Application of AISC Design Provisions for Tapered Members

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THE USE OF tapered structural members having tapered depths and/or widths was first proposed by Amirikian[1] for reasons of economy. Typically, tapered members are used in single story structures of one or more bays (Figs. 1a and 1b) and in cantilevered sections of buildings (Fig. 1c). Some low-rise structures also contain tapered members (Fig. 1d). In view of the lack of basic understanding of the behavior of tapered members, as well as design criteria, a joint WRC-CRC subcommittee was established in 1966 to coordinate research studies. In addition, a subcommittee of the MBMA Technical Committee was established in 1973 to facilitate the development of design guides for tapered members. Two previous papers[2,3] containing design recommendations were a result of the technical guidance of these subcommittees. The major thrust of these papers deal with the formulation of allowable axial stress, allowable bending stress, and combined axial-flexural compression for tapered members. These design recommendations have been incorporated into the AISC Specification as Appendix D.[4]

The philosophy behind the development of the allowable stress formulas for tapered members is to use the form of the prismatic formulas in AISC Specification Sects. 1.5.1.3, 1.5.1.4, and 1.6.1 and include a length modification factor. The appropriate factors are summarized in Ref. 2 for axial buckling and lateral buckling. They were determined based on the following approach: first, the Rayleigh-Ritz Method was used to determine the critical stress of a tapered member; then, that stress was equated to the critical stress for a prismatic member having the same cross section as the smaller end of the tapered member, but of different length. These modification factors are functions of the tapering ratios and other geometrical properties of the member. The design formulas of AISC Specification Appendix D are for linearly web-tapered members only.

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The forms of the allowable compressive stress formulas for tapered members in Sect. D2* are identical to those for AISC prismatic member design. The tapering influence is incorporated in the effective length factor. Since the formulas are intended to be used for web-tapered members, the weak-axis effective length factor can be taken the same as that for a prismatic member. For strong-axis buckling the effective length factor differs appreciably from a prismatic member. The strong-axis effective length factors for tapered members, K_γ, have been calculated for typical tapered framing situations[2] and are included in the Addenda to the Commentary on the AISC Specification (see Supplement No. 3).[4]

The allowable bending stress formulas for tapered members in Sect. D3 differ in three aspects from the AISC formulas for prismatic member design. First, the length modification factors, h_w and h_s, appear in the formulas which account for the tapering. These functions were determined in a similar way as for axial compression.[2] The adoption of these two functions was dictated by the complicated dependence of the function on the warping and St. Venant rigidities, and by the use of the double formula procedure for critical lateral buckling stress. Second, the use of the total resistance formulation[3] to determine the allowable bending stress. Third, consideration of the restraining effect of adjacent spans when a member is continuous over lateral supports. The restraining effect is included in the coefficient B.**

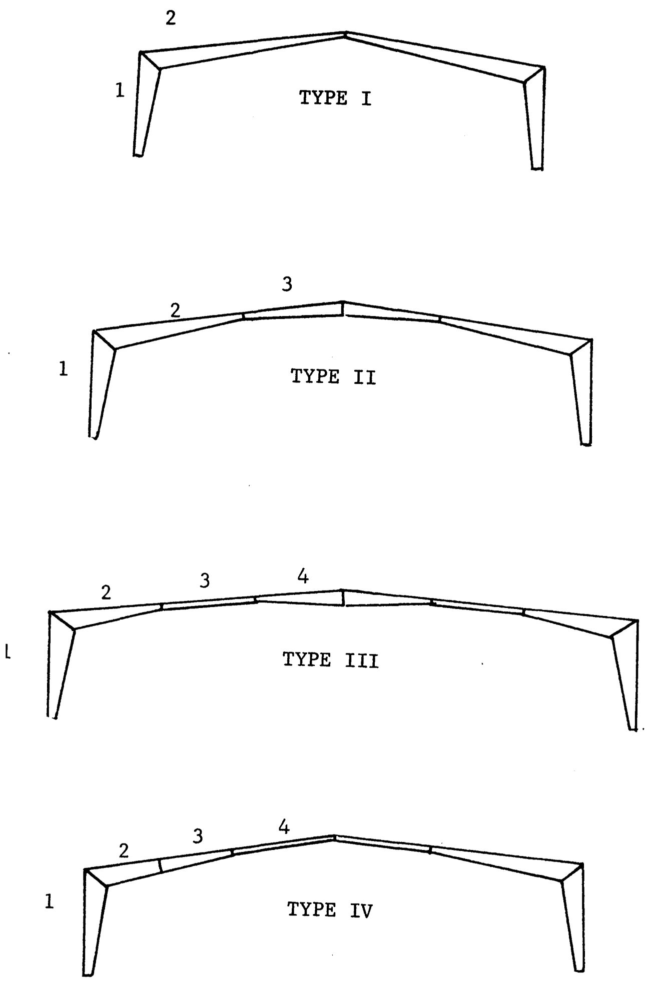
Combined axial compression and bending for tapered members is handled in the same manner as for prismatic members. This case is covered in Sect. D4.

AISC Specification design provisions applicable to tapered members can be grouped into three categories (see Table 1):

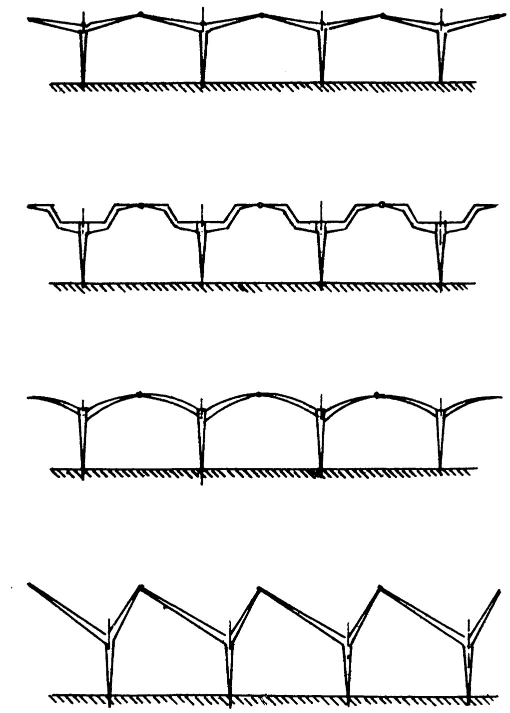
A. Prismatic member provisions that may be directly applied to tapered members, *provided maximum stresses are determined at the correct locations (usually corresponding to the smallest net cross-sectional area).*

* *Sections and equations with the prefix "D" refer to Appendix D of the AISC Specification (Supplement No. 3).*

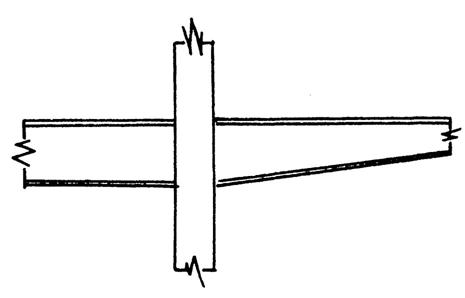
** *In Ref. 3 the restraint effect is called R_γ.*



