Seismic Drift Control Criteria

EDWARD J. TEAL

NEED FOR SEISMIC DRIFT CONTROL

ALL BUILDINGS which may be subjected to earthquake forces need drift control to insure structural integrity, and to minimize non-structural damage. Adequate drift control may be automatically obtained by a code stress design, but more often it is not. Even shear walls and braced frames may be too flexible if they are tall and narrow, especially when they are subjected to forces large enough to cause them to yield in flexure or to rock on their foundations. Stress generally will *not* govern the design of ductile moment frames, since they are expected to yield under strong seismic forces.

Threats to the structural integrity of ductile moment frames will generally not come from excessive axial forces due to frame rocking moment, but might come from excessive member or joint distortions or $P\Delta$ moment due to excessive frame flexibility. Controlling drift to a limit of 2 or 3% for probable maximum seismic forces would generally assure controlled ductile yielding, since this would limit member hinge and joint rotation to about one degree. $P\Delta$ moments would be limited to about 10% of the forces causing the drift, even for tall buildings, whose seismic response forces are generally quite low.

The safety hazard with regard to non-structural damage is generally concerned with the securing of non-structural elements to the frame so that they do not fall. To keep non-structural elements from falling, their connections to the structural frame must accommodate the difference between the distortion of the non-structural element and the frame distortion. This is made easier if the frame distortion is tightly controlled, but the connections can be made to accommodate drifts of at least 2 or 3%. Stricter control of drift to minimize interruption of service and to control economic loss is another matter. For this purpose it is desirable to control the drift as tightly as economics and other practical considerations allow.

Edward J. Teal is Director of Structural Engineering, Albert C. Martin and Associates, Los Angeles, Calif.

DIFFICULTIES IN SETTING SEISMIC DRIFT CRITERIA

Criteria for drift have been difficult to establish, parcially because of lack of knowledge about the specific tonsequences of drift, but much more because of the wide range of opinions about the possible input ground motions and the building response to these ground motions. At the present time any fixed drift limit which applies regardless of the input motion will result in widely different flexibility control for different buildings, due to widely different motion predictions for them. At the same time, the practical economic limits of flexibility control for a particular type of building frame are fairly narrow and constant. In particular, economically practical flexibility control of moment-frame buildings falls into a fairly narrow band. It is not economically prudent to design above or below that range.

Since drift is the product of the two variables, building flexibility and seismic force, and since building flexibility is determinate and predictable, whereas seismic lateral force is not, we need to separate the two variables. We need to evaluate buildings on the basis of their flexibility, a determinate building property independent of specific lateral force predictions. Since we are concerned with dynamic forces which depend on flexibility *and* weight, however, it is the *dynamic* flexibility evaluation which is needed. An index for this property is therefore suggested.

THE DYNAMIC FLEXIBILITY INDEX

The Dynamic Flexibility Index is defined as the average building drift caused by a lateral force equal to the total weight of the building, where the average building drift is defined as the lateral roof displacement divided by the building height. Since, again, this is a *dynamic* flexibility, the total lateral force must be distributed throughout the height of the building in direct proportion to the variation of dynamic response acceleration. The acceleration, relative to the base, increases approximately uniformly with height for most buildings, which leads to a triangular lateral force distribution, with the maximum lateral force at the top. This is the well known code static distribution, confirmed by many dynamic time-history computer response computations. A more accurate distribution is seldom justified, at least for the purpose of obtaining the Dynamic Flexibility Index.

Figure 1 illustrates the dimensions and symbols for drift and dynamic flexibility. This is a basic diagram for drift. It could represent a single frame or a single story, but it is intended here to represent the building as a whole. Therefore W, in this context, represents the total weight of the building, and the seismic lateral force due to the building's dynamic response to a given base motion is proportional to the dead weight of the building and the response coefficient C. A horizontal force equal to the shear coefficient C times the weight W causes a horizontal displacement Δ_H at a height H. The ratio of Δ_H to the height H is the drift coefficient. It is also the tangent of the angle θ, and, since the angle is small, it is equal to the drift angle θ, itself, in radians. Thus a 1% drift represents a drift angle of $0.01 \times 57°$, or about ½ a degree.

The drift coefficient for a given lateral force coefficient C is therefore represented by the equation

$$\theta = \Delta_H/H$$

When the displacement is computed for a horizontal force equal to the total weight of the building ($C = 1.0$), the lateral force coefficient variable is normalized. The ratio of this displacement to the height H provides a standard measure of the dynamic flexibility of a building. The displacement for a lateral force coefficient C equal to 1.0 can also be expressed as Δ_H over the coefficient used to determine Δ_H. Thus, the Dynamic Flexibility Index, represented by α, is given by the equation:

$$\alpha = \frac{\Delta_H/C}{H} \text{ or } \alpha = \frac{\theta_H}{C}$$

This concept is simple, but basic. Buildings have a fixed dynamic property by which they can be compared, independent of ground motion predictions. This property

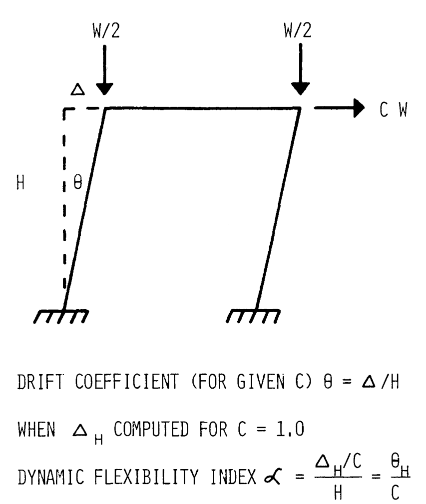

Figure 1

determines their approximate fundamental period of vibration, which determines their response to any given ground motion, which determines the distortion which would result from that response. All of these factors are linked together and stem from the *dynamic* flexibility, which is measured by the building's flexibility *and weight.*

FUNDAMENTAL PERIOD DETERMINATION USING THE DYNAMIC FLEXIBILITY INDEX

As stated above, the fundamental period of vibration is approximately determined by the dynamic flexibility. The fundamental period T is equal to a constant times the square root of the mass times the flexibility. If the period constant is represented by C_T, the mass is given by W/g, and the flexibility is given by the displacement over the force causing the displacement,

$$T = C_T \sqrt{\frac{W}{g} \cdot \frac{\Delta}{F}}$$

If the gravity acceleration g is moved into the period constant and the force is represented by CW,

$$T = C_T \sqrt{\frac{W}{CW} \cdot \Delta}$$

$$= C_T \sqrt{\Delta/C}$$

Since the Dynamic Flexibility Index is given by

$$\alpha = \frac{\Delta_H/C}{H}$$

then

$$T = C_T \sqrt{\alpha H}$$

Unfortunately the quantity C_T is a constant only for a single degree of freedom system. For multi-degree of freedom (MDF) systems, C_T varies with the deflected shape, and period calculations become complicated. The computations are so tedious that they are usually performed by a computer and this creates a mystique about MDF periods which leaves many engineers without any feel for estimating periods. A good estimate of building periods is all that is needed for seismic design, considering the accuracy of ground motion prediction and the rest of our assumptions. We can generally estimate periods with sufficient accuracy for design by considering the building as a simple cantilever member.

