Design of Columns Subject to Biaxial Bending

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TASK GROUP 3 of the Column Research Council is charged with seeking improved methods for the design of columns subject to biaxial bending. As a result of recent research, this Task Group now can recommend improved design techniques for the most commonly used type of steel column, namely the wide flange shape. It is the purpose of this paper to review the existing AISC Specification design requirements for biaxially loaded beam-columns, to present the proposed new design procedures, and to compare the size of column section required by the existing and proposed design rules for several practical design problems.

PRESENT STATUS

In the current AISC Specification, the expressions governing the design of beam columns subject to biaxial bending are as follows:

For strength (maximum bending stresses unaffected by axial load acting on deflected shape):

$$\frac{f_a}{0.6F_y} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0 \qquad (1)$$

For stability (maximum bending stresses increased by axial load acting on deflected shape):

$$\frac{f_a}{F_a} + \frac{C_{mx} f_{bx}}{\left(1 - \frac{f_a}{F'_{ex}}\right)F'_{bx}} + \frac{C_{my} f_{by}}{\left(1 - \frac{f_a}{F'_{ey}}\right)F'_{by}} \leq 1.0 \qquad (2)$$

It will be seen that these expressions essentially place stress limits on the extreme corner fibers of a column section. At a brace point, this limit originally was $0.6F_y$, giving a factor of safety against nominal first yield of 1.67. Through successive editions of the AISC Specification, minor modifications have been incorporated. The permissible bending stress has been increased from $0.6F_y$ to $0.66F_y$ in recognition of the shape factor of wide flange sections about their major axis. It will be noted that the same relationship has been assumed between the elastic and plastic moment capacities about the *y*-axis as about the *x*-axis. Of course, this is only a conservative simplifying assumption, because the shape factors for wide flange shapes are approximately 1.12 and1.50 about the major and minor axes, respectively. If the column were part of a rigid framework, the permissible bending stresses would be increased to $0.75F_y$.

Equation (6.19) of the Second Edition (1966) of the Column Research Council's *Guide to Design Criteria for Metal Compression Members* gives the following expression for the ultimate strength of beam-columns subject to biaxial bending, for a stability type failure:

$$\frac{P}{P_u} + \frac{C_{mx}M_x}{M_{ux}\left(1 - \frac{P}{P_{ex}}\right)} + \frac{C_{my}M_y}{M_{uy}\left(1 - \frac{P}{P_{ey}}\right)} \leq 1.0 \qquad (3)$$

where

C_{mx}, C_{my} = equivalent moment factors used in the AISC Specification interaction formula
M_{ux}, M_{uy} = ultimate bending moments in the absence of axial load
M_x, M_y = applied end moments
P = axial load
P_{ex}, P_{ey} = Euler buckling loads
P_u = ultimate load of axially loaded column

The conservatism mentioned previously with respect to the different plastic moment capacities about the major and minor axes is eliminated in this expression, since the correct ultimate moment capacity about each respective axis is used.

Both the AISC design expressions and CRC Eq. (6.19) are straight line interaction equations. Research has shown that the interaction of moments about the orthogonal axes is not linear; on the contrary, the interaction curve resembles more closely the quadrant of a circle (see Fig. 1). It is important to note that if a member is fully loaded under axial load and bending about one axis, then there is no spare capacity to accept moment about the other axis. However, as the loading decreases slightly below the maximum, capacity rapidly develops to accept bending about the other axis.

