottom} to be 10, a Pipe 10 Std or Pipe 8 X-Strong column would be required, resulting in an increase of more than 40 percent in column weight.

The key word to keep in mind when using the Alignment Chart is *rigidly*. The expression "rigidly attached" in this case requires that the column base connection be designed for the full moment capacity of the column. For simplicity, the anchor bolts may be sized for moment only, while checking the base plate bearing stress for combined loading.

Table 1[3]

	Unconfined Compressive Strength (tons/ft^2)	Modulus of Subgrade Reaction (lb/in.^{3})
Coarse grained soils	1 2 3	45 150 270
Fine grained soils	1 2 3	30 115 220

DESIGN EXAMPLE NO. 1 (PART 2)

Given:

Design a rigid column base connection for the column selected in Part 1.

Solution:

Try a base plate 16 in. square with 4 anchor bolts (2½-in. edge distance; 11-in. ga.):

For a Pipe 8 Std column (S = 16.8 in.^{3}):

$$M_r = 16.8 \times 22 = 369.6 \text{ kip-in.}$$

Assume that the location of critical section for bending used in the design of the base plate also defines the limit of the triangular (working stress design) compressive stress block.*

Computing n by means of the recommended design procedure on pg. 3-95 of the Manual:

$$n = \frac{16 - (0.80 \times 8.625)}{2}$$
$$= 4.55 \text{ in.}$$

Computing the moment arm of the internal couple:

$$jd = 16 - \left(2.5 + \frac{4.55}{3}\right)$$
$$= 11.98 \text{ in.}$$

$$T = C = 369.6/11.98$$
$$= 30.84 \text{ kips}$$

* *It is recognized that this is not a precise solution, but it is considered sufficiently accurate for this type of analysis.*

From AISC Manual, pg. 4-3:
Allowable tension for two $1\frac{1}{4}$-in. diam. A307 bolts:
$2 \times 19.38 = 38.76$ kips

Use: Four $1\frac{1}{4}$-in. diam. A307 anchor bolts

Concrete working stress:

$$f_c = \left(\frac{30.84}{0.5 \times 4.55 \times 16}\right) + \left(\frac{81}{16^2}\right)$$
$$= 1.164 \text{ ksi}$$

From ACI 318-71, paragraph 8.10:

$$\begin{aligned} \text{Allow. bearing} &= 0.35\,(2 \times 0.85\,\phi f'_c) \\ &= 0.35\,(2 \times 0.85 \times 0.70 \times 3.0) \\ &= 1.250 \text{ ksi} \qquad \textbf{o.k.} \end{aligned}$$

Design base plate thickness:

$$M = \left[\frac{2 \times 4.55}{3}\left(\frac{30.84}{16}\right)\right] + \left[\frac{4.55^2}{2}\left(\frac{81}{16^2}\right)\right]$$
$$= 9.12 \text{ kip-in./in.}$$

$$S = 9.12/27 = 0.34 \text{ in.}^3/\text{in.}$$

Try $1\frac{1}{2}$-in. plate:

$$S = 0.38 \text{ in.}^3/\text{in.}$$

Use: Base plate $16 \times 1\frac{1}{2} \times 1$ ft-4 in.

Similarly, the footing should be designed for the moment capacity of the column, as well as the gravity loads. In this example it will be found that a somewhat larger footing than assumed will be required. Since a larger footing will have a greater stiffness, no column design check is required.

This method provides the designer with a simple and convenient way to evaluate column base stiffness for more economic column design. It should be noted that in so doing there will be an increase in cost for the rigid column base. It remains for the designer to determine the best solution for each case.

If considered desirable to lower the cost of a fully rigid column base, a somewhat modified approach can be adopted. By stopping the foregoing method at the point of column selection and reversing the procedure, the required partial base rigidity can be determined. This is outlined as follows:

Step 1. From the column load tables for the given column and load, find the allowable effective length KL and factor K.