For a prismatic cantilever with the weight (lateral force) uniformly distributed and considering:

Bending only: $C_T = 0.26$

Shear only: $C_T = 0.29$

For a prismatic cantilever with the weight (lateral force) concentrated at the top considering:

Bending only: $C_T = 0.32$

If the building flexibility is varied to provide uniform drift regardless of lateral force distribution and drift component (bending or shear):

$$C_T = 0.23$$

It can be seen that the value of C_T is reasonably predictable. For the average building, a value of 0.25 can be assumed, yielding the formula:

$$T = 0.25\sqrt{\Delta/C} \quad (\Delta \text{ in inches})$$

The period can be expressed in terms of the Dynamic Flexibility Index (α) and the building height (H). For convenience, H is given in feet and the square root of 12 is moved into the constant. Then:

$$T = 0.89\sqrt{\alpha H}$$

THE SEISMIC DRIFT INDEX

Vibrating accelerations, and therefore forces, depend on the period of vibration, given by $T = 0.89\sqrt{H\alpha}$. Therefore, drift control for buildings of different height cannot be equated on the basis of the Dynamic Flexibility Index alone. An index which includes the influence of α *and* H on period, and therefore on lateral forces, is needed. For equal drift control independent of building height, the index must increase the dynamic flexibility with height at the same rate that the lateral force reduces with height. We therefore need to examine the variation of lateral force in terms of period.

Seismic lateral force is usually given in the form $C = C_{(T=1)}/T^n$, where $C_{(T=1)}$ represents the intensity of ground motion measured by $2\pi/g$ times the spectral velocity of a system with a fundamental period equal to 1.0. If the envelope of velocity response spectra for probable ground motions is represented by a constant velocity for a range of periods, the formula becomes $C = C_{(T=1)}/T$ for SDF systems, and $C = C_{(T=1)}/T^{3/4}$ for MDF systems of uniform drift and weight.

Buildings do not have exactly uniform drift, the spectral velocity is never constant over even the middle range of periods, and responses are not completely elastic. Therefore there is not agreement about the exponent for T. However, the spread generally falls between $T^{1/2}$ and $T^{3/4}$. Lack of agreement about the base motion intensity factor $C_{(T=1)}$ does not affect the index.

It can be seen that a Seismic Drift Index in the form $Di = \alpha/H^n$ will equate drift control independent of building height. If a compromise force formula $C = C_{(T=1)}/T^{2/3}$ is assumed, the Seismic Drift Index for equal drift control becomes $Di = \alpha/H^{1/2}$. This can be checked by noting:

If α is proportional to $H^{1/2}$:

T becomes a function of $\sqrt{H^{1/2} \cdot H} = H^{3/4}$

C becomes a function of $1/H^{3/4 \cdot 2/3} = 1/H^{1/2}$

For $\theta = \alpha C$, the H factor cancels out.

However, since it is easier to control drift with height, a Seismic Drift Index which reduces drift a little with height seems practical and more reasonable. The recommended Seismic Drift Index is therefore $Di = \alpha/H^{1/3}$.

TABLES SHOWING THE JUSTIFICATION AND APPLICATION OF THE INDEXES

The rest of this discussion will center around three tables which show the practical variation limits of the proposed Indexes by reference to a number of representative buildings, and two tables which show the effects of varying the Indexes for buildings of different heights. The definitions and bases for the variables, which were briefly covered in the introduction of the Indexes, will be discussed more thoroughly in reference to the tables to which they apply.

Table 1—This table shows the flexibilities and periods determined for 16 steel moment frame buildings located in the Los Angeles area. The buildings were analyzed by a computer program which included all components of frame drift flexibility, i.e., column and girder bending and shear, joint panel zone bending and shear, and column axial deformations. The computer model included all structural frame elements, with no allowance for non-structural elements. Neglect of non-structural elements for strong motion analysis has been validated by the San Fernando Earthquake accelerograph readings for several of the buildings. Time-history dynamic analyses for a number of earthquake ground motions were run on each building, and the flexibilities were obtained as averages from the lateral forces and the displacements computed for those ground motions. The buildings were designed by a number of different engineering offices and therefore represent a broad range of design practice.

In Column 2, H is the effective building height, i.e., the height from the building base level, where the building is restrained against significant displacement, to the roof level. Penthouse weights are considered concentrated at the roof level. The dynamic flexibilities involve only the building weight which is included in the effective height.

The period constant C_T were computed by substituting into the formula $T = C_T\sqrt{\alpha H}$ the values computed for T and α, and the height H (in inches). It can be seen that a C_T value of 0.25 for the typical building is indeed a reasonable estimate. The range of values is not great enough to significantly influence design or evaluation considering the period accuracy warranted by the cacuracy of the other seismic response variables.

