Drift Reduction Factors for Belted High-Rise Structures

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Reduction of drift in high rise structures is necessary to prevent unsightly plaster cracks and minimize the psychological effects of unnerving movements and frightening noises.

The purpose of lateral load bracing systems is to resist lateral forces due to wind and earthquake.

Wind or seismic bracing systems may be classified as X brace, K brace, knee brace, and rigid frame. These systems are used in building core regions, as between elevator or duct shafts and adjacent to passageways or corridors. Shear walls are often selected for structures with large column spacings when lateral forces can no longer be resisted by the floor slabs alone. The extent of joint fixity provided by connections becomes a primary consideration in the design of rigid frame bracing.[1]

Fleischer[2] has introduced a method by which drift for high rise steel structures can be closely estimated in a few minutes using a desk or pocket calculator. He concludes that the method is fully justified in the very important field of preliminary design and drift estimations. In a later paper,[3] Fleischer extended his method to obtain drift estimates for high-rise steel frames under the application of seismic loads as prescribed in the major building codes. Both of Fleischer's papers include a detailed application of his method to a 360-ft high structure with 30 supported levels.

Drift under factored combined load may influence the strength and stability of multistory steel frames. Recognition of this feature of structural behavior is an important conceptual contribution of plastic design. As an example, Rumpf, Hooper, and Yura[4] present an excellent plastic design of a braced multistory building which includes drift calculations.

In his recent paper "Optimum Belt Truss Locations for High Rise Structures," Taranath[5] discusses the belt truss as a means of limiting drift in high-rise buildings. The belt truss system for limiting drift embodies a great deal of foresight and ingenuity. However, the impression is left with the reader that regardless of the conditions assumed (i.e., ideal or real), there is only one significant design parameter, and that is the location of the belt truss system. It will be shown in the following mathematical development that in addition to the belt truss location another parameter is of major importance. This other parameter is a function of properties of the core as well as properties and spacing of the exterior columns of the structure.

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DERIVATION OF THE GOVERNING EQUATIONS

The mathematical development that follows is made for a simplified structure whose analytical model is shown in Fig. 1. The following simplifying assumptions, essentially the same as those in Ref. 5, were made:

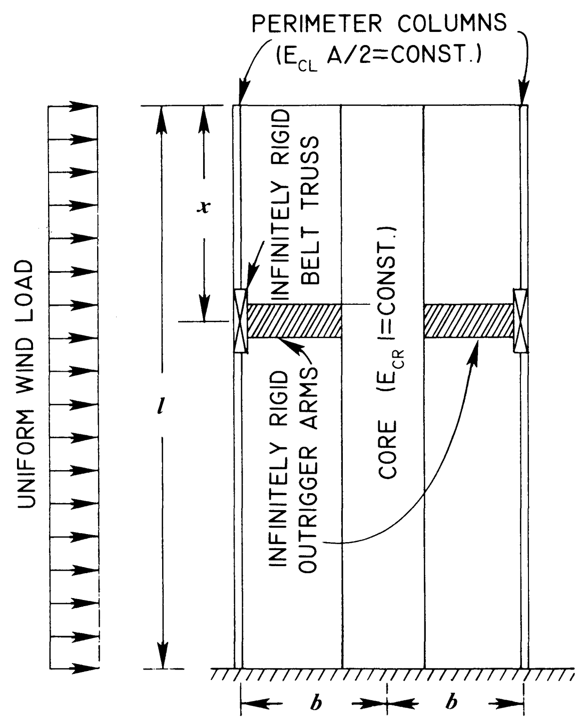
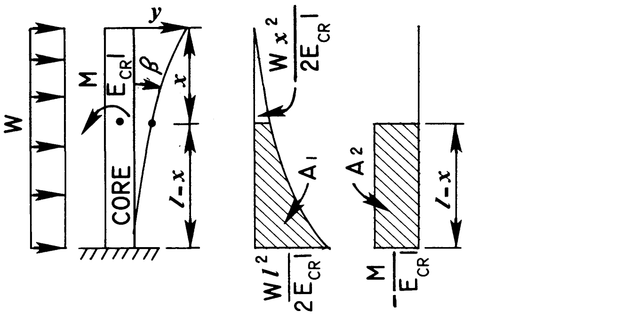
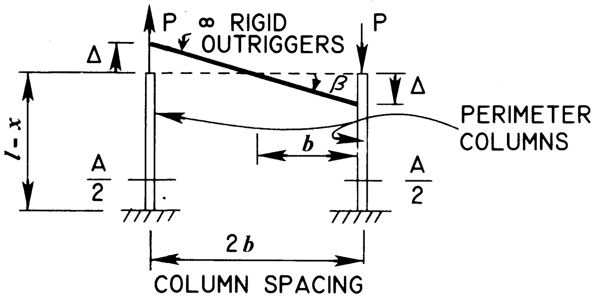