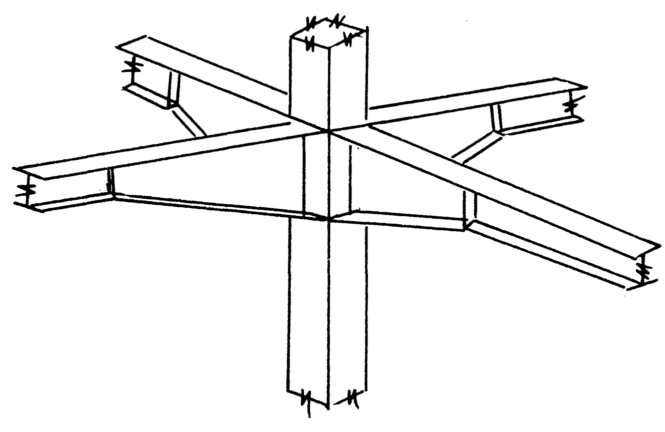
(*a*) *Multiple tapered beam gable frames*



(*b*) *Multi-bay single story tapered frames*



(*c*) *Tapered beam used in cantilevered situations*



(*d*) *Use of tapered members in low-rise frames*

Figure 1

B. Prismatic member provisions that have been modified in Appendix D for application to tapered members.

C. Prismatic member provisions that have not been adequately investigated with regard to tapered members. *Caution should be exercised in the application of these provisions to the design of tapered members.*

Table 1

	AISC Specification Section	Category
Sect. 1.5	Allowable Stresses	
1.5.1.1	Tension	A
1.5.1.2	Shear	A
1.5.1.3	Compression	B
1.5.1.4	Bending	B
1.5.1.5	Bearing	A
Sect. 1.6	Combined Stresses	
1.6.1	Axial Compression and Bending	B
1.6.2	Axial Tension and Bending	A
1.6.3	Shear and Tension	A
Sect. 1.8	Stability and Slenderness Ratios	
1.8.2	Sidesway Prevented	B
1.8.3	Sidesway Not Prevented	B
1.8.4	Maximum Ratios	C
Sect. 1.9	Width-Thickness Ratios	
1.9.1	Unstiffened Elements Under Compression	A
1.9.2	Stiffened Elements Under Compression	C*
Sect. 1.10	Plate Girders and Rolled Beams	C
Sect. 1.11	Composite Construction	C

* For web tapered elements only.

FRAME AND STRESS ANALYSIS

The in-plane analysis of certain types of single-story tapered frame assemblies has been shown to be simpler than the analysis of conventional prismatic rigid frames.[1] Advantage is taken of the fact that each tapered member in the frame (see Fig. 1a) has a pinned end condition at its smaller end and a fixed end condition at its larger end, i.e., the bending moment diagram is similar to that of a cantilevered beam. This permits deflection computations of simple frames by summing the deflections of each component member. This approach was very appealing in the early 1950's before the development of digital computational capabilities.

Other standard methods of structural analysis based on either the flexibility or stiffness approach, such as moment distribution, can be readily adapted to tapered frame assemblies (stiffness and carry-over factors for tapered beam-columns are given in the Appendix to this paper). For highly redundant frames, however, a more automated approach is desirable, and the finite element (displacement) method has distinct advantages.

For tapered member frames free from torsion, the element stiffness matrix (based on first order theory) may be found in standard matrix method books dealing with structural analysis. For problems involving warping torsion and/or second-order effects, the appropriate elemental matrices are contained in Ref. 5 with some examples. The member stiffness matrix for prismatic members can be satisfactorily used for tapered members if the tapered member is discretized by a sufficient number of elements.

The computation of stresses in tapered members is complicated by the fact that at least one new parameter must be introduced into the solution to describe the variation of cross-sectional properties along the length of the member. For members with small angles of taper, less than 15°, the normally assumed approximations of structural mechanics still can be applied, with the exception that the cross-sectional properties vary.[2] The designer must realize that, since in general both stress resultants and cross-sectional properties do vary, it is not always possible to find the points of maximum stress by inspection. By calculating the stress at several points and plotting the stress variation, the maximum stress can be determined. Also, the equation for the stress variation can be differentiated to find the maximum stress.

DESIGN EXAMPLES

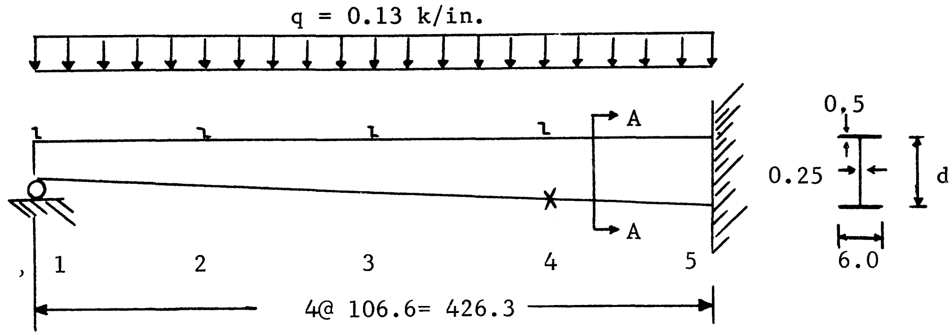
The primary intent of the paper is to give illustrations of the application of AISC Specification design provisions for tapered members, with particular emphasis on those provisions that are modified by Appendix D (category B of Table 1.) Some attention is given to provisions in Category A. Several examples are presented for this purpose, attempting to illustrate all of the options in Sects. D3 (Bending) and D4 (Combined Stresses) of Appendix D.

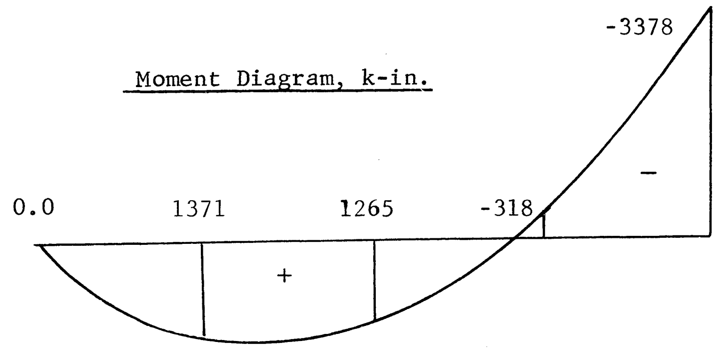
A special note should be made with regard to the lateral supports. In order to apply the design formulas, it is required that there be no lateral movement and no twisting of the cross section at the points of lateral support. Girts and purlins are used to attach siding and roofing to the frame; these supply supports only to one flange, e.g., the outside flange. Under normal situations, bracing only one flange (when in compression) will be sufficient to meet this requirement. If, however, this flange braced by a girt or purlin is in tension, then an additional support must be supplied to the other (compression) flange, indicated by an "X" in the figures.

Example 1

Given:

Determine the allowable bending stress for segment **2-3** in the tapered beam, Fig. 2, and compare it to the calculated bending stress. F_y = 42 ksi.