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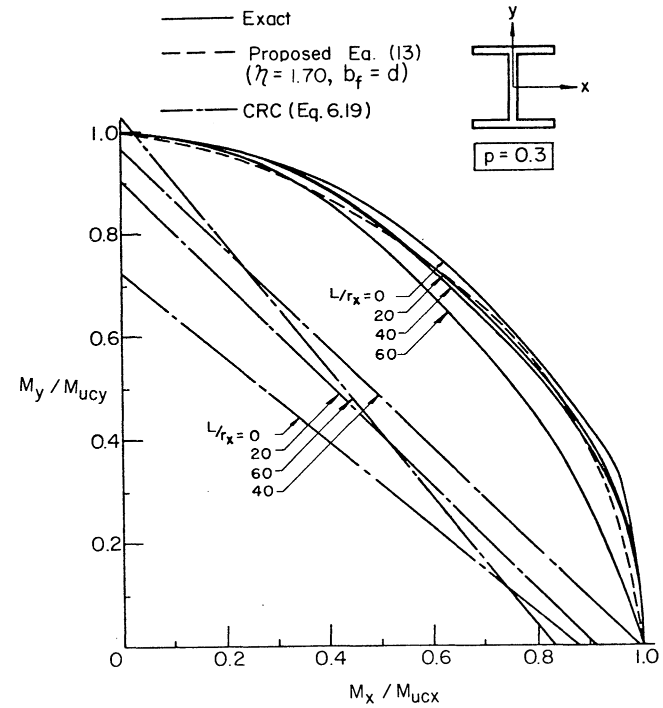

Fig. 7. Comparison of interaction curves for long columns

NATURE OF THE PROBLEM

Designers may well ask why, if a computer may be used to check the adequacy of a section, is it not possible to use complex but precise design methods? Considering the design of a section for strength only, it is relatively simple to check a wide flange section for axial load and moment about one axis only by assuming that the neutral axis lies either in the flange or in the web and parallel to one or the other, then solving for its precise location directly. However, when the section is subject to biaxial bending, neither the location nor the angular disposition of the neutral axis is known. In consequence, a simple direct design procedure cannot be followed.

The solution for stability is far more complex. In addition to the problem of locating the neutral axis, the effects of torsional buckling and the deflection of the column, due to both end moments and the effect of the axial load acting on the deflected column, must be determined to satisfy equilibrium and compatibility. Superimposed on this is the effect of residual stresses, so that once the actual stress induced by the loading plus the residual stress exceeds yield, the determination of the deflected shape must take account of the partially plastified section. Clearly, this is a difficult problem, demanding far too much computer time to be solved for each column to be designed. Such a procedure may be used only to define master interaction surfaces to which simpler empirical surfaces may be fitted in the derivation of design expressions.

RECENT DEVELOPMENTS

Several design methods have been proposed by members of CRC Task Group 3 which reflect the true capacity of a beam-column subject to biaxial bending. Ringo, McDonough, and Basehart[1] have studied the beam-column at a brace point. They were able to classify the 77 commonly occurring wide flange column sections into seven groups and to provide convenient design aids for these sections in the form of seven direct reading nomographs.

Sharma and Gaylord[2] developed a series of five design charts for biaxially loaded beam-columns, plotting dimensionless parameters against slenderness ratio. In this way they were able to offer design of the complete column, both for strength at the brace point and for stability. Design using the Sharma and Gaylord charts is quite rapid; the only disadvantage is that interpolation is required on each chart followed by interpolation between the charts.

In England, Young[3] has recently published a design method, complete with formulas and design aids, in which the section is selected such that its major axis plastic moment, reduced by coefficients reflecting the presence of axial load, buckling about major and minor axes, and torsional buckling, fulfills the design requirements.

Many designers dislike graphical procedures, although the accuracy obtained thereby probably is within the limits with which we can predict the capacity of a beam-column under biaxial loading. For this and other reasons, specifications are generally expressed in terms of algebraic formulas, where possible. Such formulas are particularly suitable for incorporation into computer design programs.

By algebraic transposition, Eq. (3) may be written in the following form:

$$\frac{M_x}{M_{ucx}} + \frac{M_y}{M_{ucy}} \leq 1.0 \tag{4}$$

where M_{ucx} and M_{ucy} are the ultimate symmetrical single curvature moment capacities of an axially loaded beam-column, about the x- or y-axis, respectively, when there is zero moment about the other axis. These denominator terms may be expressed as follows:[4]

At braced points:

$$M_{ucx} = M_{pcx} = 1.18M_{px}[1 - (P/P_y)] \tag{5}$$

$$M_{ucy} = M_{pcy} = 1.19M_{py}[1 - (P/P_y)^2] \tag{6}$$

Considering stability:

$$M_{ucx} = M_{ux}[1 - (P/P_u)][1 - (P/P_{ex})] \tag{7}$$

$$M_{ucy} = M_{py}[1 - (P/P_u)][1 - (P/P_{ey})] \tag{8}$$

In the above equations,

M_{pcx}, M_{pcy} = plastic moment capacity about the x- or y-axis, respectively, reduced for the presence of axial load