Fig. 1. Schematic diagram of simplified structure



POSITIVE MOMENT=TENSION ON WINDWARD SIDE

(a) Load, deflection, and moment diagram of core



(b) Load and deflection of perimeter columns

Figure 2

1. The connection between the columns and the outrigger arms is such that the exterior columns are subjected only to axial forces.
2. The core of the structure is heavily braced, so that its rotation due to bracing deformations may be assumed negligible.
3. The girders are assumed to be pinned to the columns, so that the entire wind load is resisted by the bending of the core of the structure and axial deformation of the perimeter columns.
4. The belt-truss is infinitely rigid.
5. The perimeter columns have a constant stiffness and the core has a constant moment of inertia.

Refer to Fig. 2a and apply the First Area Moment Theorem to obtain an equation for the rotation of the core β at the level of the belt truss, a distance x measured from the top of the high rise structure:

$$\beta = A_1 - A_2 = \frac{W}{6E_{CR}I}(l^3 - x^3) - \frac{M_x}{E_{CR}I}(l - x) \quad (1)$$

where $M_x = P(2b)$ is the moment at x, representing the outrigger and columns restraint.*

* *Complete Nomenclature is given in Appendix A.*

Since the outrigger of Fig. 2b is assumed to be infinitely rigid and the angle β is small, one may write:

$$\beta = \Delta/b \quad (2)$$

where Δ is the total axial deformation of the perimeter columns and $2b$ is the spacing of these columns.

By assumption 1, above:

$$\Delta = P(l - x)/\tfrac{1}{2}AE_{CL} \quad (3)$$

Substituting $P = M_x/(2b)$ into Eq. (3) yields:

$$\Delta = M_x(l - x)bAE_{CL} \quad (4)$$

Substitution of Eq. (4) into Eq. (2) leads to:

$$\beta = M_x(l - x)/b^2AE_{CL} = M_x/b^2K_x \quad (5)$$

where
$$K_x = AE_{CL}/(l - x) \quad (6)$$

Rotational compatibility is expressed by equating the β values of Eqs. (1) and (5) to yield:

$$\frac{W}{6E_{CR}I}(l^3 - x^3) - \frac{M_x}{E_{CR}I}(l - x) = \frac{M_x}{b^2K_x} \quad (7)$$

Solving Eq. (7) for M_x, one obtains:

$$M_x = \frac{W(l^3 - x^3)}{6(l - x)} \frac{1}{[1 + E_{CR}I/b^2AE_{CL}]} = \frac{W\alpha}{6}\left(\frac{l^3 - x^3}{l - x}\right) \quad (8)$$

where the factor α is given by

$$\alpha = \frac{1}{1 + E_{CR}I/b^2AE_{CL}} \quad (9)$$

and has been termed the drift reduction factor.

Refer to Fig. 2a and apply the Second Area Moment Theorem to write an equation for the displacement or drift y at the top of the structure:

$$y = \frac{Wl^4}{8E_{CR}} - \frac{M_x}{2E_{CR}I}(l - x)(l + x) \quad (10)$$

Substitute for M_x from Eq. (8) into Eq. (10) to obtain:

$$y = \frac{Wl^4}{8E_{CR}I} - \frac{W(l^3 - x^3)(l + x)\alpha}{12E_{CR}I} \quad (11)$$

Introduce the dimensionless drift R, given by

$$R = y\bigg/\left(\frac{Wl^4}{8E_{CR}I}\right) \quad (12)$$

Substituting Eq. (12) into Eq. (11) and using the dimensionless coordinate $\gamma = (x/l)$ gives:

$$R = 1 - (2\alpha/3)(1 + \gamma - \gamma^3 - \gamma^4) \quad (13)$$

R is clearly a function of both the drift reduction factor, α, and the dimensionless coordinate, γ.