Seismic Drift Index values vary widely, from a low of 0.0049 to a high of 0.0236. However, the low Indexes are for hospitals designed to comply with the intent of the new State of California Hospital Code. This intent is

Table 1. Computer Dynamic Analysis of 16 Steel Moment Frame Buildings

(1) Building Type	(2) H (ft)	(3) α	(4) T (sec)	(5) C_T	(6) Di	(7) θ_H for $C = 0.50/T^{2/3}$ (%)	(8) θ_H for $C = 0.18/T^{1/2}$ (%)	(9) θ_H for $C = 0.033/T^{1/3}$ (%)
School	29	0.046	1.0	0.25	0.015	2.30	0.82	0.15
Hospital	90	0.060	2.2	0.27	0.0132	1.80	0.73	0.15
Hospital	107	0.027	1.5	0.25	0.0056	1.05	0.40	0.08
Office	113	0.045	2.0	0.25	0.0093	1.12	0.57	0.12
Hospital	132	0.025	1.5	0.26	0.0049	0.98	0.37	0.07
Office	150	0.090	2.8	0.22	0.0170	2.32	0.97	0.21
Office	170	0.130	4.1	0.25	0.0234	2.46	1.18	0.28
Hospital	184	0.098	3.2	0.25	0.0136	1.80	0.77	0.17
Office	244	0.058	3.0	0.23	0.0092	1.33	0.60	0.13
Hospital	305	0.100	4.7	0.25	0.0150	1.80	0.83	0.20
Office	325	0.063	4.0	0.26	0.0091	1.20	0.57	0.13
Office	358	0.100	5.5	0.27	0.0140	1.75	0.76	0.19
Office	375	0.100	5.4	0.24	0.0152	1.75	0.84	0.21
Office	492	0.065	4.5	0.23	0.0082	1.23	0.55	0.13
Office	640	0.060	5.4	0.25	0.0069	0.96	0.46	0.11
Office	677	0.060	6.2	0.28	0.0068	0.90	0.44	0.11

to try to insure that the hospital stays operational through a strong earthquake ground motion. The highest Index is for an office building apparently designed without drift control in mind. From an examination of these buildings it would seem that the practical Seismic Drift Index range for steel moment frame buildings is from 0.005 to 0.025. Flexibility control below 0.005 is not very practical and a Seismic Drift Index limit of 0.025 bears no significant cost penalty. The *prudent* Seismic Drift Index limit depends on the probable ground motion intensity exposure and the consequences of excessive drift, mostly in relation to non-structural damage.

To indicate the range of drifts which would occur in these buildings if they were subjected to seismic forces, drifts are shown for three lateral force formulas.

The force level shown in Column 7 represents one version of a maximum credible seismic force. The ground motion intensity factor $C_{(T=1)} = 0.50$ represents a very strong earth-shaking. The variation of response given by $1/T^{2/3}$ represents an approximate MDF response to a ground velocity which is essentially constant for the period range involved. It may be noted that most spectrum response design assumes a constant velocity in this period range. The currently popular tripartite log spectrum plots generally assume this. The tripartite plots are SDF plots where the spectral acceleration (Sa) is equal to 2π times the spectral velocity (Sv) divided by T. Since Sa/g represents the lateral force coefficient C, then C is also given by $(2\pi/g)(Sv/T)$.

The use of these plots for MDF systems requires combining of modal responses on a modal participation basis. For an assumed deflected shape (generally constant drift) the MDF response can be approximated closely by modifying the exponent applied to the fundamental period T. The assumed value for $(2\pi/g)Sv$ is represented by the ground motion intensity factor $C_{(T=1)}$. A factor of 0.50 therefore assumes a damped Sv of about 2.5 ft/sec. This is a very brief explanation of the credible lateral force formula used to illustrate the approximate upper limit of drift represented by the Seismic Drift Index. It is not the intention here to espouse any particular force formula, but to provide some indication of the maximum drifts which must be considered. The form of the formula has been checked against many time-history dynamic computer analyses, and has proven a good approximation of response variation with T.

The force level shown in Column 8 is the latest SEAOC formula, with modifying factors of 0.67 for a moment frame and 4 for drift. Therefore,

$$C = \frac{0.67 \times 0.67 \times 4}{T^{1/2}} \simeq 0.18/T^{1/2}$$

The force level shown in Column 9 is the SEAOC formula which is still in general use, modified by a frame factor of 0.67. Obviously this formula represents a weak ground motion intensity and the variation of force with $1/T^{1/3}$ does not fit the response computed by dynamic analyses. This formula, factored for frame types and working stresses, was not intended to represent real forces. However, drift has commonly been checked for this force formula, if it was checked at all.

Table 2—This table shows the variation of the fundamental period (T) with variations in the Seismic Drift Index (Di) and the effective height (H). Given any two of the variables, the third can be readily estimated from this table. The designer can also readily picture the real meaning of the Seismic Drift Index. It must be very obvious that a building with a period over 1 sec. for a height of 25 ft or a period of 11 sec. for a height of 800 ft

Table 2. Variation in Fundamental Period T (sec) with Variations in H and Di*

Eff. Ht. H (ft)	Drift Index (Di)					
	0.0025	0.0050	0.0075	0.010	0.015	0.020
25	0.30	0.50	0.64	0.74	0.90	1.1
50	0.59	0.80	1.0	1.2	1.5	1.7
100	0.94	1.3	1.6	1.9	2.3	2.7
200	1.50	2.1	2.6	3.0	3.7	4.3
300	1.90	2.6	3.3	3.8	4.7	5.7
400	2.44	3.3	4.1	4.7	5.8	6.9
500		3.9	4.8	5.5	6.8	8.0
600		4.4	5.4	6.2	7.7	8.9
700		4.9	6.0	6.9	8.4	10.1
800		5.4	6.5	7.5	9.3	11.0

* $Di = \alpha/H^{1/3}$.

is a very flexible building. The Dynamic Flexibility Index could have been used in this table in place of the Seismic Drift Index, but the Seismic Drift Index was chosen because it has more general use.

Table 3—This table shows the variation of drift (θ_H) with variations in the Seismic Drift Index (Di) and the effective height (H). The lateral force formula $C = 0.50/T^{2/3}$ is the formula used in Column 7 of Table 1 and covered in the discussion for that table. Obviously the drifts can be directly factored for different base motion intensity factors ($C_{(T=1)}$). It can be seen that the drifts computed for this force formula and the Seismic Drift Index formula decrease with increase in building height. If the same Seismic Drift Index formula but a force formula in which C varies inversely with $T^{1/2}$ (instead of $T^{1/3}$) were used, the drift would not change with building height.

