Simplified Seismic Drift Analysis of High-Rise Steel Frames

WALTER H. FLEISCHER

After a simplified method for estimating motion amplitudes of high-rise steel frames under wind forces had been published in this periodical[1] about two years ago, the question invited itself if the general approach used in that paper could also be employed to obtain drift statements for high-rise steel frames under the application of seismic loads as prescribed in the major building codes.

This paper answers the question in the affirmative by demonstrating how the practicing engineer can estimate seismic drift amplitudes under code seismic forces very quickly and without the use of a computer by applying the following simplifying assumptions which have been demonstrated to be valid for use in preliminary design, yielding results of excellent accuracy:

1. The distribution of material properties (such as column stiffnesses, girder stiffnesses, cross-sectional areas of members, etc.) over the height of the bent can be expressed by "homogenized" mathematical expressions.
2. All level-to-level distances are assumed to be equidistant and to be of a magnitude equal to the building height divided by the total number of levels in the bent.
3. Each level supports the same dead load. The dead load per level is assumed to be the total load of the bent divided by the number of levels in the bent.

The working equations derived in this paper require the stress design of only three representative levels in the bent in order to obtain realistic values for the total bent drift.

NOMENCLATURE

b = Material property distribution factor
c = Material property distribution exponent
d = Story drift contribution
e = Distance from center of individual column to common center of gravity of all columns in the same bent
i = Level from top of frame (at roof: $i = 1$; at level above grade: $i = n$)
k = Column line or bay identification (see Fig. 2)
l = Length of frame component
m = Mass per level; also, column line or bay identification (see Fig. 2)
n = Number of levels in the frame
t = Top of frame (subscript)
A = Cross-sectional area
B = Bent (subscript)
C = Column (subscript)
C' = Column-moment (subscript)
C'' = Column-axial load (subscript)
D = Total drift; also Bracing (subscript)
E = Modulus of elasticity
F = Lateral seismic force against the frame
G = Girder (subscript)
H = Total frame height
I = Moment of inertia
L = Overall plan dimension of lateral force resisting system
M = Moment
P = Any cross-sectional material property
T = Total of stiffnesses of a given material property at a given level; also, Total bent (subscript)
V = Shear force
αL = Girder span
ϕ = Drift correction factor

Walter H. Fleischer is Structural Consultant, Sales Engineering Division, Bethlehem Steel Corporation, Bethlehem, Pa.

Equations are presented for both planar rigid and planar braced frames. The lateral forces used in this technique have been obtained by the "Dynamic Approach" procedure recommended by the Seismology Committee of the Structural Engineers Association of California (SEAOC),[2] which provides a rational basis for establishing equivalent static forces to simulate the conditions and stresses that will occur under seismic ground motions. Drift amplitudes will be estimated based on these static equivalent forces.

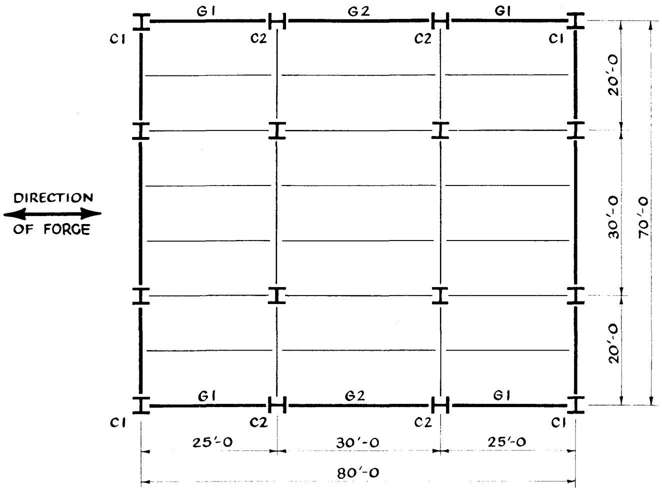

Figure 1

The designer is advised to familiarize himself with the extent and the limitations as well as the professional expert opinions on this subject, as delineated in the *Recommended Lateral Force Requirements and Commentary* by the Seismology Committee of SEAOC,[2] before using the following material.

The method will be demonstrated step-by-step by an example of a typical drift analysis in a planar rigid high-rise steel bent, including a description of the procedure for both rigid and braced frames. Following this example, the development of the method and the derivations of the working equations will be explained.

ILLUSTRATIVE EXAMPLE

Given:

The frame under consideration is 360 ft high, with 30 supported levels. The typical floor plan is shown in Fig. 1. This structure is to be analyzed as a rigid perimeter system, i.e., all lateral forces are to be resisted by spandrel girders connected rigidly to the perimeter columns. The rigid frame parallel to the direction of the seismic force will be investigated, i.e., Columns C1 and C2, Girders G1 and G2.