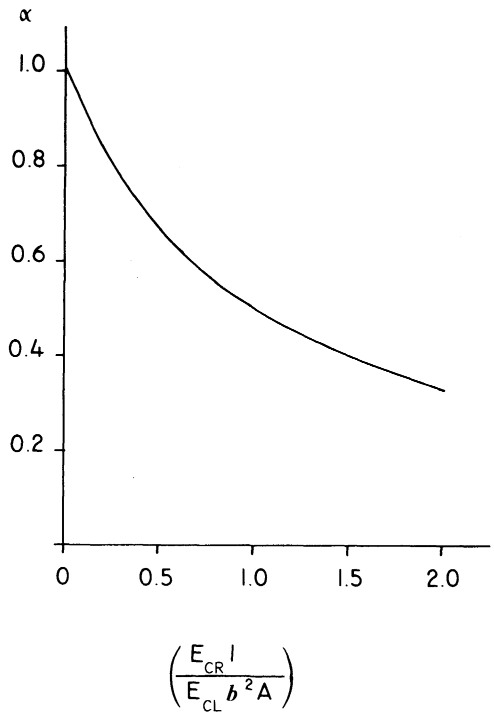


Fig. 3. Drift reduction factor

Differentiate R partially with respect to γ and equate to zero to obtain:

$$1 - 3\gamma^2 - 4\gamma^3 = 0 \tag{14}$$

This cubic equation has a single real positive root $\gamma = 0.455$. In order to minimize the dimensionless drift R, the single belt truss must be located at $x = 0.455l$. This result is independent of the drift reduction factor α and checks the value given in Ref. 5.

DRIFT REDUCTION FACTOR

The preceding mathematical developments give rise to the design parameter α which, for the sake of emphasis, has been labelled the drift reduction factor. As shown by Eq. (13), the dimensionless drift depends upon the parameter α which, as seen from Eq. (9), is a function of the properties of the core as well as the properties and spacing of the exterior columns. Equation (9) also shows that α can assume a maximum value of unity (i.e., for $E_{CR}I$ very small in comparison to b^2AE_{CL}) and decreases as the ratio $E_{CR}I/b^2AE_{CL}$ increases. The variation of the drift reduction factor α with the ratio $E_{CR}I/b^2AE_{CL}$ is shown in Fig. 3 for values of this ratio up to 2.0. For an all steel structure, $E_{CR} = E_{CL} = E$ and this latter ratio reduces to $I/(b^2A)$.

Examination of Eq. (13), which represents the dimensionless deflection at the top of the structure, reveals that this dimensionless quantity R depends upon two factors:

1. The position of the belt truss $\gamma = x/l$. For the idealized structure of Fig. 1, the value $\gamma = 0.455$ leads to the least deflection at the top of the structure.

2. The drift reduction factor. This factor depends upon the properties of the core as well as the properties and spacing of the exterior columns. The influence of the drift reduction factor on the deflection of the top of the structure is shown in Fig. 4, where the dimensionless deflection R is plotted vs. the parameter $\gamma = x/l$ for several values of the drift reduction factor α. It is evident from this family of curves that the larger the drift reduction factor, the less would be the deflection at the top of the structure. This effect is further illustrated by examining Eq. (13) differently. For constant values of the parameter γ, Eq. (13) results in a straight line relation between the dimensionless deflection R and the drift reduction factor α. Thus,

$$R = 1 - C\alpha \tag{15}$$

where C is a constant for a given value of γ and is given by the equation

$$C = \tfrac{2}{3}(1 + \gamma - \gamma^3 - \gamma^4) \tag{16}$$

Therefore, for various values of γ (i.e., for different locations of the belt truss), various values of C are obtained from Eq. (16). Equation (15) then represents a family of straight lines showing that the drift decreases linearly with increasing values of α except, of course, for