The drifts shown in the table are *average* drifts for the building heights shown. That is, they are computed on the basis of roof displacement divided by effective building height, and they imply uniform drift if the word "average" is not considered. It should be recognized that story drifts will not be uniform at any instant of time during an earthquake, and even the envelope of maximum drifts will not show uniform story drift maximums. The term "average building drift" assumes that some story drifts will be exceeding the average, while others are less than the average. The variation in story drifts cannot be anticipated by any design to any given lateral force distribution, since each earthquake generates random variable motions with their own constantly changing unique combinations of modal responses. Elastic time-history dynamic computer analyses have been run on many buildings for many ground motion inputs. A few inelastic analyses have been run for simulated ground motions factored up to force extensive inelastic yielding. These analyses indicated that some individual maximum story drifts exceeded the maximum average building drift by a factor as great as 2. These buildings were not entirely regular, but they contained no strong dynamic irregularities. All of this indicates another variable which must be considered, but which has reasonably predictable upper limits if the building has no excessively flexible stories.

Table 3. Variation in Average Drift, θ_H (%) for $C = 0.50/T^{2/3}$, with Variations in H and Di*

H(ft)	0.0025	0.0050	0.0075	0.010	0.015	0.020
25	0.69	1.1	1.4	1.7	2.2	2.8
50	0.64	1.0	1.4	1.7	2.1	2.6
100	0.62	1.0	1.3	1.6	2.0	2.3
200	0.57	0.9	1.1	1.4	1.8	2.2
300	0.55	0.9	1.1	1.3	1.8	2.1
400	0.52	0.8	1.0	1.3	1.7	2.1
500		0.8	1.0	1.2	1.7	2.0
600		0.8	1.0	1.2	1.6	1.9
700		0.8	1.0	1.2	1.6	1.9
800		0.8	1.0	1.2	1.5	1.9

* $Di = \alpha/H^{1/3}$.

Table 4—Table 4 shows the computed *elastic* Dynamic Flexibility and Seismic Drift Indexes for a number of buildings with seismic frames other than steel moment

Table 4

Concrete Moment Frame Buildings				
Building	H (ft)	α	Di	θ_H for $C = 0.18/T^{1/2}$ (%)
1	66	0.016	0.004	0.30
2	125	0.020	0.004	0.30
3	124	0.006	0.0012	0.13
4	143	0.011	0.0021	0.18
5	180	0.031	0.005	0.40
6	200	0.016	0.0027	0.24
Concrete Shear Wall Buildings				
Building	H (ft)	α	Di	θ_H for $C = 0.18/T^{1/2}$ (%)
1	67	0.013	0.0035	0.27
2	84	0.010	0.0023	0.20
3	116	0.003	0.0006	0.08
4	124	0.002	0.0004	0.06
5	161	0.005	0.0010	0.10
6	224	0.022	0.0037	0.29
Steel Braced Frame Buildings				
Building	H (ft)	α	Di	θ_H for $C = 0.18/T^{1/2}$ (%)
1	114	0.014	0.0029	0.25
2	210	0.040	0.0067	0.44
3	270	0.062	0.0092	0.60

frames. The qualification "elastic" is necessary here because the flexibility of most of these buildings begins to change radically at the level of lateral force causing yielding or rocking to start. This is in contrast to the behavior of *ductile* moment frames. A number of detailed inelastic analyses of ductile steel moment frames have shown that their "effective" average flexibility did not change markedly for ground motions more than twice the intensity of that causing first yielding. In fact, the inelastic roof displacement has almost always been less than if the building had remained elastic for the same ground motion. This is, of course, the measure of their "effective" average building flexibility. Inelastic dynamic analyses have not been run on ductile concrete moment frame buildings. Ductile concrete frame elements have been tested, and simulated inelastic analyses indicate that their effective dynamic flexibility should not increase radically until inelastic distortions become large.

Most of the buildings shown here were analyzed by simplified hand calculations, rather than detailed computer analyses. However, the Indexes obtained are believed to be accurate enough for the comparative use intended. The buildings are numbered for reference in the following discussion.

Concrete Moment Frame Buildings—Buildings listed as 1 through 4 are in California, Building 5 is in Managua, and Building 6 is in New Zealand. Buildings 1, 2, and 5 have non-ductile frames, and Buildings 3, 4, and 6 have ductile frames. The very low Seismic Drift Index of Building 3 is the result of 1 ft-6 in. x 6 ft-0 in. exterior columns at 27 ft o.c. for an 82-ft wide building. The spandrels are 1 ft-6 in. x 4 ft-0 in. and the story height is 12 ft. Shear walls resist seismic forces on the other axis. This is therefore, an example of about the lowest frame Seismic Drift Index to be expected. Buildings 4 and 6 are rather typical concrete ductile moment frames.

The Dynamic Flexibility Indexes for the non-ductile concrete frame buildings will increase rapidly after yield capacity is reached, due to the fact that the frame will begin to lose strength. The building's fundamental period can be expected to increase proportionally as the ground motion intensity increases above yield intensity. The Dynamic Flexibility Index can therefore be expected to increase roughly with the square of the increase of ground motion intensity above that at frame yielding. Of course the frame would lose all drift control with any serious deterioration of strength.

It can be seen that concrete frames generally control drift well up to yield capacity, but non-ductile frames will rapidly lose drift control for forces greater than those causing yield.

Concrete Shear Wall Buildings—Buildings 1 and 6 are in Managua. The other buildings listed are in California. It can be seen that the two Managua Shear Wall buildings are not materially less flexible than the Managua Concrete Frame building. This comparison is an important use of the Dynamic Flexibility Index. Shear wall Building 6 was little damaged by the recent earthquake, while the adjacent Concrete Frame building (5) was heavily damaged. Without a standard flexibility index, it has been widely assumed that the difference in performance was largely due to the difference between the flexibility of a shear wall building and a moment frame building. This then leads to overconfidence in the drift control of a shear wall building, per se, as opposed to the drift control of a moment frame, per se. The real reasons for the difference in performance of these two buildings are readily seen if the examination is not stopped because of an assumption about flexibility. It is important to be able to compare earthquake performance on the basis of a standard index.

It is also noted that Shear Wall Building 2 has a Dynamic Flexibility Index similar to Concrete Moment Frame Buildings 4 and 6. This emphasizes the danger in being too quick to generalize regarding the potential for drift control of different framing systems.

The Dynamic Flexibility Indexes shown for all Shear Wall buildings applies only until the shear walls start to rock. A wall will start to rock when the resistance to rocking is exceeded. When a shear wall starts to rock, it will start to lean on the building frame and the building flexibility will start to progress from that of the shear wall to that of the frame. The drift control will start to depend on the flexibility and strength of the frame. The only other control is the balance between kinetic energy input and the potential energy involved in raising the c.g. of the wall due to rocking. This potential energy control is reasonably effective at limiting drift only for low velocity motions. Since the energy is related to the velocity squared, it takes a large rotation to balance high velocities.