M_{px}, M_{py} = plastic moment capacities

M_{ux} = plastic moment capacity about the x-axis, reduced for the presence of lateral torsional buckling, if necessary[6]

$$= M_{px}\left[1.07 - \frac{(L/r_y)\sqrt{F_y}}{3160}\right] \leq M_{px} \quad (9)^*$$

P_y = axial load at full yield condition

RECOMMENDED METHOD

The following interaction equation was suggested by Chen and Atsuta[5,6] and Tebedge and Chen.[7]

$$\left(\frac{M_x}{M_{ucx}}\right)^{\text{exponent}} + \left(\frac{M_y}{M_{ucy}}\right)^{\text{exponent}} \leq 1.0 \qquad (10)$$

By deriving the correct exponents, an interaction curve could be defined which would fit the actual strength curve of a biaxially loaded beam-column of a particular cross section, such as a wide flange shape.

At a braced section, the capacity was determined by a superposition technique and interaction curves plotted. For stability, Tebedge and Chen determined empirical interaction curves for wide flange columns at various levels of axial stress. Exponent expressions giving a good fit to these curves were developed.

Based on the suggestions of Refs. 5, 6, 7, and 8, the complete recommendations of Task Group 3 are as follows:

At a braced location, the following equation should be satisfied:

$$\left(\frac{M_x}{M_{pcx}}\right)^{\zeta} + \left(\frac{M_y}{M_{pcy}}\right)^{\zeta} \leq 1.0 \qquad (11)$$

For wide flange shapes having a flange width to web depth ratio from 0.5 to 1.0:

$$\zeta = 1.6 - \frac{P/P_y}{2 \ln (P/P_y)} \qquad (12)$$

where ln indicates the natural logarithm.

To check stability between braced points, the following equation should be satisfied:

$$\left(\frac{C_{mx}M_x}{M_{ucx}}\right)^{\eta} + \left(\frac{C_{my}M_y}{M_{ucy}}\right)^{\eta} \leq 1.0 \qquad (13)$$

**Slenderness ratios greater than $\sqrt{2\pi^2 E/F_y}$ do not appear to have been considered in the development of Eq. (9). Beyond this ratio, values of M_{ux} could be based on 1.67 times the allowable stress given in Sect. 1.5.1.4.6a of the AISC Specification. The effect of lateral torsional buckling generally is to reduce the axial load capacity only slightly.*

in which,

C_{mx}, C_{my} = equivalent moment factors used in the AISC Specification interaction formula,

i.e., $$C_m = 0.6 - 0.4\frac{M_1}{M_2} \geq 0.4 \qquad (14)$$

where M_1/M_2 is the ratio of the smaller to larger moments at the ends of that portion of the member unbraced in the plane of bending under consideration. M_1/M_2 is positive when the member is bent in reverse curvature and negative when it is bent in single curvature.

M_x, M_y = the greater of the moments applied at one or the other end of the beam-column

$$\eta = 0.4 + P/P_y + b_f/d \geq 1.0 \quad \text{when } b_f/d \geq 0.3 \qquad (15)$$
$$= 1.0 \text{ when } b_f/d < 0.3$$

b_f = flange width of W or I section

d = depth of W or I section

and in which M_{ucx} and M_{ucy} can be determined from Eqs. (7) and (8), respectively.

The AISC Specification recommends a value of 0.85 for C_m for compression members in frames subject to joint translation (sidesway). This approach should not be used in combination with the method recommended here, the development of which is based on constant end eccentricities up to maximum load. To use the recommended method in sway frames, the end moments should be determined by a second order analysis, i.e., the P-Δ effects at ultimate load should be included.[9,10,11]

The advantages of the design interaction equation suggested by Chen et al are that both strength and stability are covered by the same basic equation and the equation has universal applicability, for if the exponent for the particular section being designed has not been determined, it may conservatively be assumed to be unity.