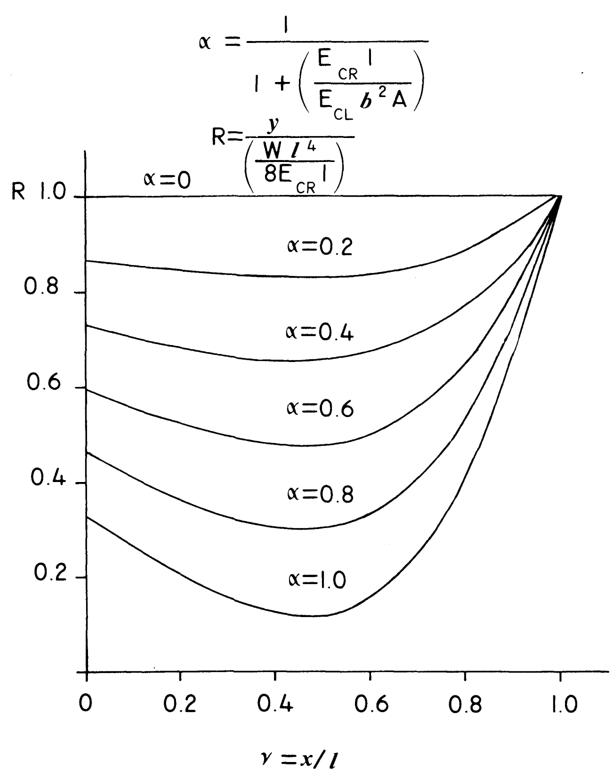


Fig. 4. Dimensionless deflection vs. γ for various values of α

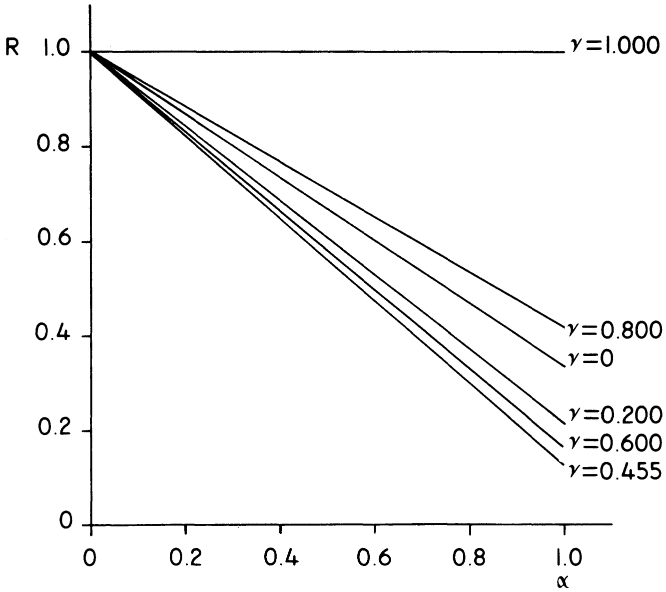


Fig. 5. Dimensionless deflection vs. α for various values of γ

the trivial case when $\gamma = 1$ (i.e., when the belt truss is placed at the very bottom of the structure). These relations are shown for a few selected values of γ in Fig. 5, including the case for $\gamma = 0.455$ which leads to the least deflection possible with a single belt truss.

REAL VS. IDEAL STRUCTURES

The preceding relations were obtained on the basis of simplified assumptions listed above. If one attempts to treat a real structure (i.e., one with flexible outriggers and with changing cross-sectional areas and moments of inertia, etc.), the resulting relations would be extremely complex, necessitating a computer solution. Taranath[5] has done this for a single belt truss as well as for a two belt truss system. He concerned himself only with the belt truss locations and did not explore the influence of the parameter α, the drift reduction factor. For comparison purposes, the curve from Fig. 5 of Ref. 5 labelled "Single Belt Truss" has been replotted in Fig. 6 of this paper, along with one of the curves ($\alpha = 0.75$) obtained on the basis of the simplified structure. The choice of the value $\alpha = 0.75$ was made such that the two curves would coincide at $\gamma = 0$ (i.e., the simplified structure and the real structure would have the same deflections if the belt trusses were placed at the top). It is evident that, with the exception of the two end points, the two curves are very different and the reason for the differences is found in the basic assumptions made in deriving the two curves. However, the essential point being made here is the fact that, whether one is dealing with an ideal or a real structure, the shape of the R vs. γ curve as well as the minimum drift depends upon the drift reduction factor α.

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APPENDIX A

NOMENCLATURE

A	Cross-sectional area of perimeter columns
α	Drift reduction factor
β	Core rotation at level of belt truss
$2b$	Perimeter column spacing
C	A function of γ which is constant for a given value of γ
Δ	Axial deformation of perimeter columns
E	Modulus of elasticity of steel
E_{CR}	Core modulus of elasticity
E_{CL}	Perimeter columns moduli of elasticity
γ	Dimensionless coordinate $= x/l$
I	Moment of inertia of the core
K_x	Equivalent spring constant for perimeter columns
l	Total height of the high-rise structure
M_x	Moment representing the outrigger and columns restraint
P	Axial force in one half of the perimeter columns
R	Dimensionless deflection measured at the top of the high rise structure
W	Uniform wind loading applied to the structure
x	Coordinate which locates the belt truss with respect to the top of the structure
y	Deflection measured at the top of the high-rise structure