It might be worthwhile to stop and discuss the term "rocking moment" vs. the term "overturning moment." The term "overturning moment," which is standard, presents problems in dealing with laymen and the profession. Laymen are startled by the specter of the whole building overturning. The profession knows that buildings of any size never overturn, and elements of buildings seldom overturn. There is a tendency on the part of engineers to depreciate overturning forces on the basis that buildings have not been known to overturn. The term "rocking moment" might dispel some of the alarm on the part of laymen, and more engineers might agree that elements such as shear walls or braced frames will rock when the resistance to rocking is exceeded. If the walls are short enough and lightly loaded they might even overturn, as did the shear elements of the Four Seasons apartment building in Anchorage, Alaska. If their support fails, they may overturn as did the stair towers of the Olive View Hospital in San Fernando. Whole buildings have overturned, but because of complete loss of foundation support due to liquification, as at Nigata, Japan. But mostly they will just rock.

Steel Braced Frames—The comments made for shear wall systems apply generally to steel braced frames as well. The three buildings listed have quite different flexibilities, as shown, and they have different rocking moment resistance capacities. Building 1 is a hospital building designed to the new State of California Hospital Code on the basis of a site evaluation which required a design lateral force C of 0.45. The 48-ft wide braced bays have 8-ft diameter deep belled caissons designed for a rocking moment axial force of about 6000 kips. Obviously this building's Index represents almost a lower limit of flexibility for a braced frame. The flexibility of Building 1 is designed to remain constant over the full range of probable earthquake shaking.

The braced frames of Building 2 are used in combination with some moment frames. The braced frames have rocking moment resistance for a lateral force C of about 0.08. At this force level the braced frames will start to rock and lean on the moment frames. After rocking starts, the Dynamic Flexibility Index depends mostly on the flexibility of the moment frames.

Building 3 has no moment frames to back up the braced frames. Control of flexibility after rocking of these frames starts (at $C \simeq 0.04$) will depend only on the energy balance between the velocity energy and the potential energy change involved in rocking.

The very definite possibility of seismic forces greatly exceeding code required design forces should be considered in regard to drift as well as strength.

Table 5—Three buildings subjected to strong earthquake ground motions during the 1971 San Fernando earthquake were analyzed for their change in flexibility when their elastic capacity was exceeded. The first two buildings had accelerographs which recorded the building motion at three levels. The change in period and flexibility was determined from the accelerograph recordings. The change in flexibility was also computed by applying the inelastic considerations noted in the discussion of Table 4, and good correlation with the instrument records was obtained.

Concrete Moment Frame Building—The non-ductile concrete frame building has a yield capacity lateral force coefficient which indicates an elastic response up to a ground motion intensity factor of $C_y \simeq 0.18$ ($T = 1$). The 10% damped response spectrum for the accelerograph record indicates that the building was subjected to a lateral force given by a ground motion intensity factor of $C_{SF} \simeq 0.26$ ($T = 1$). Period measurements from the instrument record indicate three stages of building response.

Table 5

Concrete Frame Building (Effective Height 66′)

	T (sec)	α	Di	C_y ($T = 1$)	C_{SF} ($T = 1$)
Elastic	0.90	0.016	0.004	0.18	—
Inelastic	1.5	0.036	0.009	—	0.26

Shear Wall Building (Effective Height 161′)

	T (sec)	α	Di	C_y ($T = 1$)	C_{SF} ($T = 1$)
Elastic	0.8	0.005	0.0010	0.06	—
Rocking	1.0	0.008	0.0015	—	0.12
Frame	2.5	0.054	0.010	—	—

Shear Wall Building (Effective Height 84′)

	T (sec)	α	Di	C_y ($T = 1$)	C_{SF} ($T = 1$)
Elastic	0.8	0.010	0.0023	0.16	—
Rocking	1.25	0.025	0.0055	—	0.26
Frame	2.5	0.054	0.012	—	—

The first short section of the record shows the effect of non-structural elements in reducing the period to below the elastic period of the frame. The next section of the record shows the building vibrating at the frame elastic period. The third section of the records shows an increase in period from $T = 0.90$ to $T = 1.5$. This change in period is approximately equal to the ratio between the maximum ground motion intensity and the ground motion intensity causing frame yield. Relative roof displacement, as obtained from a comparison of the integration of the ground and roof acceleration records, should show an increase approximately equal to the square of the period change. It did. The result is shown as a change in the Seismic Drift Index from 0.004 to 0.009. For ground intensities greater than that of the San Fernando earthquake, the drift control would deteriorate rapidly, seriously threatening the structural integrity, as well as causing serious non-structural damage.

161-ft-high Shear Wall Building—This building was also instrumented, and recorded three stages of vibration during the San Fernando earthquake. The first short section again showed effects of non-structural damping. The second section shows a period which is close to that derived analytically for the elastic dynamic flexibility of the shear walls. A third stage shows the period increasing to about 25% greater than the elastic period. This, then, increases the Dynamic Flexibility Index by about $(1.25)^2$. The change in flexibility can be traced analytically by deriving the elastic periods for the wall and for the frame (acting, not design), and determining ground motion intensity factors when the wall starts to rock and at the maximum ground-shaking. Since the rocking resistance of the wall determines the maximum moment force it can take, while increases in building lateral shear force will still be taken almost entirely by the wall near its base, the distribution of shear up the wall must be the variable. The frame will start to pick up a portion of the lateral force in the upper part of the building. The building flexibility and period will gradually move from that of the shear wall to that of the frame as more of the lateral force shear is dumped off to the frame due to rocking of the wall. It can be seen that the building flexibility was still low at maximum ground-shaking, but the maximum ground motion intensity factor was also low. If the maximum ground motion intensity factor was as great as the other two buildings, the Seismic Drift Index would have been high for the non-ductile, low lateral strength frame and serious damage may have occurred.

84-ft-high Shear Wall Building—There were no accelerographs in this building, so that the dynamic performance of the building during the San Fernando earthquake is not known in detail. However, the physical performance correlates with an inelastic analysis similar to that made for the instrumented shear wall building. The important point is that the performance for greater ground motion intensity factors than that to which the building has been subjected can be estimated on the basis of projected distortions for the greater motions.