The assumptions made in the proposed design procedure are:

1. The design is for an isolated member, not for a part of a framework in which the forces redistribute as ultimate load is approached.

2. The design method is valid for beam-columns in which the axial load and end moments are known, being determined by a first or second order elastic analysis, as appropriate to braced or unbraced frames. The adoption of biaxial plastic design is not suggested.

3. The sections are compact, i.e., premature failure will not occur due to local buckling of flanges or webs prior to the member attaining its ultimate strength.

4. The column is initially twisted and out-of-straight, contains residual stresses, and the material is elastic/perfectly plastic.

In using the proposed interaction equation, the determination of axial load capacity of the column should be as realistic as possible. Heretofore, in working strength design, the tangent modulus curve has been modified by a variable factor of safety as slenderness increases.

It is suggested that column capacity be based on an approach such as ultimate strength,[3,9] rather than the unmodified tangent modulus critical load given by CRC Eq. (2.9). Part 2 of the AISC Specification recommends 1.7 times the working stress allowable compression, not the CRC basic column stress. This procedure was used in the examples and comparisons in this text.

COMPARATIVE DESIGNS

The following design examples have been deliberately chosen to demonstrate the difference in column section required for strength (Example 1) and for stability (Example 2). The examples have been solved on the basis of the following design methods:

(a) AISC Specification (working stress design):
See Eqs. (1) and (2) for strength and stability formulas, respectively.

(b) CSA S16.1-1974 (limit states design):*
(Based on ultimate load equations, using an overall factor of 1.667.)

For strength:

$$\frac{P}{P_u} + \frac{0.85M_x}{M_{px}} + \frac{0.6M_y}{M_{py}} \leq 1.0$$

$$\frac{M_x}{M_{px}} + \frac{M_y}{M_{py}} \leq 1.0$$

For stability: Use CRC Eq. (6.19). [See Eq. (3).]

(c) Interaction equations recommended by Chen et al:[5,6,7,8]

For strength, see Eq. (11).

For stability, see Eq. (13).

(Similar equations appear in an Appendix to CSA S16.1, with restrictions on flange width to beam depth ratio.)

(d) Nomographs developed by Ringo et al.[1]

(e) Charts developed by Sharma and Gaylord.[2]

(f) Young's procedure.[3]

**Canadian Standards Association.*

Example 1—Design a W12 section in A36 steel, effective length 15 ft-0 in., to resist an ultimate axial load of 375 kips, subject to equal and opposite end moments of 160 and 60 kip-ft about the x- and y-axes, respectively, inducing antisymmetric double curvature about both axes.

For this case, since reverse curvature is induced by the end moments, maximum bending stresses will occur at the ends and will be unaffected by the axial load acting upon the deflected shape of the column. The strength-type analysis is found to control in this example.

The column section required for strength according to each design method is shown in Table 1, along with a factor indicating the relative economy of each section.

Table 1. Comparison of Solutions to Design Examples 1 and 2 Using Various Design Methods

	Example 1 (Strength Design)		Example 2 (Stability Design)	
Method	Column Section	Economy Factor	Column Section	Economy Factor
a. AISC	W12×106	1.00	W12×133	1.00
b. CSA S16	W12× 85	0.80	W12×120	0.90
c. Chen	W12×72	0.68	W12×99	0.74
d. Gaylord	W12×72	0.68	W12×92	0.69
e. Ringo	W12×72	0.68	*	*
f. Young	W12×72	0.68	W12×92	0.69

* Not applicable (braced points only).

Example 2—Same as Example 1, except that the end moments induce single curvature about both axes.

For this case, since single curvature is induced by the end moments, maximum bending stresses will occur between the ends of the column, as a result of the axial load acting upon the deflected shape. Therefore, a stability type analysis must be made.

The column section required for stability according to each design method is shown in Table 1, along with a factor indicating the relative economy of each section.

To illustrate the use of the Chen method, following are the required calculations for solving Example 2:

Chen Solution:

Try W12 × 99:

$A = 29.09$ in.^{2} $\quad Z_x = 152$ in.^{3} $\quad r_x = 5.43$ in.

$E = 29000$ ksi $\quad Z_y = 69.5$ in.^{3} $\quad r_y = 3.09$ in.

