Bending Under Seated Connections

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THE SEAT, OR BRACKET, is a common type of connection in structural steel. A bracket carried by a column or beam web or, in general, by a vertical plate, throws a bending moment into the supporting plate, and if the plate is relatively thin it may be overstressed. No data about this important effect are given in the AISC Manual of Steel Construction or in readily available reference works; yet this stress may be critical and govern the design of the connection.

The problem of evaluating these stresses arose in the analysis of numerous heavily loaded brackets used in the Vertical Assembly Building at Cape Kennedy, designed by Urbahn-Roberts-Seelye-Moran (URSAM). A theoretical investigation of plate bending under brackets has been carried out by both an elastic method and a plastic method. This paper presents the results of these analyses, including a rule of thumb as well as more accurate methods of computation.*

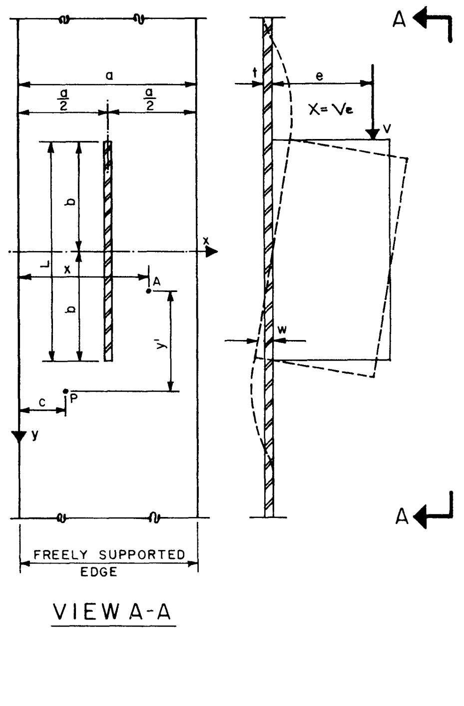

Figure 1

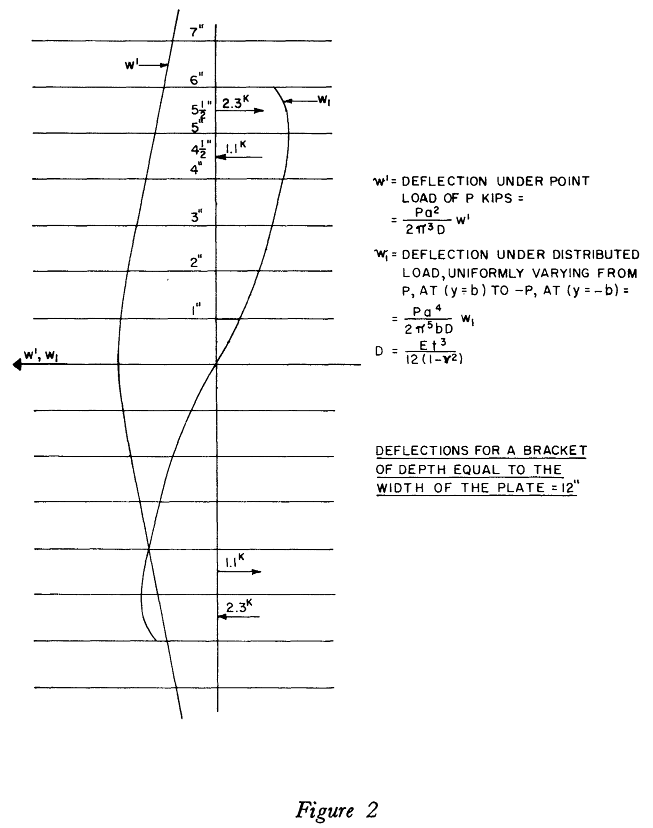

Figure 2

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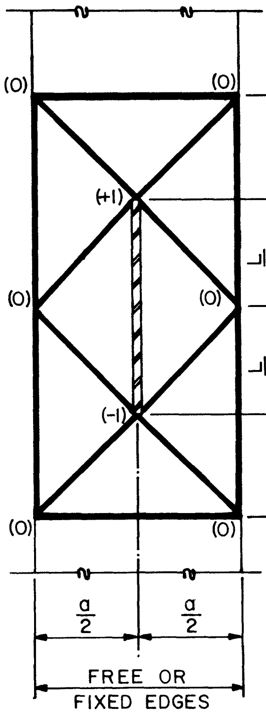
* *Computation sheets containing mathematical derivations of formulas in this paper have been prepared by Mr. Abolitz and are available from AISC.*