Sect.#	d	A	I_x	S_x	r_x	r_y	r_T
1	25.0in.	12.00in.^{2}	1188in.^{4}	95.1in.^{3}	9.95in.	1.23in.	1.50in.
2	28.5	12.88	1610	112.9	11.18	1.18	1.47
3	32.0	13.75	2109	131.8	12.39	1.15	1.45
4	35.5	14.63	2694	151.7	13.57	1.11	1.43
5	39.0	15.50	3368	172.7	14.74	1.08	1.40

Fig. 2. Design Examples 1 and 2: design of segments 2–3 and 4–5 of propped cantilever; F_y = 42 ksi, dimensions shown in inches

Solution:

For compression on extreme fibers of tapered flexural members, Formulas (D3-1) and (D3-2) apply. The following quantities are needed to calculate h_s and h_w:

$$\gamma_{2-3} = (d_3 - d_2)/d_2 = (32.0 - 28.5)/28.5 = 0.12$$

$$ld_2/A_f = 106.6 \times 28.5/(0.5 \times 6.0) = 1013$$

$$l/r_{T_2} = 106.6/1.47 = 72.30$$

$$\therefore h_s = 1.0 + (0.0230 \times 0.12 \times \sqrt{1013}) = 1.090$$

$$h_w = 1.0 + (0.00385 \times 0.12 \times \sqrt{72.30}) = 1.004$$

Next, the two terms $F_{s\gamma}$ and $F_{w\gamma}$ are computed:

$$F_{s\gamma} = 12{,}000/(1.090 \times 1013) = 10.87$$

$$F_{w\gamma} = 170{,}000/(1.004 \times 72.30)^2 = 32.26$$

Before $F_{b\gamma}$ in Formula (D3-2) can be calculated, the term B, which includes moment gradient and/or end restraint effects, needs to be determined from subparagraphs D3a, b, c, or d. Case D3a can be used, since segment **2-3** has a span of equal length on each side and the maximum moment for these three segments occurs on segment **2-3**:

M_2 = 1508 kip-in. (occurs at 152 in.)

M_1 = −318 kip-in.

$M_1/M_2 = 0$*

$$\therefore B = 1.0 + 0.37(1.0 - 0) + 0.50(0.12)(1.0 - 0)$$
$$= 1.43$$

* *See Spec. Appendix D, footnote to subparagraph D3a.*

Subsequently, from Formula (D3-2):

$$F_{b\gamma} = 1.43 \sqrt{(10.87)^2 + (32.26)^2} = 48.68 \text{ ksi}$$

However, this value is greater than $\frac{1}{3}F_y = 14.0$ ksi. Therefore, Formula (D3-1) should be used.

Formula (D3-1):

$$F_{b\gamma} = \frac{2}{3}\left[1.0 - \frac{42}{6 \times 48.68}\right] 42$$

$$= 24.0 \text{ ksi} < 0.6F_y = 25.2 \text{ ksi}$$

The maximum computed bending stress for segment **2-3** is taken as:

$$f_b = \frac{M_2}{S_{x_2}} = \frac{1508}{121} = 12.4 \text{ ksi} < F_{b\gamma}$$

$\therefore$ segment is **o.k.**

Example 2

Given:

Determine the allowable bending stress for segment **4-5** in the tapered beam, Fig. 2, and compare it to the calculated bending stress. $F_y = 42$ ksi.

Solution:

For compression on extreme fibers of tapered flexural members, Formulas (D3-1) and (D3-2) apply. The following quantities are needed to calculate h_s and h_w:

$$\gamma_{4\text{-}5}(d_5 - d_4)/d_4 = (39.0 - 35.5)/35.5 = 0.10$$

$$ld_4/A_f = 106.6 \times 35.5/(0.5 \times 6.0) = 1261$$

$$l/r_{T_4} = 106.6/1.43 = 74.8$$

$$\therefore h_s = 1.0 + (0.0230 \times 0.10 \sqrt{1261}) = 1.081$$

$$h_w = 1.0 + (0.00385 \times 0.10 \sqrt{74.8}) = 1.003$$

Next, the two terms $F_{s\gamma}$ and $F_{w\gamma}$ are computed:

$$F_{s\gamma} = 12{,}000/(1.081 \times 1261) = 8.80$$

$$F_{w\gamma} = 170{,}000/(1.003 \times 74.8)^2 = 30.20$$

Before $F_{b\gamma}$ in Formula (D3-2) can be calculated, the term B, which includes moment gradient and/or end restraint effects, needs to be determined from subparagraphs D3a, b, c, or d. Either case D3b or D3c may apply, but first the following stresses must be calculated:

$$f_{b_3} = \frac{M_3}{S_{x_3}} = \frac{1265}{131.8} = 9.60 \text{ ksi}$$

$$f_{b_5} = \frac{M_5}{S_{x_5}} = \frac{3378}{172.7} = 19.56 \text{ ksi}$$

Since the larger stress, in magnitude, occurs at the larger end, case D3b applies.*

$$B = 1.0 + 0.58\left[1.0 + \frac{9.60}{19.56}\right] - \left\{0.70 \times 0.10\left[1.0 + \frac{9.60}{19.56}\right]\right\}$$

$$= 1.76$$

Subsequently, from Formula (D3-2):

$$F_{b\gamma} = 1.76 \sqrt{(8.80)^2 + (30.20)^2} = 55.36 \text{ ksi}$$

However, this value is greater than $\frac{1}{3}F_y = 14.0$ ksi. Thus, Formula (D3-1) should be used.

Formula (D3-1):

$$F_{b\gamma} = \frac{2}{3}\left[1.0 - \frac{42}{6 \times 55.36}\right] 42$$

$$= 24.46 \text{ ksi} < 0.6F_y = 25.2 \text{ ksi}$$

The maximum computed bending stress for segment **4-5** is

$$f_{b_5} = M_5/S_5 = 19.56 < F_{b\gamma}$$

$\therefore$ segment is **o.k.**

Example 3

Given:

Determine the adequacy of the column **1-2** in Fig. 3. The column is pinned in both weak- and strong-directions at its base, and braced at the top against lateral movement. There is no intermediate bracing. The column is not prevented from swaying in the plane. The column is A36 steel ($F_y = 36$ ksi.)

Solution:

First, the following maximum stresses are computed:

Axial stress:

$$f_a = P_{12}/A_1 = 87.5/23.6 = 3.71 \text{ ksi}$$

Shear stress:

$$f_v = V_{12}/A_{v1} = 20.1/(10.9 \times 0.5) = 3.69 \text{ ksi}$$

Bending stress:

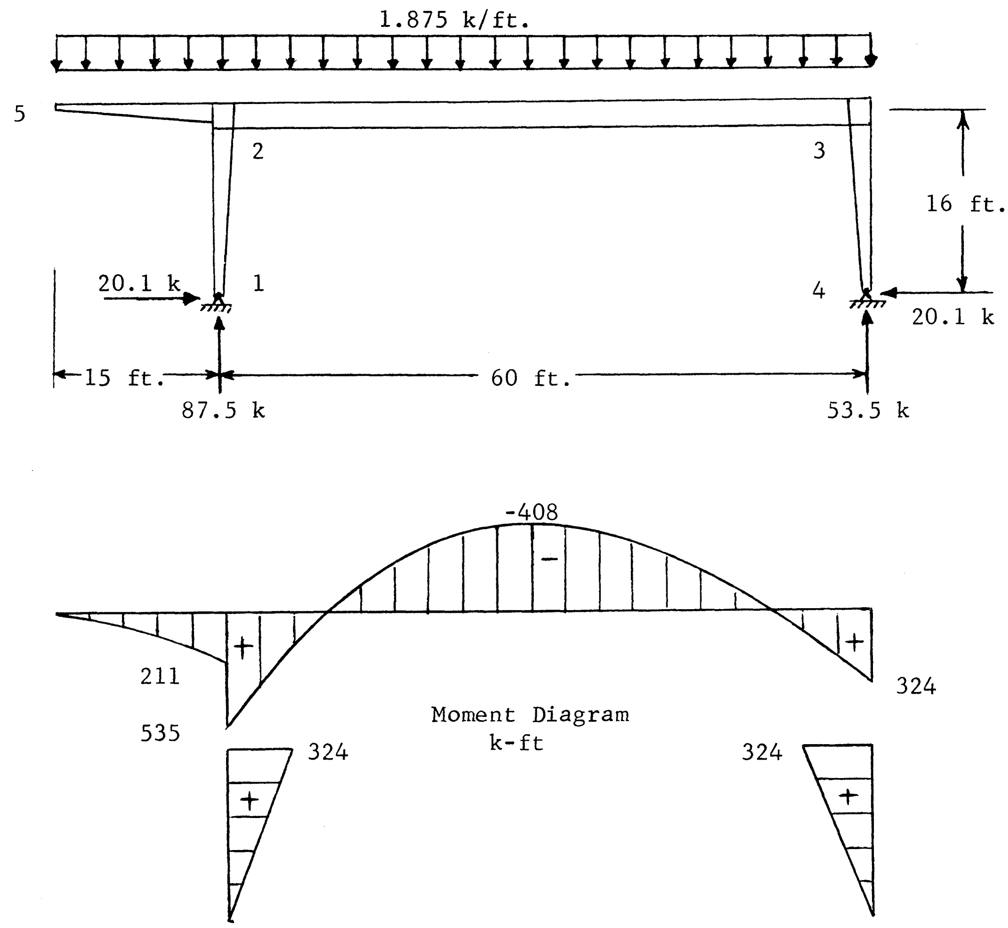
$$f_b = M_{21}/S_{x_{21}} = 324 \times 12/254 = 15.30 \text{ ksi}$$

Next, the allowable stresses are obtained:

Allowable shear stress (AISC Spec. Sect. 1.5.1.2):

$$F_v = 0.4F_y = 14.5 \text{ ksi} > f_v = 3.69 \text{ ksi} \qquad \textbf{o.k.}$$

* *The ratio for f_{b3}/f_{b5} is positive for reverse curvature.*



Member:	d_o	d_ℓ	t_w	b	t_f
1-2	12.4 in.	24.8 in.	0.50 in.	12.1 in.	0.75 in.
2-3		W 30 x 116			
3-4	12.4	24.8	0.50	12.1	0.75
2-5	8.0	24.0	0.42	7.0	0.60

Fig. 3. Design Example 3: design of member 1–2 of a frame with tapered columns; $F_y = 36$ *ksi*

Allowable compression stress (AISC Spec. Appendix D, Sect. D2):

The effective length factor for the weak axis is obtained by treating the weak direction as prismatic. For a pin ended column, $K_y = 1.0$. Figure CD1.5-11, AISC Commentary Sect. D2,[4] can be used to determine K_x.

$\gamma_{1\text{-}2} = (d_2 - d_1)/d_1 = (24.8 - 12.4)/12.4 = 1.0$

$G_T = b_T I_1/L I_T = 720 \times 671/(192 \times 4930) = 0.51$

$G_B = 10$ (AISC recommended value for pinned end)

From Fig. CD1.5-11:

$$K_x = K_\gamma = 1.35$$

Slenderness ratios for the weak and strong axes:

$$(Kl/r_0)_y = 1.0 \times 192/3.06 = 62.7$$

$$(Kl/r_0)_x = 1.35 \times 192/5.33 = 48.6$$

Thus, the weak axis governs.

$F_{a_\gamma} = 17.17$ ksi (AISC Spec. Table 1-36)

$$> f_a = 3.71 \text{ ksi}$$

Allowable bending stress (AISC Spec. Appendix D, Sect. D3):

$$\gamma_{1\text{-}2} = 1.0 \text{ (see above)}$$

$$ld_0/A_f = 192 \times 12.4/(12.1 \times 0.75) = 262.4$$

$$l/r_{T_0} = 192/3.33 = 57.64$$

$$\therefore h_s = 1.0 + (0.0230 \times 1.0\sqrt{262.4}) = 1.373$$

$$h_w = 1.0 + (0.00385 \times 1.0\sqrt{57.64}) = 1.029$$

$$F_{s_\gamma} = 12{,}000/(1.373 \times 262.4) = 33.31$$

$$F_{w_\gamma} = 170{,}000/(1.029 \times 57.64)^2 = 48.32$$

$$B = 1.75/(1.0 + 0.25\sqrt{1.0}) = 1.40$$

since there are no adjacent spans and the moment at the smaller end is zero (see subparagraph D2d).

Subsequently, from Formula (D3-2):

$$F_{b_\gamma} = 1.40\sqrt{(33.31)^2 + (48.32)^2} = 82.16 \text{ ksi}$$

However, this value is greater than $\frac{1}{3}F_y = 12$ ksi. Thus, Formula (D3-1) should be used.

$$F_{b_\gamma} = \frac{2}{3}\left[1.0 - \frac{36}{6 \times 82.16}\right]36 = 22.25 \text{ ksi} > 0.6F_y$$

$$\therefore F_{b_\gamma} = 0.6F_y = 22 \text{ ksi} > f_b = 15.30 \text{ ksi}$$

Combined axial compression and bending (AISC Spec. Appendix D, Sect. D4):

From previous calculation, $f_a/F_{a_\gamma} = 0.22$; thus, both Formulas (D4-1a) and (D4-1b) must be checked. Since bending is about the strong axis,

$$F'_{e_\gamma} = 63.2 \text{ ksi (AISC Spec. Table 2)}$$

Also, since the column is allowed to sway in its own plane,

$$C_m = 0.85$$

Finally, Formula (D4-1a) gives:

$$\frac{3.71}{17.17} + \left(\frac{0.85}{1.0 - \frac{3.71}{63.16}} \times \frac{15.30}{22}\right) = 0.84 < 1.0 \quad \textbf{o.k.}$$

and Formula (D4-1b) gives:

$$\frac{3.71}{22} + \frac{15.30}{22} = 0.86 < 1.0 \quad \textbf{o.k.}$$

Example 4

Given:

Determine the allowable bending stress for segment **8-9** in Fig. 4 (A440 steel, $F_y = 50$ ksi). For compression on the extreme fibers of tapered flexural I-shaped members, Formulas (D3-1) and (D3-2) apply.

Solution:

The following quantities are needed to calculate h_s and h_w.

$$\gamma_{8\text{-}9} = (d_8 - d_9)/d_9 = 0.17$$

$$ld_9/A_f = 63.25 \times 14.375/5 \times 0.1875 = 969.8$$

$$l/r_{T_9} = 63.25/1.22 = 51.8$$

$$\therefore h_s = 1.0 + (0.0230 \times 0.17\sqrt{969.8}) = 1.125$$

$$h_w = 1.0 + (0.00385 \times 0.17\sqrt{51.8}) = 1.005$$

Next the two terms F_{s_γ} and F_{w_γ} are computed:

$$F_{s_\gamma} = 12{,}000/(1.125 \times 969.8) = 11.00$$

$$F_{w_\gamma} = 170{,}000/(1.005 \times 51.8)^2 = 62.73$$

Before F_{b_γ} in Formula (D3-2) can be calculated, the term B, which includes moment gradient and/or end restraint effects, needs to be determined from subparagraph D3a, b, c or d. Either D3b or D3c might apply in this case, but first the following bending stresses are needed at points **7** and **9**. Case D3a is not applicable because the tapering ratio is not linear.

$$f_{b_7} = M_7/S_{x_7} = 5.68 \text{ ksi}$$

$$f_{b_9} = M_9/S_{x_9} = 20.1 \text{ ksi}$$

Since the larger stress occurs at the smaller end of a two-span segment, subparagraph D3c applies.* Then,

$$B = 1.0 + 0.55\left[1.0 - \frac{5.68}{20.1}\right] + \left\{2.2 \times 0.17 \times \left[1.0 - \frac{5.68}{20.1}\right]\right\}$$

$$= 1.66$$

Subsequently, from Formula (D3-2):

$$F_{b_\gamma} = 1.66\sqrt{11.00^2 + 62.73^2} = 105.7 \text{ ksi.}$$

However, this value is greater than $\frac{1}{3}F_y = 16.7$ ksi. Therefore, Formula (D3-1) should be used.