Euler buckling stress $= \pi^2 E/(L/r)^2$

$L/r_x = 33 \quad F_{ex} = 260$ ksi $\quad P_{ex} = 7563$ kips

$L/r_y = 58.2 \quad F_{ey} = 84.3$ ksi $\quad P_{ey} = 2452$ kips

From the AISC column curve, $F_a = 17.60$ ksi.

Using a load factor of 1.667, $P_u = 853$ kips.

$$M_{ux} = M_{px}\left[1.07 - \frac{(L/r_y)\sqrt{F_y}}{3160}\right] \leq M_{px}$$

$$= (152 \times 36)\left[1.07 - \frac{58.2 \times 6}{3160}\right]$$

$$= 5{,}250 \text{ kip-in.} = 438 \text{ kip-ft}$$

$$M_{ucx} = M_{ux}(1 - P/P_u)(1 - P/P_{ex})$$
$$= 438(1 - 375/853)(1 - 375/7563)$$
$$= 233 \text{ kip-ft}$$

$$M_{ucy} = M_{py}(1 - P/P_u)(1 - P/P_{ey})$$
$$= 69.5 \times 36(1 - 375/853)(1 - 375/2452)$$
$$= 1{,}189 \text{ kip-in.} = 99 \text{ kip-ft}$$

$$\frac{C_{mx}M_x}{M_{ux}} = \frac{1.0 \times 160}{233} = 0.687$$

$$\frac{C_{my}M_y}{M_{uy}} = \frac{1.0 \times 60}{99} = 0.606$$

$$P/P_y = 375/(29.09 \times 36) = 0.358$$

Since $b_f = d$, find $\eta = 1.76$ from Fig. 2; alternatively, from Eq. (15), $\eta = 0.4 + P/P_y + b_f/d = 1.758$.

From Fig. 3, enter with appropriate moment ratios, finding that the required value of η is $1.6 < 1.76$; therefore, the W12×99 section is adequate. This may be verified by solution of the interaction equation:

$$0.687^{1.76} + 0.606^{1.76} = 0.93 < 1.0 \quad \textbf{o.k.}$$

Comparisons of Results—In Table 1, the most striking features of the comparative designs are the remarkable reduction in column section required by methods (c) through (f), and the agreement of their results.

In Table 2, comparison is made with an example published by Young.[3] The ultimate major axis moment capacity predicted by various methods is given, assuming fixed values for axial load and minor axis moment.

It is most encouraging to observe that for single curvature, the Young and Chen predictions are in very close agreement, even though their origins are different.

Table 2. Comparison* of Major Axis Biaxial Moment Capacities (Kip-Ft) Predicted by Various Design Methods

C_m	1.0	0.6	0.4
AISC**	43†	43†	43†
CSA S16.1	117	157†	157†
Chen	167	209†	209†
Young	167	183	198

*Based on Young's example (Ref. 3): W12×65, $L = 118''$, $P = 0.3P_y$, $M_{y(\text{top})} = 0.4M_{py}$, $M_{y(\text{bot})} = 0$.

**Determined by applying 0.6 times ultimate thrust and minor axis moment, and multiplying resulting major axis working stress design moment, by 1.67 for direct comparison.

†Strength controls ("stocky" column).

Similar comparisons are made in Table 3 for uniaxial bending.

VERIFICATION WITH TESTS

Birnstiel[12] carried out refined testing of 5x5 and 6x6 W-shaped beam-columns loaded in single curvature biaxial bending.[11] Two comparisons of design equations with these tests have been made. In the earlier comparison by Pillai[13] Chen's equation was not evaluated. Springfield's evaluation[14] of the Chen equation vs. Eq. (4) showed that, for the Birnstiel tests, Chen's equation was extremely reliable (Mean 1.01, SD 0.074), while Eq. (4) was conservative (Mean 1.20, SD 0.085).

A further verification of Chen's equation is its good agreement with Birnstiel's incremental analytical procedure.[15] Aside from one result, in which the error was 7% conservative, all the other values agree to within 3%.