FREE EDGES

$u = \frac{a}{2}\sqrt{2}$

$k = \frac{2a}{L} + \frac{2L}{a} + 4\sqrt{2}$;

for $L = a$, $k = 9.66$

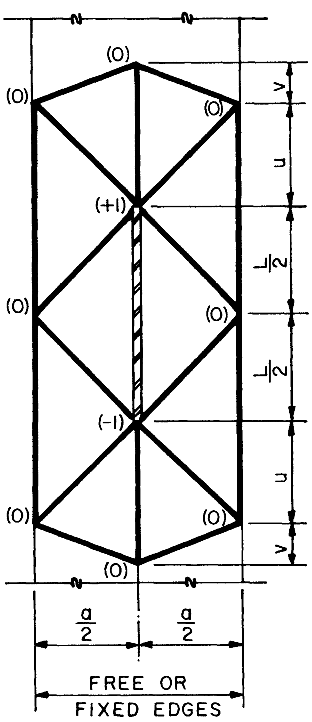
FIXED EDGES

$u = \frac{a}{2}$

$k = \frac{2a}{L} + \frac{4L}{a} + 8$;

for $L = a$, $k = 14.00$

Figure 3



FREE EDGES

$u = \frac{3a}{14}\sqrt{7}$

$v = \frac{a}{14}\sqrt{7}$

$k = \frac{2a}{L} + \frac{2L}{a} + 2\sqrt{7}$;

for $L = a$, $k = 9.29$

FIXED EDGES

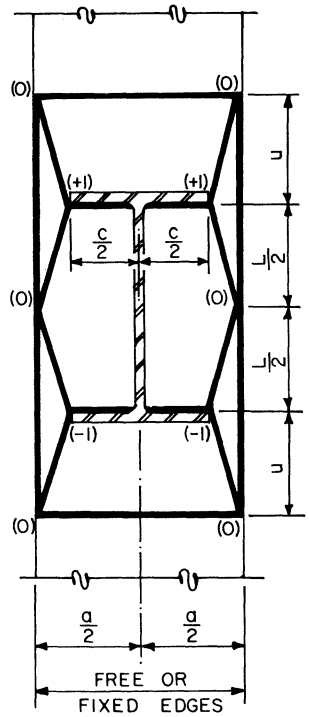
$u = \frac{a}{6}\sqrt{3}$

$v = \frac{a}{6}\sqrt{3}$

$k = \frac{2a}{L} + \frac{4L}{a} + 4\sqrt{3}$;

for $L = a$, $k = 12.93$

Figure 4



FREE EDGES

$u = \frac{1}{2}\sqrt{2a(a-c)}$

$k = \frac{2a}{L} + \frac{2L}{a-c} + 4\sqrt{\frac{2a}{a-c}}$;

for $L = a$ and $c = \frac{a}{2}$,

$k = 14$

FIXED EDGES

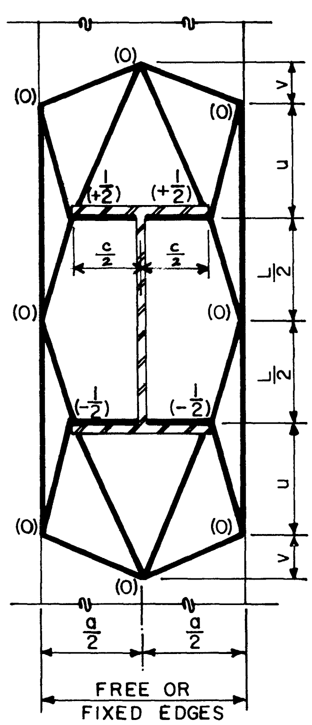
$u = \frac{1}{2}\sqrt{a(a-c)}$

$k = \frac{2a}{L} + \frac{4L}{a-c} + 8\sqrt{\frac{a}{a-c}}$;

