Simplified Solution to Interaction Equation

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ACCORDING TO the requirements of the AISC Specification, there are three equations, Formulas (1.6-1a), (1.6-1b), and (1.6-2), which must be considered in designing a steel column subjected to combined loading. In Formula (1.6-1a), the combined axial and bending stress is limited by stability, while yielding controls the combined stress in Formula (1.6-1b).

The bending stress f_b in Formula (1.6-1a) is increased by a factor of $1/(1 - f_a/F_e')$. This factor is appropriately known as the amplification factor. The computed bending stress is required to be increased due to the additional bending stress induced by a secondary moment. This moment is caused by the compressive force and the deflection of the column due to moments applied at the ends and/or a transverse loading on the column.

The purpose of this paper is to facilitate the calculations required for obtaining the modified bending amplification factor, $C_m/(1 - f_a/F_e')F_b$, along with the allowable bending stresses of various steels available. It is hoped that with the use of the proposed charts, the time and effort of the designer will be greatly reduced.

The charts presented in this paper enable the designer to get values for the modified bending amplification factor, $1/(1 - f_a/F_e')F_b$, for steels with F_y of 36 ksi and 50 ksi. For the charts presented, C_m is assumed to be unity and F_b is taken to be 60% of F_y. If, in designing a column, C_m and/or F_b are not equal to these assumed values, a simple calculation using a slide rule is all that is necessary to correct this. In the charts, Kl_b/r_b is plotted as the ordinate and the modified bending amplification factor is plotted as the abscissa for a given value of f_a.

If $F_b = 0.66F_y$ is applied instead of $0.6F_y$, multiply the chart value by 0.909. For F_b other than 0.6 or $0.66F_y$, appropriate modification should be made to the chart value by the ratio of $0.6F_y$ to the given F_b. For values of $C_m \neq 1.0$, the actual modified bending amplification factors can be calculated by multiplying the chart values by the actual C_m values. Values of Kl_b/r_b greater than 200 and modified amplification factors greater than 6.0 are not plotted, since they are rarely encountered. For values of f_a between the plotted values of f_a on the charts, one need only interpolate. The error introduced by this approximation is insignificant.

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SAMPLE CALCULATIONS

The following examples demonstrate the use of the charts. Examples 1, 2, and 3 correspond to dashed lines ①, ②, and ③ drawn in the charts.

Example 1—Using A36 steel, design a column by the following data:

$$l_x = l_y = 20.0 \text{ ft}$$
$$K_x = 1.6;\ K_y = 1.0$$
$$M_x = 100 \text{ kip-ft};\ M_y = 0$$
$$P = 500 \text{ kips};\ C_{mx} = 0.85$$

Solution: Try W14×142

$$A = 41.8 \text{ in.}^2;\ S_x = 227.0 \text{ in.}^3;\ d/A_f = 0.895 \text{ in.}^{-1}$$
$$r_x = 6.32 \text{ in.};\ r_y = 3.97 \text{ in.};\ r_T = 4.33 \text{ in.}$$
$$b_f = 15.5 \text{ in.}$$

$$\frac{K_x l_x}{r_x} = \frac{1.6\ (20)(12)}{6.32} = 60.7$$

$$\frac{K_y l_y}{r_y} = \frac{1.0\ (20)(12)}{3.97} = 60.5$$

$$F_a = 17.36 \text{ ksi (AISC Spec. App. A, Table 1-36)}$$

$$f_a = P/A = 500/41.8 = 11.95 \text{ ksi}$$

$$\frac{f_a}{F_a} = \frac{11.95}{17.36} = 0.69 > 0.15$$

∴ Check Formulas (1.6-1a) and (1.6-1b).

Enter Chart No. 1 with $(K_x l_x/r_x) = 60.7$ and proceed to the right to $f_a = 11.95$, then down to the axis for $F_y = 36$. The modified bending amplification factor is found to be 0.066.

$$f_{bx} = \frac{M_x}{S_x} = \frac{(100)\ (12)}{227} = 5.3 \text{ ksi}$$

$$\frac{76 b_f}{\sqrt{F_y}} = 196.3 < l_b = 240 \text{ in. } \therefore \text{ not compact}$$

AISC Formula (1.5-7):

$$F_{bx} = \frac{12{,}000\ (1.0)}{240\ (0.895)} = 55.9 > 0.6F_y$$

Therefore, use $F_b = 0.6F_y = 22$ ksi.