LIMITATIONS OF THE PROPOSED METHOD

Before adopting designs according to the proposed interaction equation, some general thought should be given to the extent to which yielding under service load is likely and, in view of this, whether further restrictions on the design are necessary. In Table 4, for each of two ratios of actual axial load to yield axial load and three ratios of M_x/M_{pcx}, the corresponding ratios of M_y/M_{pcy} have been determined from the proposed interaction equation, Eq. (11). In evaluating M_x and M_y, a W12×65

Table 3. Comparison* of Major Axis Uniaxial Moment Capacities (Kip-Ft) Predicted by Various Design Methods

C_m	1.0	0.6	0.4
AISC**	93	116†	116†
AISC (plastic)	93	136	136
CSA S16.1	93	137†	137†
Chen	93	137†	137†
Young	101	130	132

*Based on Young's example (Ref. 3): W12×65, $L = 118''$, $P = 0.6P_y$.

**Determined by applying 0.6 times ultimate thrust and minor axis moment, and multiplying resulting major axis working stress design moment by 1.67 for direct comparison.

†Strength controls ("stocky" column).

Table 4*

P/P_y	$\frac{M_x}{M_{pcx}}$	$\frac{M_y}{M_{pcy}}$	f_a	f_{bx}	f_{by}	$\frac{f}{1.7}$
0.3	0.2	0.963	10.8	6.55	52.6	41.1
	0.5	0.812	10.8	16.4	44.3	42.0
	0.8	0.516	10.8	26.2	28.2	38.3
0.7	0.2	0.994	25.2	2.81	32.9	35.8
	0.5	0.932	25.2	7.02	30.9	37.1
	0.8	0.726	25.2	11.24	24 1	35.6

*Based on a W12×65 section, A36 steel, Eq. (11).