for $L = a$ and $c = \frac{a}{2}$,

$k = 21.31$

Figure 5



FREE EDGES

$u = \frac{3a+c}{2}\sqrt{\frac{a-c}{7a+c}}$

$v = \frac{a}{2}\sqrt{\frac{a-c}{7a+c}}$

$k = \frac{2a}{L} + \frac{2L}{a-c} + 2\sqrt{\frac{7a+c}{a-c}}$;

for $L = a$ and $c = \frac{a}{2}$,

$k = 13.75$

FIXED EDGES

$u = \frac{a+c}{2}\sqrt{\frac{a-c}{3a+c}}$

$v = \frac{a}{2}\sqrt{\frac{a-c}{3a+c}}$

$k = \left(\frac{2a}{L} + \frac{4L}{a-c} + 4\sqrt{\frac{3a+c}{a-c}}\right)$;

for $L = a$ and $c = \frac{a}{2}$,

$k = 20.58$

Figure 6

ELASTIC THEORY APPROACH

A typical plate-supported bracket is shown in Fig. 1. In the elastic theory, the deflections of a plate under a specified loading are given by Lagrange's equation.*

The distribution of compression and tension between bracket and plate is not known. An assumption is often made that the pressure distribution is linear, from a maximum positive value at the bottom of the bracket to a maximum negative value at the top. This results in plate deflection w_1, shown in Fig. 2, which is decidedly non-linear. Yet, as can be intuitively grasped from Fig. 1, the deflection must be linear or nearly so. Therefore, the usual assumption of a linear pressure distribution is erroneous.

Taking as an example a 12-in. deep bracket on a 12-in. wide column web, a loading resulting in an approximately linear deflection is represented by pressures of +2.3 kips in the middle of the last inch, −1.1 kips in the middle of the preceding inch, and equal and opposite forces near the top of the bracket, with the central part of the bracket not transmitting any load to the plate (see Fig. 2). The bracket moment corresponding to this loading is 15.4 in.-kips. With a linear pressure distribution, as is frequently assumed, the maximum compression and tension produced by this moment would be 0.64 kips per inch, a much lower value than 2.3 kips per inch, given above. In passing, it is pointed out that the high localized values of the pressure should be considered in the design of the welds, particularly where they are in tension or subject to fatigue effects.

Reverting to the example, the maximum bending moments in the plate, as computed by elastic theory, are 0.38 in.-kips per inch at a point ½ in. below the bottom of the bracket, and 0.26 in.-kips per inch at a point 1½ in. below the bottom. The high value of 0.38 in.-kips per inch represents a localized condition, and may safely be neglected. Further calculations are therefore based on a plate moment of 0.26 in.-kips per inch.

Denote by t the plate thickness, and by S its section modulus per unit width. (According to elastic theory $S = \frac{1}{6} t^2$.) Denote by f the allowable bending stress, conservatively taken as 22 ksi for A36 steel,** and by m the allowable moment in the plate, in.-kips per inch of width. Then

$$m = fS = \tfrac{1}{6} f t^2 \qquad (1)$$

Further, denote by X the allowable bracket moment (in.-kips) to which the plate may be subjected, by L the depth of the bracket, and by a the horizontal width of the plate, which is assumed to be of indefinite length vertically, and simply supported along its edges in this example (see Fig. 1).

$$\text{Let } X = kmL \qquad (2)$$

In this expression the coefficient k varies somewhat with the ratio L/a but is independent of t.

In the example shown above, $X = 15.4$ in.-kips, $m = 0.26$ kips, and $L = 12$ in. Therefore $k = 4.9$ and, for A36 steel, $X = 4.9\, mL = 18\, Lt^2$.

For brackets provided with a flange similar to Fig. 7, which mitigates the bending stresses in the plate, the following rule of thumb is suggested for figuring the allowable bracket moment X on a plate of A36 steel t in. thick, the bracket being L in. long:

$$X = 24Lt^2 \text{ (in.-kips)} = 2\, Lt^2 \text{ (ft-kips)} \qquad (3)$$

(Brackets for the Vertical Assembly Building were figured for moments expressed by the formula: $X = 20\, Lt^2$. More detailed analysis and study gives the evidence that the coefficient of 24 as given in Equation (3) seems to be more reasonable.)