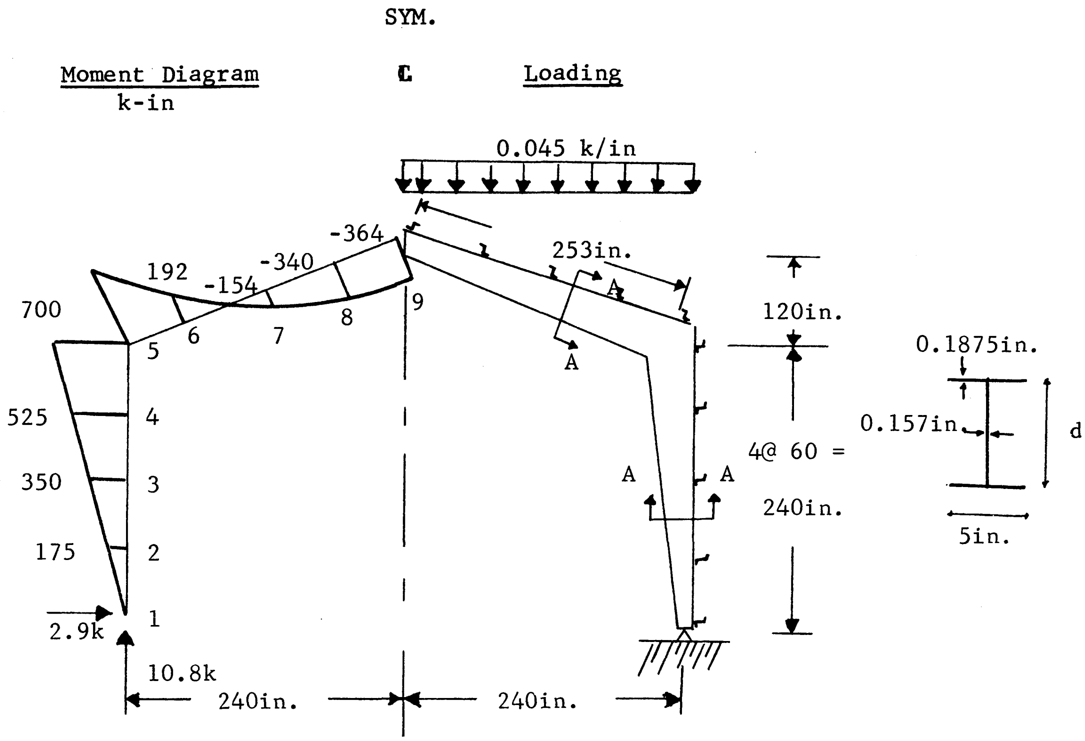
Formula (D3-1):

$$F_{b_\gamma} = \frac{2}{3}\left[1.0 - \frac{50}{6 \times 105.7}\right]50$$

$$= 30.70 \text{ ksi} > 0.6F_y$$

$$\therefore F_{b_\gamma} = 0.6F_y = 30.0 \text{ ksi} > f_{b_9} = 20.1 \text{ ksi} \quad \textbf{o.k.}$$

* *The ratio f_{b7}/f_{b9} is considered as negative when producing single curvature.*



Sect.#	d	A	I_x	S_x	r_x	r_y	r_T
Column:							
1	8.375in.	3.13in.^{2}	38.1in.^{4}	9.1in.^{3}	3.49in.	1.12in.	1.31in.
2	12.375	3.76	92.2	14.9	4.95	1.02	1.25
3	16.375	4.39	176.4	21.5	6.34	0.94	1.20
4	20.375	5.02	295.7	29.0	7.68	0.88	1.16
5	24.375	5.64	455.1	37.3	8.98	0.83	1.12
Beam:							
5	24.375	5.64	455.1	37.3	8.98	0.83	1.12
6	21.875	5.25	350.5	32.0	8.17	0.86	1.14
7	19.375	4.86	262.3	27.1	7.35	0.90	1.17
8	16.875	4.46	189.3	22.4	6.51	0.94	1.20
9	14.375	4.07	130.3	18.1	5.66	0.98	1.22

Fig. 4. Design Examples 4 and 5: design of segments 8–9 and 4–5 of a tapered member gable frame; F_y = 50 ksi

Example 5

Given:

Determine the adequacy of the column **1-5** in Fig. 4. The column is pinned at its base in both weak and strong directions and laterally braced at 5-ft intervals. The column is not prevented from sidesway. The column yield stress is 50 ksi. Each segment should be checked to find the critical one. After having done this, segment **4-5** satisfied the design criterion most closely. The adequacy of only this segment is presented.

Solution:

First, the following maximum stresses are computed:

Axial stress:

$$f_a = P_1/A_1 = 10.8/3.13$$
$$= 3.45 \text{ ksi}$$

Bending stress:

$$f_b = M_5/S_{x_5} = 700/37.3$$
$$= 18.8 \text{ ksi}$$

Next, the allowable stresses are obtained:

Allowable compression stress (AISC Spec. Appendix D, Sect. D2):

The effective length factor for the weak axis is obtained by treating the weak direction as prismatic; therefore $K_y = 1.0$. The charts in the AISC Commentary, Sect. D2,[4] can be used to determine K_x. Since the column is restrained at the top by a tapered member, an equivalent moment of inertia needs to be determined. The equivalent moment of inertia, I_e, can be defined by equating the stiffness coefficient of a prismatic beam with a pinned end $(3EI_e/L)$ to that for a tapered beam and solving for I_e:

$$I_e = \frac{b_T}{3E} K_{AA}(1 - C_{AB}C_{BA})$$

where b_T is the length of beam **5-9**, K_{AA} is the stiffness coefficient at **A** (larger end) of a tapered beam with end **B** (smaller end) fixed, C_{AB} is the carry over factor from **A** to **B**, and C_{BA} is the carry over factor from **B** to **A**. The stiffness and carry over factors are contained in Appendix A. The following quantities are needed for the beam **5-9**:

$\gamma_{5\text{-}9} = d_5/d_9 - 1 = 0.7$

$P_5 = 7.0$ kips

$P_{ex_9} = \pi^2 EI_{x_9}/b_T^2 = 185.5$ kips

$P/P_{ex_9} = 0.04$

$\therefore K_{AA} = 2.3 \times 4EI_9/b_T$ (Fig. A1)

$C_{AB} = 0.4$ (Fig. A3)

$C_{BA} = 0.65$ (Fig. A4)

$$\therefore I_e = 2.3 \times 4 \times 130.3\,(1 - 0.4 \times 0.65)/3 = 296 \text{ in.}^4$$

Using the charts in AISC Commentary Sect. D2:

$\gamma_{1\text{-}5} = d_5/d_1 - 1 = 1.91$

$G_T = b_T I_1/LI_e = 268 \times 38.1/(240 \times 296) = 0.14$

$G_B = 10$ (AISC recommended value for pinned end)

$\therefore K_x = K_\gamma = 0.92$

Slenderness ratios for the weak and strong axes are:

$(KL/r_4)_y = 1.0 \times 60/0.88 = 67.9$

$(KL/r_1)_x = 0.92 \times 240/3.49 = 63.3$

Thus, the weak-axis governs.