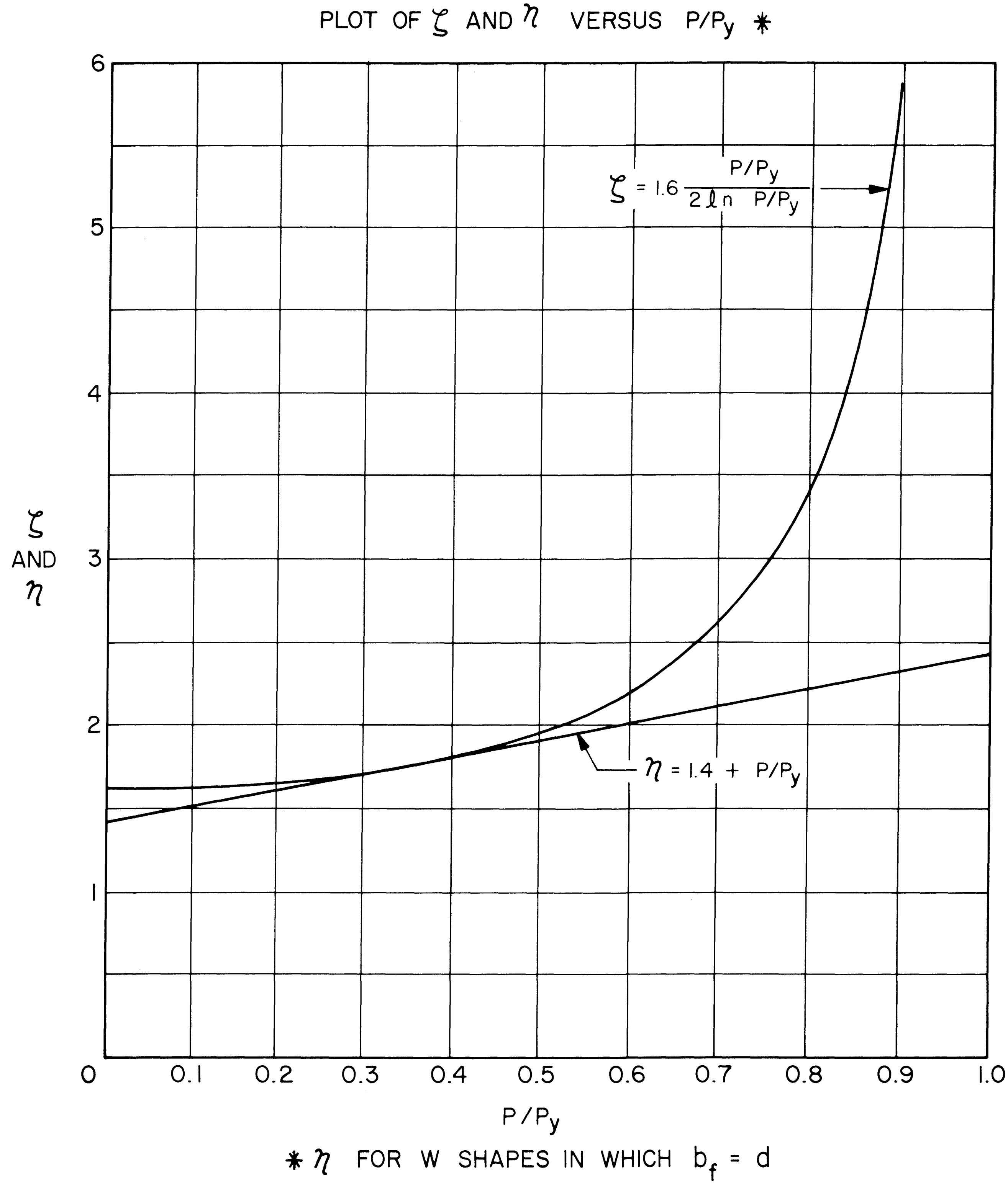

Fig. 2. Plot of ζ and η vs. P/P_y

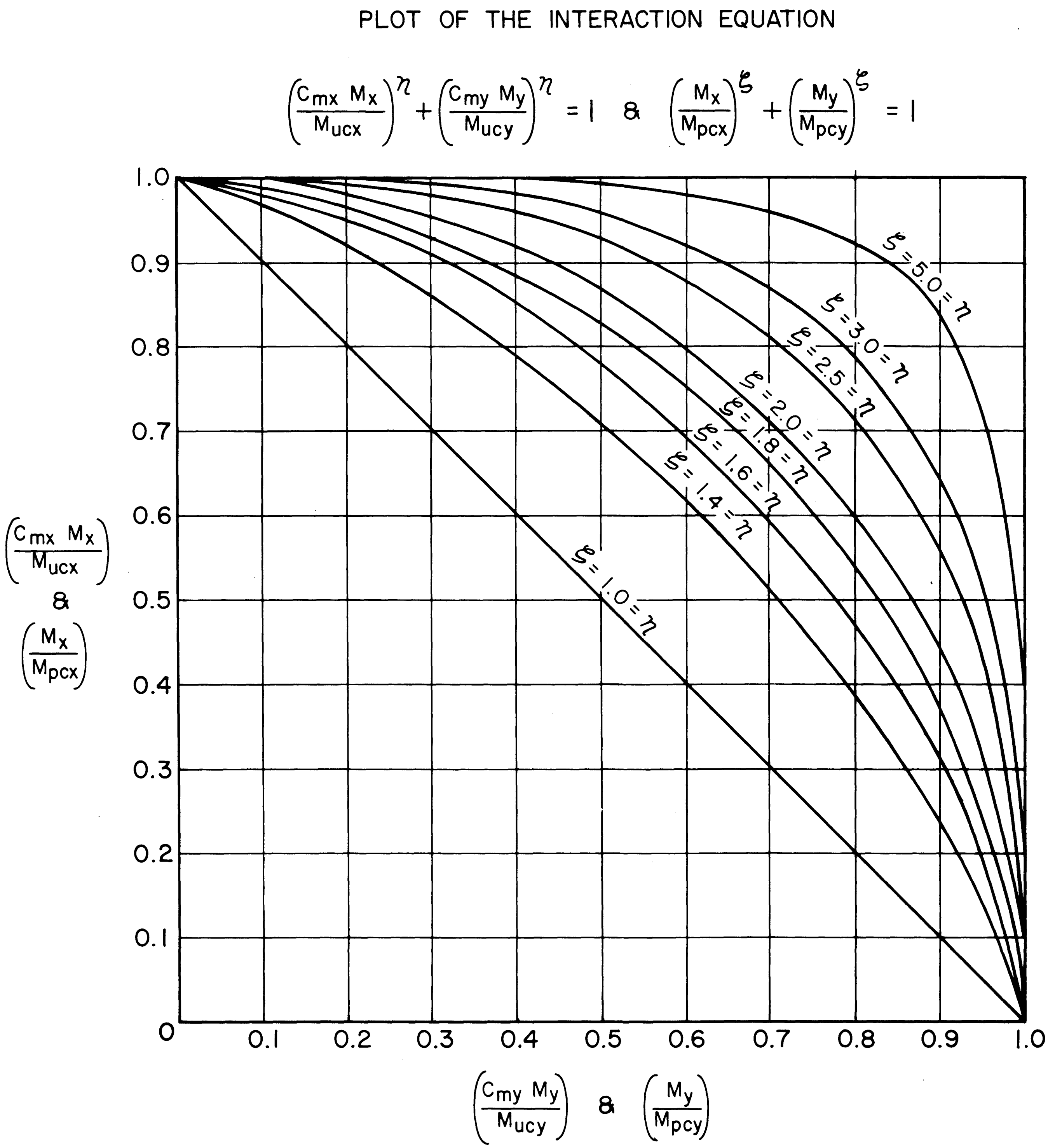

Fig. 3. Plot of Interaction Equations (11) and (13)

section of A36 steel has been assumed and the extreme corner fiber stress has been calculated elastically and divided by a load factor of 1.7. It will be noted that, for the lower ratio of axial stress, the yield stress of 36 ksi has been exceeded by an appreciable margin. This rather simple calculation demonstrates that the onset of yielding under service loads is a real probability for sections designed according to the proposed interaction equation.

In the writer's view, three precautions are desirable when designing beam-columns by the proposed procedure. These are:

1. Sections should be proportioned so that influences such as wind or earthquake, which are reversible, should not, when taken alone, cause stresses in extreme fibers which exceed the nominal yield strength.

2. Sections should be proportioned such that influences which are variable, such as combined wind plus live load (taken at a load factor of 1.5 reduced to account for probability of occurrence, say by 0.7), should not in themselves cause extreme fibre stresses which exceed the nominal yield strength of the material.

3. Keep in mind the fact that the procedure was developed on the assumption that the section is compact.

It appears that further advances in the design of biaxially loaded beam-columns must recognize shape dependency. In the writer's view, this creates no problem, since a very large portion of the columns being designed in buildings are of wide flange shape. If we are able to develop the exponent required for Chen's proposed equation for all wide flange shapes and for square and rectangular tubular sections, almost the whole practical field will have been covered.

CONCLUSIONS

1. The interaction equations presently in use are not sacrosanct. Indeed, their origin appears to be intuitive and, over the years, various modifications for improvement have been proposed.