PLASTIC DESIGN APPROACH

While "Rule of Thumb" Equation (3) is suitable for routine cases, it may be too conservative for brackets with wide flanges, for plates fixed at their edges, and for other conditions. A rigorous application of the elastic method involves a lengthy computation of the coefficient k in Equation (2) for each bracket shape, plate edge condition, etc. While these computations can be programmed on a digital computer, the required expenditure of time and money would in general not be justified for an individual designer.

These drawbacks of the elastic theory can fortunately be circumvented by means of the yield line method.* This method, developed by K. W. Johansen in the Scandinavian countries and used mostly in the design of reinforced concrete slabs, is a useful tool in steel design as well, the yield lines being thought of as continuous plastic hinges.

Several bracket diagrams, yield line configurations, and expressions and values for the coefficient k in Equation (2), (or rather, Equation (2a) below), are given in Figs. 3 through 7 and are summarized in Table I.

It must be understood that these coefficients should generally be multiplied by a reduction factor for use in design. The principal reasons for the reduction are: (1) A modification of the yield line layout, including the effects of the so-called corner levers, may be more critical than the assumed configuration, and should therefore govern. (2) Yielding and overstress will generally occur in some portion of the yield line pattern before it takes

* *See Timoshenko and Woinowsky-Krieger* Plates and Shells *or any advanced text on structural mechanics.*

** *Allowable working stress for a rectangular section could be assumed to be 27 ksi for A36 steel. Considering the shape factor of a rectangular plate, the unit stress of 0.75 F_y could be justified.*

* *See, for example, L. L. Jones* Ultimate Load Analysis of Reinforced and Prestressed Concrete Structures *Chatto and Windus, London, 1962.*

place in the remaining portions. This effect is particularly important in brackets subject to fatigue.

Introducing into Equation (2) a reduction factor ϕ, which should be selected by engineering judgment from about 60 to about 90 percent, depending on the circumstances, Equation (2) may be written as

$$X = k\phi mL \qquad (2a)$$

EXAMPLE

A bracket consisting of a ST12I45 has been symmetrically welded to the web of a 14W 150 column, and has to carry a 25 kip reaction from a beam parallel to the web and located 9 in. away. The material is A36 steel. Check the column web for bending.

Given: a = width of plate = T-distance of 14W column = 11.375 in. L = length of vertical weld = 11 in. c = width of horizontal weld = 7 in. t = web thickness of column = 0.695 in. Bracket moment $X = 25 \times 9 = 225$ in.-kips. To be on the conservative side the column web will be assumed simply supported by the column flanges, f will be taken as 22 ksi (not 27 ksi per AISC Specification Item 1.5.1.4.8.), S will be taken as $\frac{1}{6}t^2$ (not $\frac{1}{4}t^2$ of plastic theory), and ϕ will be taken as 75 percent.

Solution: Using the "Rule of Thumb", Equation (3), $X = 24\,Lt^2 = 127$ in.-kips <225. **NG**

Using Table I, $k_0 = 10.6$ (k from the formula in Fig. 7 is 10.9).

$$m = \tfrac{1}{6} ft^2 = 1.77 \text{ kips} \qquad (1)$$

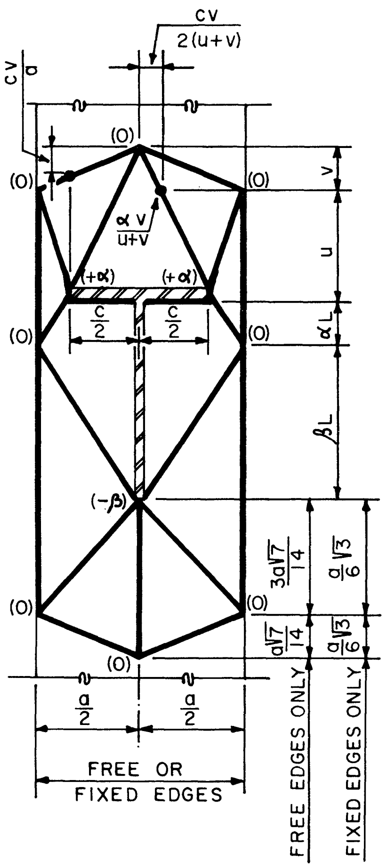
$$X = k\phi mL = 10.6 \times 75 \text{ percent} \times 1.77 \times 11$$
$$= 155 < 225 \textbf{ NG} \qquad (2a)$$

Suggested remedy: Weld a $7 \times \frac{1}{2}$ in. plate to form a bottom flange to the bracket. $c = 7$ in.