Step 2. Enter the Alignment Chart with the G_{top} previously determined and the allowable K; read G_{bottom} required.

Step 3. Design the anchor bolts, base plate, and footing for the partially rigid resisting moment corresponding to the G_{bottom} required.

Step 4. Check the base plate and footing thus designed for adequate stiffness.

Note that Step 3 is no simple matter. The author knows of no direct solution; however, the following formula is proposed:

$$M = \frac{M_r \text{ (column)}}{G_{bottom\ reqd}} \tag{3}$$

Thus, if it is necessary that G_{bottom} be 1.0 in order to justify the use of a chosen column, the column base must be designed for the full moment capacity of the column; on the other hand, if the column is sufficient when G_{bottom} is 10, the column base moment requirement is one-tenth the column moment capacity, or effectively negligible.

It should be noted that this proposal is only an approximate solution. It does, however, appear to be conservative, at least for the conditions of this paper, i.e., one story columns in unbraced frames with welded base plates.

It has been shown[4] that frame stability can be achieved with other than fully rigid connections. The moment-rotation characteristic of a partially rgid connection can be taken into consideration by increasing the effective length of column. Conversely, if a specific column is satisfactory for a greater effective length than that based on rigid connections, the connection stiffness can be reduced.

All partially rigid connections have a moment-rotation (M-ϕ) characteristic. The shape of an M-ϕ diagram during initial loading is a curve, whose precise slope can only be determined by test. However, through subsequent loading cycles the relationship is linear (i.e., elastic), the slope being parallel to the tangent to the initial loading curve, though offset from it.[5] Thus, a directly proportional relationship is established between moment and connection stiffness.

DeFalco and Marino have graphically illustrated in Ref. 4 that the relationship between connection stiffness and the effective stiffness of a bracing member is not linear. The above proposed Eq. (3) is therefore not an exact solution.

DESIGN EXAMPLE NO. 2

Given:

Using the column selected in Design Example No. 1, redesign the column base for only the connection stiffness required.

Solution:

Step 1. For Pipe 8 Std column and a load of 81 kips, determine that the allowable effective length, $KL = 30.4$ ft (AISC Manual, pg. 3-38). For $L = 20$ ft, $K = 1.52$.

Step 2. For $G_{top} = 0.26$ and $K_{allowable} = 1.52$, determine that $G_{bottom} = 4.5$ (AISC Manual, pg. 3-5).

Step 3. The moment requirement, using Eq. (3), is then:

$$M = 369.6/4.5 = 82.2 \text{ kip-in.}$$

Repeating the calculations shown in Design Example No. 1 with this moment, it is found:

Use: Four ¾-in. diam. A307 anchor bolts (minimum)
Base plate 15 × 1 × 1 ft-3 in.
Footing 5 ft-0 in. square

Step 4. Check base stiffness to verify $G_{bottom} \leq 4.5$:

Footing, using Eq. (1):

$$\frac{I}{L} = \frac{150 \times 60^4}{2{,}088 \times 10^6} = 0.931 \text{ (controls)}$$

Base plate, using Eq. (2):

$$\frac{I}{L} = \frac{15^3}{72 \times 9} = 5.21$$

$$G_{bottom} = \frac{(72.5/20) \times 12}{0.931} = 0.33 < 4.5 \quad \textbf{o.k.}$$

SUMMARY

The Alignment Chart does not provide the designer with a method of making an engineered evaluation of column bases, merely giving limit recommendations. Presented are simple and approximate methods, which allow the designer to do this. Although not included for the sake of simplicity of illustration, these methods are equally suitable for the design of columns with wind or other lateral loads.

REFERENCES

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4. *DeFalco, Fred and Marino, Frank J.* Column Stability in Type 2 Construction *Engineering Journal, American Institute of Steel Construction, April, 1966.*
5. *Disque, Robert O.* Wind Connections with Simple Framing *Engineering Journal, American Institute of Steel Construction, July, 1964.*