2. Any improvement in the prediction of the capacity of axially loaded columns will improve the prediction of the capacity of beam-columns.

3. Any improvements in the prediction of the capacity of beam-columns must be dependent on cross-sectional properties.

4. Several acceptable methods of design for biaxially loaded beam-columns have been developed by CRC Task Group 3.

5. The design expressions suggested by Chen et al are reliable compared with precise design methods, with tests, and with a design method independently developed in England.

6. The exponential interaction equations appear mathematically complex, but with the design aids provided in this paper their evaluation is simple. (Their evaluation is equally simple for designers using electronic calculators with exponent and log functions.) At the same time, the exponential design equations represent an upper limit to the degree of refinement in design and predictability of capacity.

7. Considerable economy of column section can result by designing in accordance with the procedure recommended by Task Group 3.

RECOMMENDATIONS

1. The interaction equations suggested by Chen et al are recommended for design of compact W shape beam-columns under biaxial eccentric load.

2. For any framework in which lateral deflection is critical, an evaluation of the stresses at service load should be made to assess the validity of calculated elastic lateral deflection.

3. Further primary research is recommended to expand the range of applicability of the equations to other beam-column shapes, including square and rectangular tubular sections.*

4. Secondary research is recommended to evaluate further the applicability of the equivalent moment coefficients presently used, and to check the apparent conservatism of the present lateral torsional buckling major axis bending stress reductions.

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