AISC Formula (1.6-1a):

$$\frac{f_a}{F_a} + \frac{C_{mx} f_{bx}}{(1 - f_a/F_{ex}')F_{bx}} + \frac{C_{my} f_{by}}{(1 - f_a/F_{ey}')F_{by}} \leq 1.0$$

$0.69 + (0.85)(5.3)(0.066) + 0 = 0.98 < 1.0$ **o.k.**

By inspection, Formula (1.6-lb) does not control.

Example 2—Instead of using A36 steel, design a column using A572 Gr. 50 steel.

Solution: Try W12×133

$$A = 39.1 \text{ in.}^2;\ S_x = 183.0 \text{ in.}^3;\ d/A_f = 0.875 \text{ in.}^{-1}$$
$$r_x = 5.59 \text{ in.};\ r_y = 3.16 \text{ in.};\ r_T = 3.47 \text{ in.}$$
$$b_f = 12.365 \text{ in.}$$

$$\frac{K_x l_x}{r_x} = \frac{1.6\ (20)(12)}{5.59} = 68.7$$

$$\frac{K_y l_y}{r_y} = \frac{1.0\ (20)(12)}{3.16} = 76.0$$

$F_a = 19.80$ ksi (AISC Spec. App. A, Table 1-50)

$$f_a = P/A = 500/39.1 = 12.8 \text{ ksi}$$

$$\frac{f_a}{F_a} = \frac{12.8}{19.8} = 0.646 > 0.15$$

Enter Chart No. 1 with $K_x l_x/r_x = 68.7$ and proceed to the right to $f_a = 12.8$, then down to the axis for $F_y = 50$. The modified bending amplification factor is found to be 0.056.

$$f_{bx} = \frac{M_x}{S_x} = \frac{(100)\ (12)}{183} = 6.6$$

$$\frac{76 b_f}{\sqrt{F_y}} = 133.1 < l_b = 240 \text{ in.} \therefore \text{ not compact}$$

AISC Formula (1.5-7):

$$F_{bx} = \frac{12{,}000\ (1.0)}{240\ (0.875)} = 57.1 > 0.6F_y$$

Therefore, use $F_b = 0.6F_y = 30$ ksi

AISC Formula (1.6-1a):

$$\frac{f_a}{F_a} + \frac{C_{mx} f_{bx}}{(1 - f_a/F_{ex}')F_{bx}} + \frac{C_{my} f_{by}}{(1 - f_a/F_{ey}')F_{by}} \leq 1.0$$

$0.646 + (0.85)(6.6)(0.056) + 0 = 0.96 < 1.0$ **o.k.**

Again, Formula (1.6-1b) does not control.

Example No. 3—Using A572 Gr. 50 steel, design a column for the loading below:

$$l_x = 14.0 \text{ ft};\ l_y = 15.0 \text{ ft}$$
$$K_x = 1.6;\ K_y = 1.2$$
$$M_x = 100 \text{ kip-ft};\ M_y = 40 \text{ kip-ft}$$
$$P = 500 \text{ kips}$$
$$C_{mx} = C_{my} = 0.85$$

Solution: Try W14×136

$$A = 40.0 \text{ in.}^2;\ S_x = 216 \text{ in.}^3;\ S_y = 77.0 \text{ in.}^3$$
$$d/A_f = 0.941 \text{ in.}^{-1}$$
$$r_x = 6.31 \text{ in.};\ r_y = 3.77 \text{ in.};\ r_T = 4.12 \text{ in.}$$
$$b_f = 14.74 \text{ in.}$$

$$\frac{K_x l_x}{r_x} = \frac{1.6\ (14)(12)}{6.31} = 42.6$$

$$\frac{K_y l_y}{r_y} = \frac{1.2\ (15)(12)}{3.77} = 57.4$$

$F_a = 23.16$ ksi (AISC Spec. App. A, Table 1-50)

$$f_a = P/A = 500/40 = 12.5 \text{ ksi}$$

$$\frac{f_a}{F_a} = \frac{12.5}{23.16} = 0.540 > 0.15$$

Enter Chart No. 2 with $(K_x l_x/r_x) = 42.6$ and proceed to the right to $f_a = 12.5$, then down to the axis for $F_y = 50$. The modified bending amplification factor is found to be 0.039.