FLEXIBILITY CALCULATIONS

The dynamic flexibility should be determined for all buildings which may be subjected to strong earthquake forces. An elastic Dynamic Flexibility Index is all that is needed for Ductile Moment Frame buildings. Shear Wall and Braced Frame buildings generally have two stages of flexibility, that before rocking moment resistance capacity is reached and that after the wall or braced frame starts to rock. The upper level of lateral force for which the flexibility is determined should be based on the response to real possible ground motion intensities, not code factored-down static forces.

For Ductile Moment Frames, the following drift components need to be considered in the flexibility calculations:

1. Girder and column bending
2. Girder and column shear
3. Joint panel zone bending and shear
4. Building chord drift due to rocking moment axial forces

Generally, consideration of girder and column bending on a frame center line basis will reasonably approximate the drift obtained using clear span bending plus panel zone contributions. For unusual member length-to-depth proportions, this should be checked. Building chord drift is not usually significant for frame aspect ratios less than 1 or 2. For preliminary design calculations and for simple building flexibility checks, the flexibility can often be determined with sufficient accuracy using portal shear distribution assumptions and checking a single typical bay at a few stories. Simplified chord drift calculations should be made if the frame aspect ratio is greater than 1 or 2.

For buildings with shear walls or braced frames, the elastic flexibility and the rocking moment resistance of the walls or braced frames should be determined. In addition, the flexibility of the frames that will be distorted if the wall or braced frame starts to rock should be determined. Shear transfer to the frame at the maximum credible ground motion intensity should then be estimated on a rational, simple, if not a detailed, basis. Finally, both the capacity of the frame to resist the shear transferred to it and the Dynamic Flexibility Index at this most critical stage should be computed.

STORY DRIFT CONTROL VS. AVERAGE DRIFT CONTROL

The Dynamic Flexibility Index, as noted, represents the average dynamic flexibility for the full building height. To control building dynamic response, as well as to control local damage, the excess of any story dynamic flexibility over the average flexibility needs limiting. As for the computation of the average flexibility, the assumed distribution of seismic forces up through the building should generally be on a static triangular basis, with the maximum force factor at the roof. Local story dynamic flexibilities computed for this force distribution should probably be held to not more than 25% above the average building dynamic flexibility. It should be recognized, however, that this degree of uniformity in dynamic flexibility does not insure an equal uniformity in drift from actual earthquake ground motion response. The unique combining of modal responses for some earthquakes may cause local story drifts to exceed the average building drift by a factor on the order of 2, even with the building uniformity suggested. This should be considered when evaluating drift criteria.

CONCLUSIONS

Because building drift coefficients have been locked in with building dynamic response to many different earthquake ground motions and many opinions about probable, credible, and possible ground motions, this very important index of building seismic performance has been difficult to evaluate and standardize. It is suggested that the known part of the drift problem, the building flexibility and weight, be separated out into a Dynamic Flexibility Index. In order to compare buildings of different heights in relation to their dynamic response, which also varies with height, a Seismic Drift Index is suggested.

A number of representative buildings have been analyzed for these indexes. These buildings show some indication of the practical index limits and set the stage for standardizing flexibility evaluation and criteria. Continued use of standardized evaluation indexes should provide a clearer picture of the practical limits involved and a ready means of evaluating performance on an equal comparative basis.

Guide criteria can be established in relation to building type, height, and occupancy. This may indicate that for certain building heights and occupancies, only certain types of structural systems will be practical. This is a better way of dictating systems than any generalized or arbitrary criteria.

With standardized drift evaluation related to practical limits, researchers can be provided with testing distortion limits. The areas of concern can be limited so that the significant aspects of the earthquake problem can receive more attention.

Advances in seismic design are seriously hampered by the lack of limits to the definition of the problem, often because too much emphasis is placed on academic accuracy in an area where the forces are so unpredictable that limits have more meaning than exactness. It is hoped that the indexing system proposed here will lead to better evaluation of seismic performance and thence to better seismic solutions.

APPENDIX A

SEISMIC DESIGN TERMINOLOGY

The key to understanding any subject is the knowledge of the vocabulary involved. The following general definitions of the terms used in seismic design are presented as an aid to understanding seismic design. Simplicity and clarity are given precedence over exactitude in these definitions.

Building Periods—Every structure will vibrate in accordance with the laws of harmonic motion as determined by its own dynamic characteristics. The dynamic characteristics are a function of its weight and stiffness. A building's response to the motion of its base is determined by those dynamic characteristics.

The period of vibration, T, is the time necessary to complete one cycle of oscillation and is the reciprocal of the *natural* frequency of vibration, f. The *natural* frequency is equal to the *circular* frequency, ω, divided by 2π. The circular frequency of a single degree of freedom structure is proportional to the square root of the stiffness divided by the mass. The equations are:

$$\omega = \sqrt{\lambda/m}, \text{ where } \lambda = \text{stiffness and } m = \text{mass}$$

$$f = \omega/2\pi$$

$$T = 1/f = 2\pi/\omega = 2\pi\sqrt{m/\lambda}$$

Expressing mass as W/g and stiffness as F/Δ (force over deflection),

$$T = 2\pi\sqrt{W\Delta/gF}$$

This is the formula for a single-degree of freedom (SDF) system. An SDF system has one lumped mass and can be represented by a single weight mounted on top of a slender vertical cantilever rod. This system will have a period of vibration dependent on the stiffness of the rod and the size of the weight on top. A series of such rods, of different height but with the same rod section and top weight size, will illustrate a spectrum of periods, because the stiffnesses will vary with the length of the rod. A structure with several lumped masses is a multi-degree of freedom (MDF) system and its vibration will be a combination of the vibrations due to the several lumped masses. There will be as many *modes* of vibration as there are lumped masses. Each *mode* has its own period and can be represented by an SDF system of the same period.

Approximate formulas for the first or fundamental mode of vibration of structures are set up in the code. For shear wall buildings the code formula is about as accurate as can be developed for this highly variable type of building. For moment frame buildings the code formula is not a good approximation for a wide range of buildings.