$F_{a_\gamma} = 21.3$ ksi (AISC Spec. Table 1-50) $> f_a = 3.45$ ksi

Allowable bending stress (AISC Spec. Appendix D, Sect. D3):

The following quantities are needed to calculate h_s and h_w:

$\gamma_{1\text{-}5} = 1.91$

$ld_4/A_f = 60 \times 20.375/5 \times 0.1875 = 1304$

$l/r_{T_4} = 60/1.16 = 51.7$

$\therefore h_s = 1.0 + (0.0230 \times 1.91\sqrt{1304}) = 1.163$

$h_w = 1.0 + (0.00385 \times 1.91\sqrt{51.7}) = 1.005$

Now,

$F_{s_\gamma} = 12{,}000/(1.163 \times 1304) = 7.91$

$F_{w_\gamma} = 170{,}000/(1.005 \times 51.7)^2 = 62.97$

Before F_{b_γ} in Formula (D3-2) can be calculated, the term B, which includes moment gradient and/or end restraint effects, needs to be determined. Since segment **4-5** has both axial and bending stresses, Sect. D4 will have to be checked later. When $f_a/f_{a_\gamma} > 0.15$ the value of B shall be unity (see the last paragraph in Sect. D3). For segment **4-5**, $f_a/F_{a_\gamma} = 0.165$; therefore $B = 1.0$. Subsequently from Formula (D3-2):

$$F_{b_\gamma} = 1.0\sqrt{(7.91)^2 + (62.97)^2} = 63.46 \text{ ksi}$$

However, this value is greater than $\frac{1}{3}F_y = 16.7$ ksi; thus, Formula (D3-1) applies.

$$F_{b_\gamma} = \frac{2}{3}\left(1.0 - \frac{50}{6 \times 63.46}\right)50 = 28.9 \text{ ksi} < 0.6F_y = 30.0 \text{ ksi}$$

Combined axial compression and bending (AISC Spec. Appendix D, Sect. D4):

From previous calculations, $f_a/F_{a_\gamma} = 0.165$; thus, both Formulas (D4-1a) and (D4-1b) must be checked. Since bending is about the strong axis,

$F'_{e_\gamma} = 37.3$ ksi (AISC Spec. Table 2)

Also, since the column is permitted to sway in its plane,

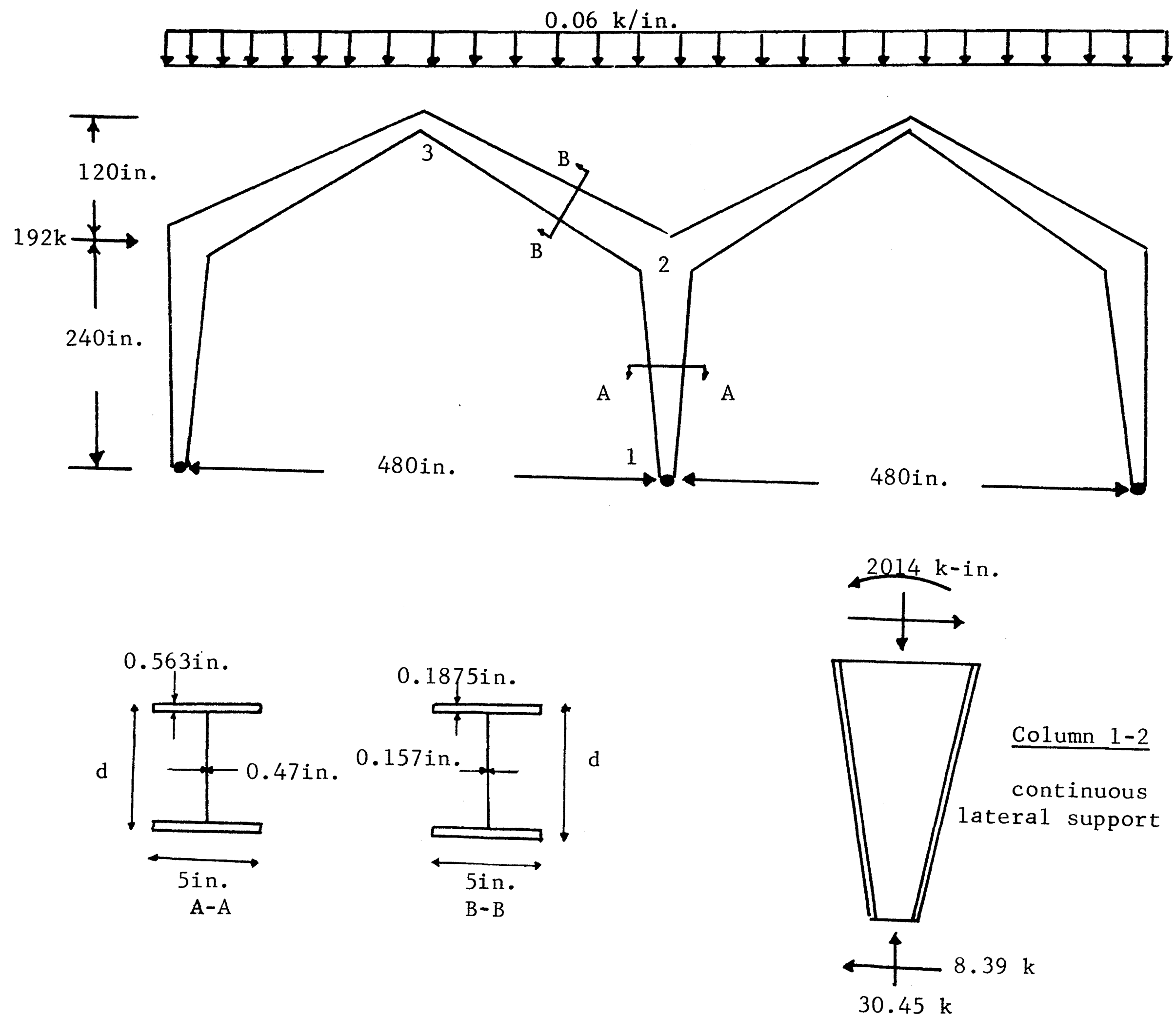
$$C_m = 0.85$$

Finally, Formula (D4-1a) gives:

$$\frac{3.45}{21.32} + \left[\frac{0.85}{\left(1.0 - \frac{3.45}{37.3}\right)} \times \frac{18.8}{28.9}\right] = 0.77 < 1.0 \quad \textbf{o.k.}$$

and Formula (D4-1b) gives:

$$\frac{3.45}{30} + \frac{18.8}{28.9} = 0.77 < 1.0 \quad \textbf{o.k.}$$



Center Column:

Sect.#	d	A	I_x	S_x	r_x	r_y	r_T
1	8.375in.	9.04in.^{2}	101in.^{4}	24.1in.^{3}	3.34in.	1.14in.	1.32in.
2-1	24.375	16.6	1290	106	8.83	0.85	1.13
2-3	24.375	5.64	455.1	37.3	8.98	0.83	1.12
3	14.375	4.07	130.3	18.1	5.66	0.98	1.22

Fig. 5. Design Example 6: design of the central column of a two-bay gable frame; F_y = 42 ksi

Example 6

Given:

Determine the adequacy of the central column **1-2** in the double bay gable frame, Fig. 5. The column is assumed to be pinned in the strong direction at its base. The frame is not braced against sidesway. The weak direction has a continuous brace along the shear center axis. The material yield stress is 42 ksi.