Enter Chart No. 2 with $(K_y l_y/r_y) = 57.4$ and proceed to the right to $f_a = 12.5$, then down to the axis for $F_y = 50$. The modified bending amplication factor is found to be 0.046.

$$f_{bx} = \frac{M_x}{S_x} = \frac{(100)(12)}{216} = 5.55 \text{ ksi}$$

$$f_{by} = \frac{M_y}{S_y} = \frac{(40)(12)}{77} = 6.24 \text{ ksi}$$

$$\frac{76 b_f}{\sqrt{F_y}} = 158 < l_{bx} = 168 \text{ in.} \therefore \text{ not compact}$$

AISC Formula (1.5-7):

$$F_{bx} = \frac{12{,}000(1.0)}{168(0.941)} = 75.9 > 0.6F_y$$

Therefore, use $F_{bx} = 0.6F_y = 30$ ksi

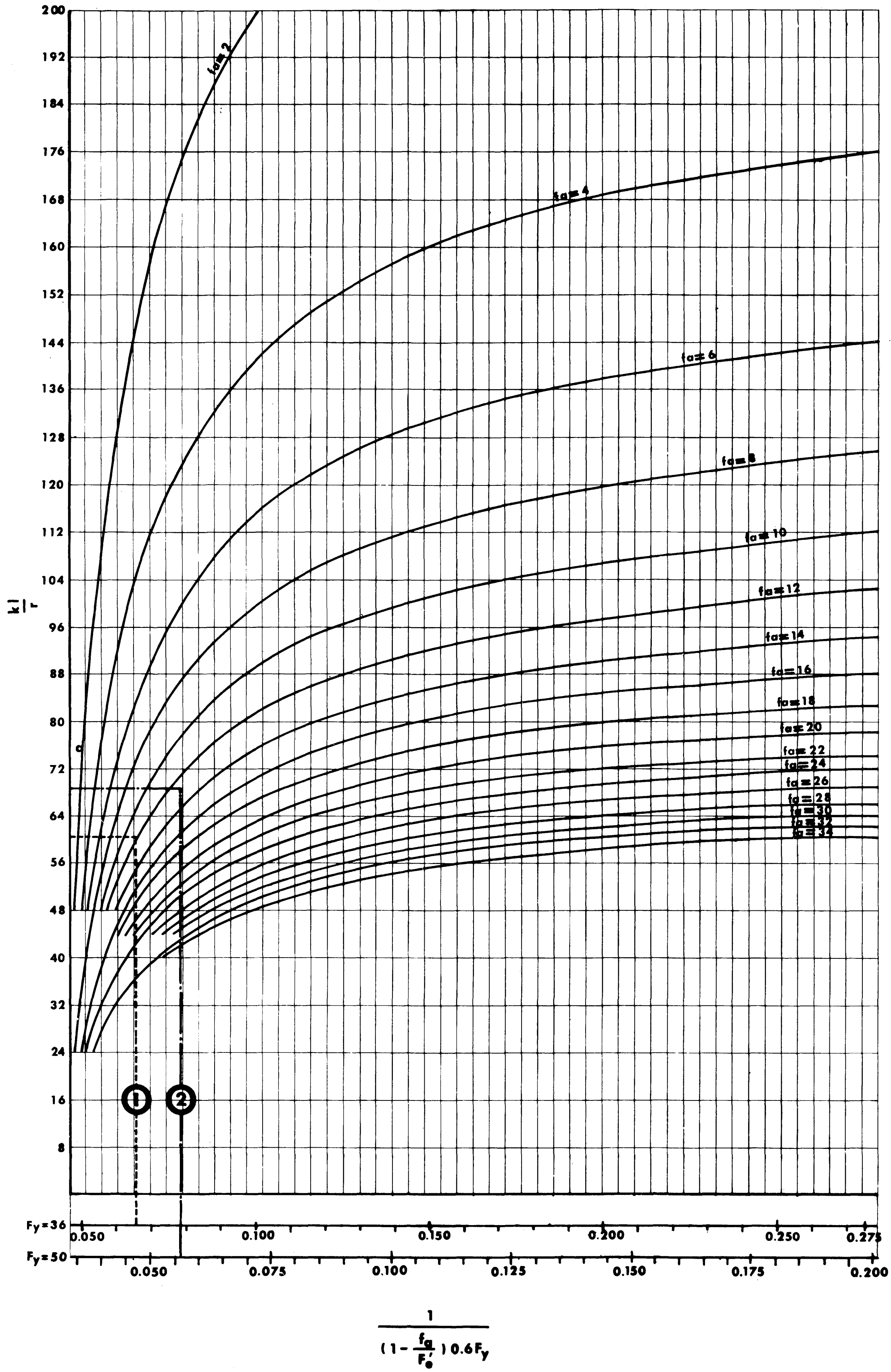
$F_{by} = 0.75F_y = 37.5$ ksi (Spec. Sect. 1.5.1.4.3)

AISC Formula (1.6-1a):

$$\frac{f_a}{F_a} + \frac{C_{mx} f_{bx}}{(1 - f_a/F_{ex}')F_{bx}} + \frac{C_{my} f_{by}}{(1 - f_a/F_{ey}')F_{by}} \leq 1.0$$

$0.54 + (0.85)(0.039)(5.55) +$
$(0.85)(0.046)(6.24)(30/37.5) = 0.92 < 1.0$ **o.k.**

By inspection, Formula (1.6-1b) does not govern.