Then, from Fig. 6,

$$k = 2a/L + 2L/(a-c) + 2\sqrt{(7a+c)/(a-c)}$$
$$= 16.0$$

$$X = 16.0 \times 75 \text{ percent} \times 1.77 \times 11$$
$$= 234 > 225 \textbf{ OK} \qquad (2a)$$



FREE EDGES

$$\alpha = \frac{a-c}{2a-c}\left[1 - \frac{a}{4L}\left(\sqrt{\frac{7a+c}{a-c}} - \sqrt{7}\right)\right]$$

$$\beta = 1 - \alpha$$

$$u = \frac{3a+c}{2}\sqrt{\frac{a-c}{7a+c}}$$

$$v = \frac{a}{2}\sqrt{\frac{a-c}{7a+c}}$$

$$k = \frac{2}{2a-c}\left[\left(1 + \frac{a\sqrt{7}}{4L}\right)\sqrt{(a-c)(7a+c)} + \frac{a(a-c)}{4L} + 2L + a\sqrt{7}\right];$$

for $L = a$ and $c = \frac{a}{2}$, $k = 10.65$

FIXED EDGES

$$\alpha = \frac{a-c}{2a-c}\left[1 - \frac{a}{4L}\left(\sqrt{\frac{3a+c}{a-c}} - \sqrt{3}\right)\right]$$

$$\beta = 1 - \alpha$$

$$u = \frac{a+c}{2}\sqrt{\frac{a-c}{3a+c}}$$

$$v = \frac{a}{2}\sqrt{\frac{a-c}{3a+c}}$$

$$k = \frac{2}{2a-c}\left[\left(2 + \frac{a\sqrt{3}}{2L}\right)\sqrt{(a-c)(3a+c)} + \frac{a(a-c)}{2L} + 4L + 2a\sqrt{3}\right];$$

for $L = a$ and $c = \frac{a}{2}$, $k = 15.33$

Figure 7

APPLICATION OF THE EQUATIONS

Treating the example of Fig. 1 by yield line methods, and referring to Fig. 4, $X = 9.3\ \phi\ mL$. Previously, by elastic methods, the value $X = 4.9\ mL$ has been obtained. To make the two results coincide, in this example ϕ must be taken as 4.9/9.3, or 53 percent. This value of ϕ appears to be too low; in other words, the results of the elastic method are too conservative, perhaps by 20 or 30 percent in this example.

The yield line patterns and the resulting coefficients may be seriously affected by the presence of other connections, stiffeners, holes, or similar features near the bracket, or by asymmetry.

TABLE I. Table of Coefficients
k_0 ($=k$, **with** $L = a$ **and** $c = 0.5\ a$) **for Equation** *(2a)*

Shape of bracket	Web only	Top flange	Top and bottom flanges
Yield line pattern	Fig. 4	Fig. 7	Fig. 6
k_0-free edges	9.3	10.6	13.8
k_0-fixed edges	12.9	15.3	20.6

The equations given above hold equally well in cases where bending is applied to plates by connections other than brackets; for example, by a fixed-ended connection of a beam which develops negative moment. On the other hand, cases where the bending is applied partly to a plate and partly to another structural element fall outside the scope of the present discussion. Important examples are:

(1) Brackets fixed to a column flange rather than to a column web.

(2) T-brackets connected to a column web, with the bracket flange welded to the column flanges, or to the column web close to the toe of the fillet. Fixing some part of the bracket to a stiffer structural element will relieve the bending of the plate. At the same time it will put heavy stress into the weld connecting the bracket to the stiff element.

It should be noted that the methods of computation presented in this paper are based on theoretical analysis and conservative assumptions. The authors believe that research on this subject and the significance of high localized stresses would be desirable, and would probably justify less conservative criteria.