Effective Length of Columns in Gable Frames

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USUALLY, single-story, shed-type buildings utilizing gable frames are braced normal to these frames or otherwise receive lateral support so that sidesway in this direction is inhibited. On the other hand, single story gabled frames are not braced against sidesway parallel to the plane of the frames. In the design of columns, the 1963 AISC Specification requires that an effective length factor larger than unity be used when computing the slenderness ratio in a plane where the frame is not prevented from sidesway. This factor, often denoted by K, is determined by considering the sidesway buckling strength of the structure and is defined by the expression[1]

$$Kh = \pi \sqrt{\frac{EI_c}{P_{cr}}} \qquad (1)$$

in which h is the column length (Kh is therefore the effective column length), E the Young's modulus, I_c the moment of inertia and P_{cr} the magnitude of the axial force in the column when sidesway buckling occurs. The buckling load P_{cr} depends on the dimensions of the frame, the stiffness of the members, and the loading condition. Obviously, a complete buckling analysis is required in order to determine the correct values of P_{cr} or K. This is often a tedious task if the frame is subjected to loads which cause primary bending moment in the members.[2] The purpose of this short paper is to furnish K values for some typical gable frames and to illustrate the validity of an approximate method which may be used in design calculations.

Figure 1 shows a pinned-base gable frame subjected to a uniformly distributed load on the rafter. Consider first the case when the moment of inertia of the rafter and column is the same, that is $I_r = I_c$. The critical value of the applied load, W_{cr}, can be determined by the method suggested by M. G. Puwein.[3] The required effective length factor can then be found from Equation (1) by letting $P_{cr} = W_{cr}/2$. Computations have been
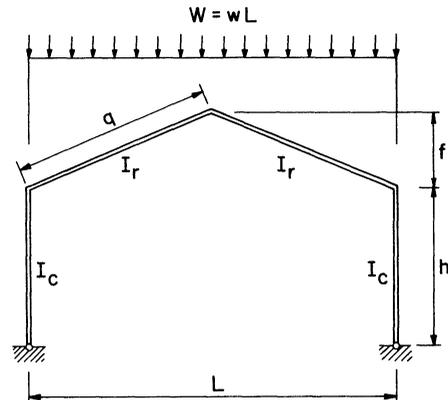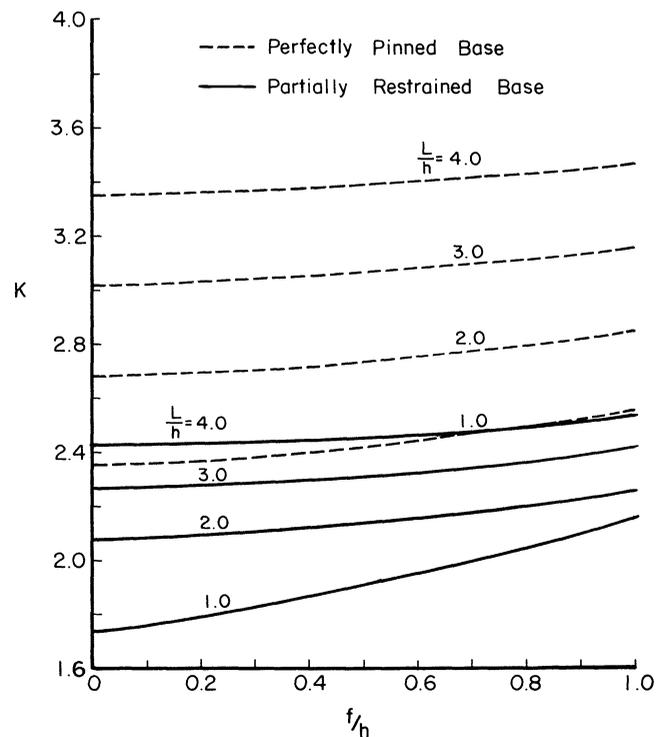
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Fig. 1. Frame dimensions and loading
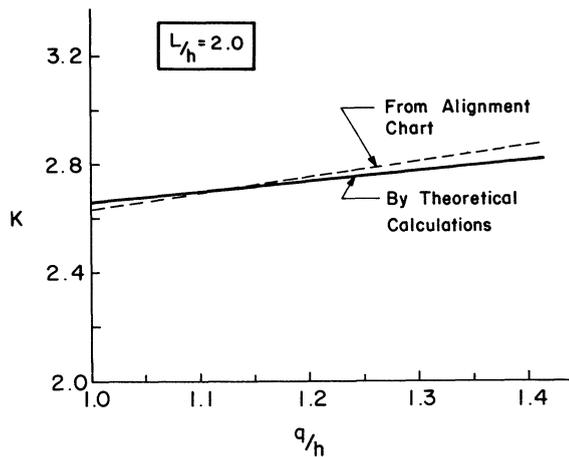

Fig. 2. Effective length factors

Figure 3

made for four L/h ratios with f/h varying from 0 to 1.0, and the resulting K values are given as the dashed lines in Fig. 2. It can be seen that for a constant column height the effective length factor increases as the span length and the height of the gable increase and that in pinned-base frames the effective column length could be as much as four times the actual length.

For practical frames, column bases are seldom pinned and sufficient friction usually develops between the footing and the foundation soil to permit the designer to assume the columns are partially restrained at the bases. The solid lines in Fig. 2 give the modified effective length factors when partial base restraint is considered. The restraining factor used is in accordance with that recommended in the Commentary on the AISC Specification,[4] i.e. $G_B = G_{bottom} = 10$.

For frames having different member sizes for the rafter and column or having L/h or f/h ratios not covered in Fig. 2, an approximate method is available for determining the K factors of the columns.[5] The method is to find an equivalent portal frame whose span length is equal to twice the rafter length, q (see Fig. 1). The effective length of the columns in this frame can be determined from the alignment chart given in the Commentary by using the following restraint factors:

$$G_{top} = \frac{I_c/h}{I_r/2q} \text{ and } G_{bottom} = \infty$$

The value of $G_{bottom} = \infty$ is used only when the column base is actually pinned. For columns supported by footings a $G_{bottom} = 10$ is recommended by the Commentary.

The validity of this method can be checked by comparing the K values obtained from the alignment chart and those given in Fig. 2. Such a comparison is shown in Fig. 3 for frames with $L/h = 2.0$. It is seen that the effective length factors determined by the approximate method are in satisfactory agreement with the theoretical values.

REFERENCES

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