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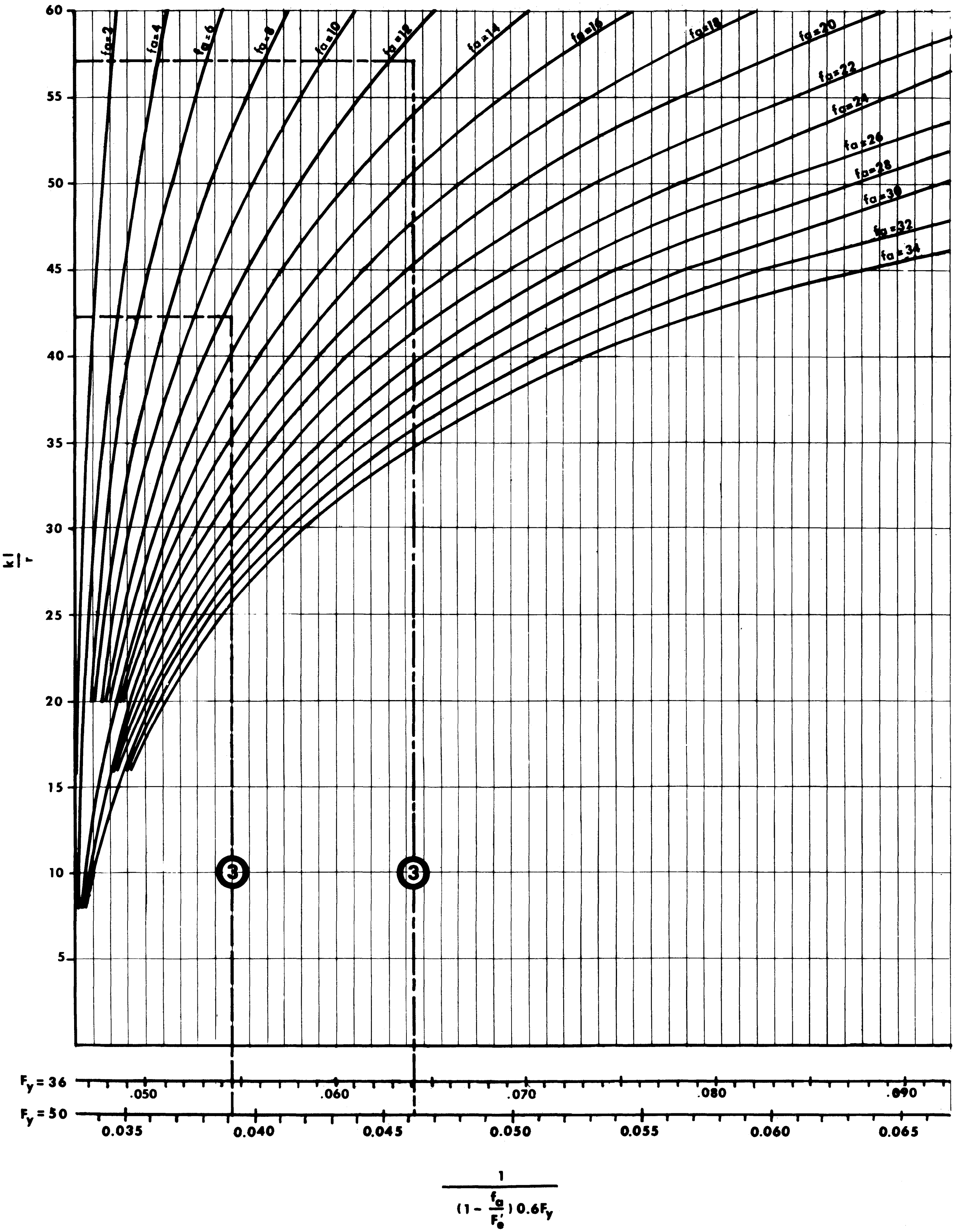
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