Response Spectra—The vibration of an SDF system due to a continuously varying base motion will, at any time, be the summation of the effects of the base motion impulses to that time. The maximum vibration reached during any length of time after the base motion starts is its spectral (maximum) value. If a series of SDF systems is subjected to the same base motion, there will be a series of maximum values related to SDF system periods, which will form a spectral curve. Thus, any given irregular motion will produce an individual response curve or response spectrum. Knowing the base motion and the SDF period, the maximum vibration can be picked off the appropriate curve, measured in terms of acceleration, velocity, or displacement.

For MDF systems this information cannot be used directly. Even when the period of vibration for each mode is determined, it is still necessary to know the relative participation of each mode toward the total vibration. Reasonable approximations of this can be determined but, since each mode vibrates independently with time, the maximums are not reached at the same time. Other approximations have to be made to account for this. The SDF spectra, though representing a great advance in seismic design, have therefore still been only a general indicator for MDF systems.

If a typical relation between the periods of the modes of vibration is assumed, based on typical building stiffness and weight distribution, the computer can run the response of all modes simultaneously, and algebraically add them at each point in time. The maximum for the algebraic sum over the entire time history of the base motion can therefore be plotted in relation to the first mode period of MDF systems. This is a true MDF spectral response for a building whose dynamic characteristics fit the assumptions noted, a very useful seismic design tool.

Damping—A perfectly elastic system, set into vibratory motion, would continue to vibrate forever if the vibration were not stopped by an outside force. However, no system is perfectly elastic, and the vibratory motion will die out due to loss of energy resulting from internal strains. This loss of energy is called damping. Damping is generally expressed as a percentage of "critical damping," the damping which would stop the vibratory motion in one swing after free vibration starts. The first small percentages of damping greatly reduce peak responses, because peak responses are generally associated with short time durations and therefore involve little energy. Damping represents energy losses from many factors and therefore can be of a number of types as related to vibration. This is a highly complex subject, but recent earthquake analyses indicate that relatively simple assumptions give good results. Most building response calculations are based on an assumed viscous damping for all modes of vibration.

Base Shear—This is the total horizontal seismic shear at the base of a structure and is a function of the acceleration of each of the masses of the structure relative to the base, the effects added algebraically at any instant. For a static design the base shear is determined as an assumed relative acceleration times the total mass of the structure.

Shear Distribution—The deflected shape of a structure for any single mode of vibration is always the same for that structure, regardless of the magnitude of the vibration. In other words, though the amplitude of the displacement changes with time, the relation between displacements throughout the height remains constant. The distribution of accelerations for a single mode of vibration therefore remains constant. If a building is assumed to vibrate in a single mode, with the deflection curve a straight line (uniform drift), the amplitude of vibration is proportional to the height. The shear will then vary linearly from zero at the base to a maximum at the top. The code triangular distribution of shear is based on this

Figure 2

general assumption. Some confusion enters here because the word shear is used to represent a force at a floor and also the sum of the forces down to that floor. The triangular loading pattern and the shape of the building shear envelope due to that loading are, of course, very different (see Fig. 2).

The triangular loading is sometimes spoken of as "throwing weight to the top." Obviously this is not true. It is a matter of assuming higher accelerations at the top. Actually, for most buildings of any significant height-to-width ratio, the acceleration varies more than linearly due to the effect of the higher modes. Statistically this can be represented simply by placing a portion of the total base shear at the roof and distributing the rest triangularly.

Rocking Moment (**Overturning Moment**)—The rocking moment is the algebraic sum of the moments of all the forces above the base multiplied by their heights above the base. If the forces are represented by an envelope of maximums reached at different times, the rocking moments will be overestimated. However, they are not greatly overestimated, since the first mode is dominant for these moments and the forces for the first mode do all reach an algebraic maximum at the same time.

It should be recognized that rocking moments are almost never a threat to overturn buildings because the transitory nature of the loading does not allow time enough for the building to move past its center of rotation. However, this type of relief is not assumed to help much in regard to the generation of axial forces in columns.

Story Drift Coefficient—The story drift coefficient is the ratio of inter-story horizontal displacement to story height, usually expressed as a percent. Thus, for a 12 ft (144 in.) story height a 1% drift coefficient is a very important quantity, becoming constantly more important as an indicator of building stiffness, and a measure of hinge rotation demand and $P\Delta$ effects.

Ductility—Present measures of ductility are confused. Energy capacity demands of earthquakes are measured in terms of force times displacement, leading to a ductility measure defined as the ratio of yield level horizontal displacement to the displacement required to meet a given energy demand. A more significant measure, as far as member and joint performance, is the ratio between the yield point strain for each of those elements and the maximum demand strain. If the element (joint or member) which develops a plastic hinge contributes only 20% of the horizontal drift, then a ductility factor of 2 by the first definition requires a ductility factor of 10 by the second definition.

It seems that the significant factor in determining the inelastic capacity limit of a member or joint is the *actual rotation demand*, not the *ratio* between elastic and inelastic. From test frames we can evaluate the potential for real failure if we know the hinge rotation demand. For instance, a 1% rotation will show little distress in either a concrete or a steel member or joint. If we deduct the drift to yield from the total drift demand, we get the yield rotation demand for whatever element develops a plastic hinge. This is defined by the demand drift coefficient less the yield point drift coefficient, since all of the yield drift results from joint rotation. The drift coefficient measures the joint rotation in radians because it measures the tangent of a very small angle. Thus, a drift coefficient demand of 1% indicates a plastic hinge rotation demand of 0.57 degrees if the elastic portion of the drift is neglected. If the elastic portion of the drift coefficient were 0.5%, the plastic rotation demand would be only 0.29 degrees.

Drift coefficients give our best measure of the actual ductility demands since they measure strain demands which we can relate to test data.

Elastic Response—Most of our present dynamic computer building analyses are run on elastic response programs, that is, the frames are assumed to remain elastic, with stress and strain proportionality constant. These programs input a base motion and record the changing distortion of the building during the duration of the motion, and the associated building forces and motions involved. When the forces generated are greater than the elastic capacity of the structural frame, the results of an elastic analysis are not directly applicable. However, because of the complexities involved in an inelastic dynamic analysis, most computer programs so far developed for inelastic response are restricted to single frames. Comparisons of inelastic response to elastic response for these simple frames indicate that the most important factor, frame distortion, is generally predicted with reasonable accuracy by an elastic analysis, though the frame is forced past the elastic range.