Solution:

First the following maximum stresses are computed:

Axial stress:

$$f_a = P_{12}/A_1 = 30.45/9.04 = 3.37 \text{ ksi}$$

Shear stress:

$$f_v = V_{12}/A_{v_1} = 8.39/(7.25 \times 0.47) = 2.46 \text{ ksi}$$

Bending stress:

$$f_b = M_{21}/S_{x_{21}} = 2014/106 = 19.0 \text{ ksi}$$

Next, the allowable stresses are obtained:

Allowable shear stress (AISC Spec. Sect. 1.5.1.2):

$$F_v = 0.4F_y = 17 \text{ ksi} > f_v = 2.46 \text{ ksi} \qquad \textbf{o.k.}$$

Allowable compression stress (AISC Spec. Appendix D, Sect. D2):

Since the column is continuously supported along the weak-axis, $K_y = 0$. The charts in the AISC Commentary Sect. D2, can be used to determine K_x. Noting that the column is restrained at the top by two tapered beams, an equivalent moment of inertia needs to be determined in the identical way as in Example 5. Note that both beams in Fig. 5 are the same.

$$\gamma_{2\text{-}3} = (d_2 - d_3)/d_3 = 0.7 \text{ (for the beam)}$$

$$P_{ex3} = \pi^2 EI_3/b_T^2$$

$$P/P_{ex3} = 0 \text{ (axial force in beam is small)}$$

From the figures in the Appendix:

$$K_{AA} = 2.3 \times 4EI_3/b_T \text{ (Fig. A1)}$$

$$C_{AB} = 0.4 \text{ (Fig. A3)}$$

$$C_{BA} = 0.65 \text{ (Fig. A4)}$$

$$\therefore I_e = 2.3 \times 4 \times 130\,(1 - 0.4 \times 0.65)/3 = 296 \text{ in.}^4$$

Now, using the charts in the AISC Commentary, Sect. D2:

$$\gamma_{1\text{-}2} = (d_2 - d_1)/d_1 = 1.9 \text{ (for the column)}$$

$$G_T = \frac{I_1}{L}\sum\frac{b_T}{I_T} = \frac{101}{240}\left(\frac{252}{296} + \frac{252}{296}\right) = 0.72$$

$$G_B = 10 \text{ (AISC recommended value for pinned end)}$$

$$\therefore K_x = K_\gamma = 1.25$$

The slenderness ratios for the weak and strong axes are:

$$(KL/r_1)_y = 0$$

$$(KL/r_1)_x = 1.25 \times 240/3.34 = 89.8$$

Thus, the strong axis governs.

$$F_{a_\gamma}\ 15.58 \text{ ksi (AISC Spec. Table 1-42)} > f_a = 3.37 \text{ ksi}$$

Allowable bending stress (AISC Spec. Appendix D, Sect. D3):

For a continuously lateral supported beam the allowable bending stress is:

$$F_{b_\gamma} = 0.6F_y = 25 \text{ ksi}$$

Combined axial compression and bending (AISC Spec. Appendix D, Sect. D4):

From previous calculations, $f_a/F_{a_\gamma} = 0.22$; thus, both Formulas (D4-1a) and (D4-1b) must be checked. Since bending is about the strong axis,

$$F'_{e_\gamma} = 18.52 \text{ ksi (AISC Spec. Table 2)}$$

and for the case when the column is allowed to sway in its plane,

$$C_m = 0.85$$

Finally, Formula (D4-1a) gives:

$$\frac{3.37}{15.58} + \left(\frac{0.85}{\left(1.0 - \dfrac{3.37}{18.52}\right)} \times \frac{19.0}{25}\right) = 1.006 \qquad \textbf{o.k.}$$

and Formula (D4-1b) gives:

$$\frac{3.37}{0.6 \times 42} + \frac{19.0}{25} = 0.89 \qquad \textbf{o.k.}$$

CONCLUSIONS

The examples presented here serve to illustrate the use of the design specifications for tapered members as required by AISC Specification Appendix D. There are still several unanswered questions regarding the rational design of tapered member. Three topics of current interest are (1) local buckling limitations, (2) tension flange support requirements,[6] and (3) the C_m factor for beam-columns in unbraced frames. The designer must still use experience and sound engineering judgement regarding design procedures not provided by Appendix D of the Specification.

ACKNOWLEDGMENTS

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REFERENCES

1. *Amirikian, A.*, Wedge-Beam Framing *Trans. ASCE, Vol. 117, p. 596 (1952).*
2. *Lee, G. C., M. L. Morrell, and R. L. Ketter* Design of Tapered Members *WRC Bulletin No. 173, June 1972.*
3. *Morrell, M. L. and G. C. Lee* Allowable Stress for Web-Tapered Beams with Lateral Restraints *WRC Bulletin, No. 192, Feb. 1974.*

4. *American Institute of Steel Construction* Supplement No. 3 to the Specification for the Design, Fabrication & Erection of Structural Steel for Buildings *June 12, 1974 (including Addenda to the Commentary on the Specification.)*
5. *Lee, G. C. and M. L. Morrell* Stability of Space Frames Composed of Thin-Walled Members *Conf. Preprint, ASCE National Conference-Structural Engineering, April 22–26, 1974, Cincinnati, Ohio*
6. *Lee, G. C. and M. L. Morrell* Allowable Stress for Web-Tapered Beams with Tension Flange Bracing *(manuscript in preparation)*.

APPENDIX:

STIFFNESS FACTORS AND CARRY-OVER FACTORS FOR WEB-TAPERED MEMBERS

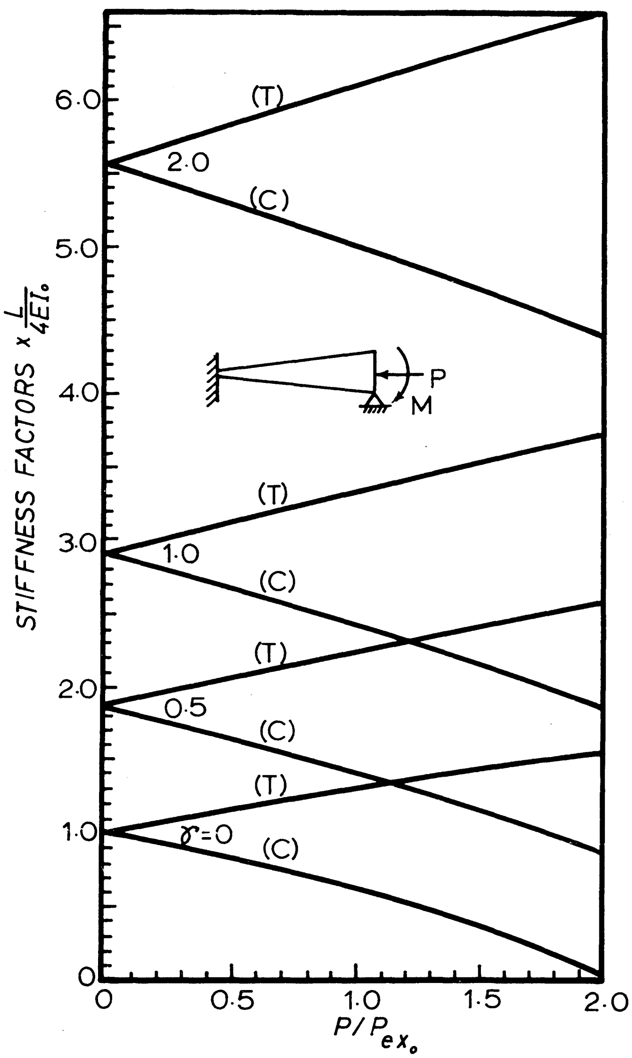

Fig. A1. Stiffness factors: small end fixed

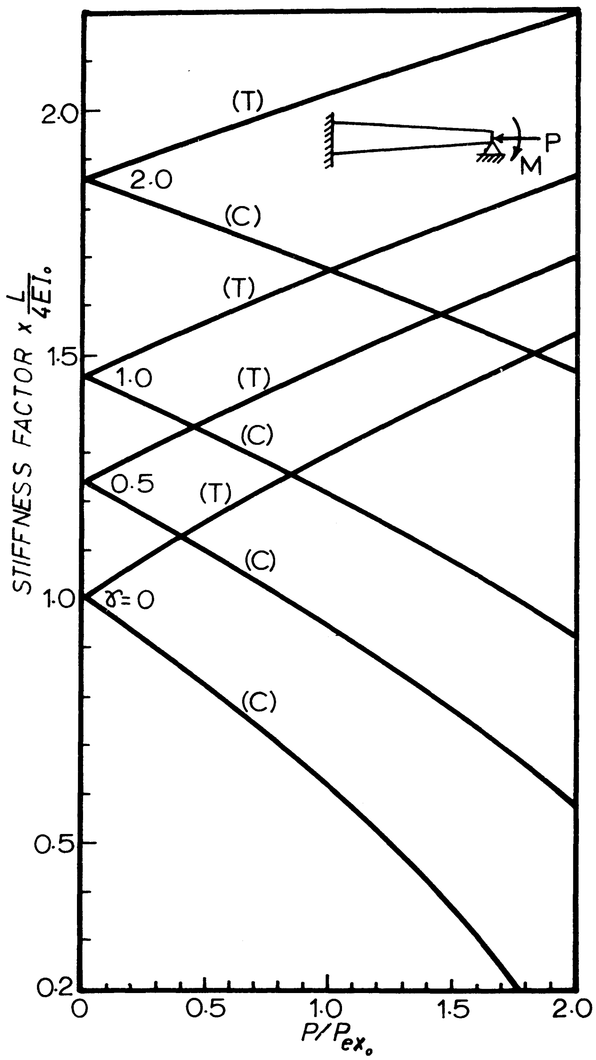

Fig. A2. Stiffness factors: large end fixed

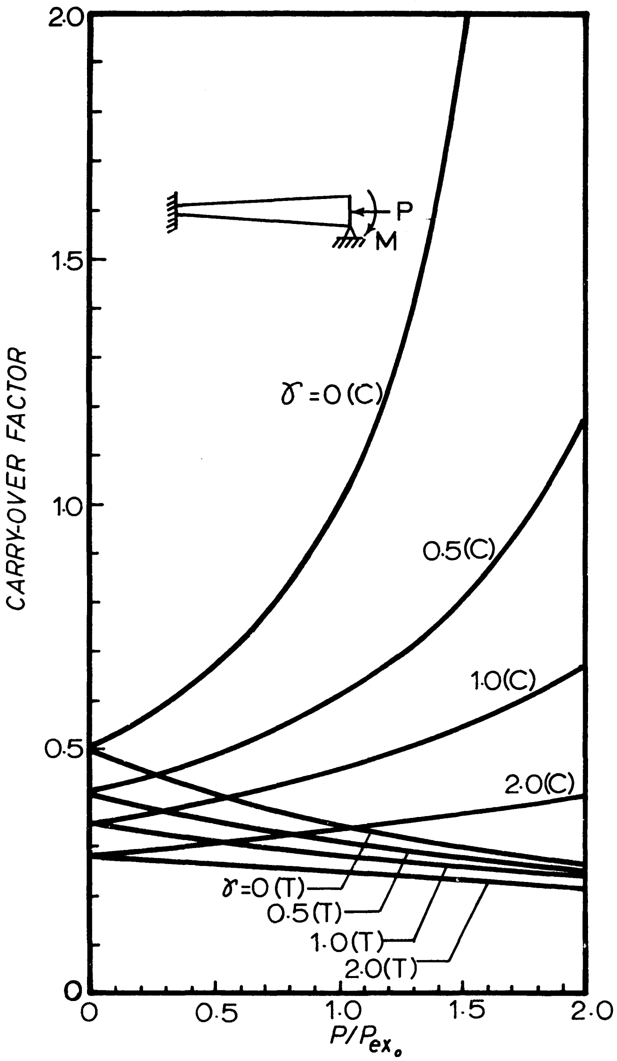

Fig. A3. Carry-over factors: small end fixed

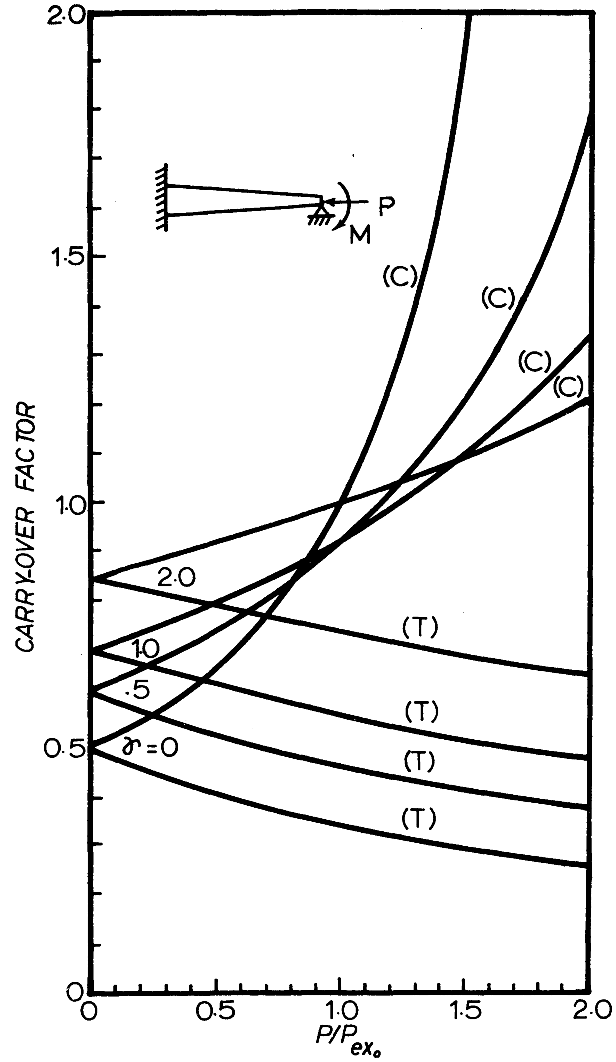

Fig. A4. Carry-over factors: large end fixed