Most of the measures of motion developed by the computer analysis are *relative* to the base motion. It is relative motion that is involved with vertical frames. The motion which is significant for horizontal diaphragms and bracing and for the things supported on floors or roofs is the *absolute* motion, that is, motion related only to their original stationary condition. Dynamic computer programs develop the absolute motion at all levels. Since the absolute motion at any level becomes the base motion for all of the structure above that level, this is very useful information. Spectral responses for this motion can be developed to evaluate its effect on the structure above, the floor or roof itself, and things supported on the floor or roof.

Inelastic Response—All of the above definitions pertain to, or are related to, elastic response derived by assuming a constant modulus of elasticity. While portions of a frame exceed the yield point (approximate proportionality limit), the stiffness of the frame is changing and the problem of dynamic analyses becomes much more complex. To solve even small inelastic problems, the modulus of elasticity must be assumed to change according to some simple curve. The curve may be assumed (1) *elastoplastic*, i.e., the modulus suddenly becomes zero when the stress is above yield stress, (2) *bilinear*, i.e., the modulus suddenly reduces to a small quantity when the stress is above yield stress, or (3) a *Ramberg Osgood* function, i.e., the modulus gradually falls off near and above the yield stress.

The Ramberg Osgood function closely represents the plots of load vs. strain for test members loaded gradually from zero up to yield point and into the inelastic range. This type of test loading, called "monotonic," is the only test loading applied for usual structural design data. For seismic design, the load is applied in one direction until a given strain is reached, and then the load is reversed to a strain in the opposite direction. The loading is then cycled to determine an energy loop and to determine if the energy loop is stable or deteriorating. The area under the curve (within the loop) does represent energy absorption because it represents force times distance. See Fig. 3. If the plots for reverse loading cycles follow the first cycle plot closely, the stiffness and capacity of the test specimen is not deteriorating with load reversals. If the slope of the loop decreases, the stiffness is deteriorating. If the maximum ordinate decreases, the capacity is deteriorating.

Elastic systems, and the elastic portion of inelastic systems, depend on storing the energy input to the building when the base is moved in one direction, and then releasing the energy during reverse motions. Inelastic systems absorb the energy in plastic hinge action. The

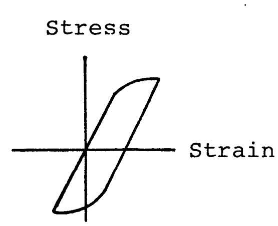

Fig. 3. Ramberg Osgood energy loop

energy loop for plastic hinges in ductile frames can provide the required energy absorption.

Many non-structural elements will also absorb energy. This is cited not only as the reason why low capacity frames survive earthquakes, but also as justification for underdesigning structural frames. It does not seem either necessary or prudent to rely on this undependable type of resistance. It seems that we can provide all of the energy absorbing capacity needed to survive earthquakes in properly designed structural frames.

***P*Δ Effects and Instability**—When a frame sways, a vertical load rocking moment develops which is given by the equation $M = P\Delta$. If this moment ever increased faster than the restoring force from the frame stiffness, instability would occur. For the vertical load problem which is the concern of low earthquake risk areas, any instability threat is involved with very flexible frames. For these frames, if the frame is not stayed against sidesway and the load is continuously increased, the frame will eventually buckle out from under the load. In other words, the P/A stresses plus the bending stress due to $P\Delta$ will reach yield, and, since the load is constant, sidesway will continue until failure. The force necessary to stay the frame against sidesway is very small, the stiffness generally being the critical factor. For vertical load stability some nominal X-bracing or shear walls will provide the stiffness and strength needed for staying. If no X-bracing or shear walls are feasible, the design axial stress is kept low on the basis of some very arbitrary column and joint stiffness factors; these factors involve a lot of design work and do not lead to rational design.

For frames designed to large lateral forces with controlled drift, the arbitrary columns strength reduction factors specified by codes for unbraced frames should not apply.

Real problems of instability due to P_Δ effects in seismic frames are few, and are connected only with large forces which would cause inelastic response. Definition of inelastic instability problems of SDF systems have now been developed in terms of time and intensity of base motion, elastic strength, and stiffness. For MDF systems simple definition is probably not possible and only an inelastic dynamic analysis will identify problems. However, there is no indication that any problem will exist for structures with relatively uniformly distributed mass and stiffness.

Earthquake Ground Motions—It is useful to define three levels of earthquake ground motion in regard to seismic design:

Probable Maximum Earthquake Motion—This represents the general level of intensity which it is believed can be associated with *significant* probability. This means that there is enough probability of this intensity of motion to require the design of a structural frame whose response to that motion will be *very* predictably adequate. Considering the risks involved, this probability does not need to be very great.

Maximum Credible Earthquake Motion—This represents the maximum motion intensity which can be predicted as credible according to presently available data and theory. A structural frame should be able to survive this motion with *reasonable* predictability. Predictability is, of course, a matter of how far we extend our predictions based on presently available dynamic analytical data and physical test analogies.

Maximum Possible Earthquake Motion—No one can say positively what maximum possibilities exist beyond our current theory and knowledge. It is assumed that frame capacity above that which is reasonably predictable will go a long way toward insuring a building's survival for motion intensities beyond those which are considered really credible.

The intensity of motion represented by all earthquake motion records obtained up to this time is, of course, less than the "maximum credible" value. Cal Tech has added to these records a series of simulated records based on theory as an aid in more fully defining credible ground motions.

Maximum Capacity—We need new definitions for maximum capacity, ultimate capacity, failure, etc. Common usage has led to thinking of these terms as a limiting condition in regard to collapse. In most cases these terms really mean a limit to some certain condition such as stress-strain linear proportionality. Safety factor working stress design is based on keeping the stress at working loads below some given safe percentage of these arbitrary limiting conditions. Design in general is working away from this somewhat irrational approach, and seismic design in particular cannot afford it. We have to establish new upper limits. Until this is done on a general basis, we will have to do the best we can toward converting working stress values to capacity limits. This is particularly difficult for bolt values, weld values, and some stresses such as shear. Our assumptions must be rational if we are to achieve rational and practical seismic designs.

We need also to consider the capacity of a frame in terms of the full capacity of all of the members which must reach full plastic yielding before the maximum lateral capacity of the frame is reached. For example, if the columns in a story are critical to story shear, the story shear capacity is not reached when one portion of one column cross section reaches yield stress. That is the point of incipient yielding, but the full frame yield capacity is not reached until a full plastic hinge forms at the top and bottom of every column in the story.