The frame is to be checked for total drift. The frame grid identification nomenclature is shown in Fig. 2.

Note that in this paper the symbols and notations of the SEAOC recommendations have been adjusted where necessary to make the nomenclature of this paper compatible with that of the author's earlier paper on wind drift analysis.[1] All working equations required in the solution are listed in Tables 1 and 2.

Table 1. Summary of Working Equations

Equation	Description	Formulation[a,b]
(7)	Material Property Distribution Factor	$b = \frac{nP_1}{P_n}$
(9)	Material Property Distribution Exponent	$c = \frac{\log\left[\frac{P_i - P_1}{P_n - P_1}\right]}{\log\left[\frac{i-1}{n-1}\right]}$
(8)	Magnitude of Material Property at any Level	$P_i = \frac{Pn}{n}\left[b + (n-b)\left(\frac{i-1}{n-1}\right)^c\right]$
(15)	Girder Moment Drift	$D_G = \frac{VH^2}{134nT_{G(n)}}(\varphi_G)$
(18)	Column Moment Drift	$D_{C'} = \frac{VH^2}{134nT_{C(n)}}(\varphi_{C'})$
(21)	Column Chord Drift	$D_{C''} = \frac{V}{I_{B(n)}}\left(\frac{H}{112}\right)^3(\varphi_{C''})$
(27)	Bracing Drift (X-Bracing)	$D_D = \frac{n\ V}{1610T_{D(n)}}(\varphi_D)$
(29)	Bracing Drift (K-Bracing)	$D_D = \frac{nV}{805T_{D(n)}}(\varphi_D)$

[a] All equations are based on customary engineering design units:
D = in., H = ft, L = ft, l = ft, I_G = in.^{4}, I_C = in.^{4}, I_B = ft^4,
A = in.^{2}, E = 29,000 ksi, V = kips

[b] Subscripts in parentheses refer to frame grid identification nomenclature (see Fig. 2).

Table 2. Summary of Material Property References

Line	Material Property	[a] Formulation for Values of P in Eqs. (7), (8), and (9)	Frame Constants	
			b	c
(1)	Total of all girder stiffnesses at any given level	$T_{G(i)} = \sum_{k=1}^{k=m} \frac{I_{G(k,i)}}{\alpha_k L}$	b_G	c_G
(2)	Total of all column stiffnesses immediately below any given level	$T_{C(i)} = \sum_{k=0}^{k=m} \frac{nI_{C(k,i)}}{H}$	$b_{C'}$	$c_{C'}$
(3)	Moment of Inertia of bent immediately below any given level	$I_{B(i)} = \sum_{k=0}^{k=m} A_{C(k,i)} e_k^2$	$b_{C''}$	$c_{C''}$
(4)	Total of all bracing "stiffnesses" immediately below any given level[b]	$T_{D(i)} = \sum_{k=1}^{k=m} \frac{A_{D(k,i)}(\alpha_k L)^2}{(l_{D(k)})^3}$	b_D	c_D

[a] Subscripts in parentheses refer to frame grid identification nomenclature (see Fig. 2).

[b] The summation shown includes bracing members stressed in tension only; this represents only half the total number of all bracing members.

Procedure and Solutions:

Let it be assumed that the requirements of the building code have established a seismic base shear of 252 kips per bent. The following design parameters are now known from the given information:

Total frame height (H) = 360 ft
Total frame length (L) = 80 ft
Number of levels in frame (n) = 30
Total seismic base shear of frame (V) = 252 kips

Step 1—Stress design levels 1, 15 and 30:

The steel sections as shown in Fig. 3 have been the result of this design. Step 2—Establish all material property values (P) at levels 1, 15 and 30 (see Table 2, lines 1–3; line 4 is only required for braced frames):

a. Sum of girder stiffnesses:

$$T_{G(i)} = \sum_{k=1}^{k=m} \frac{I_{G(k,i)}}{\alpha_k L}$$

$$T_{G(1)} = \frac{300}{25} + \frac{513}{30} + \frac{300}{25} = 41.1 \text{ in.}^4/\text{ft}$$

$$T_{G(15)} = \frac{2100}{25} + \frac{2370}{30} + \frac{2100}{25}$$

$$= 247 \text{ in.}^4/\text{ft}$$

$$T_{G(30)} = \frac{4000}{25} + \frac{4930}{30} + \frac{4000}{25}$$

$$= 484 \text{ in.}^4/\text{ft}$$

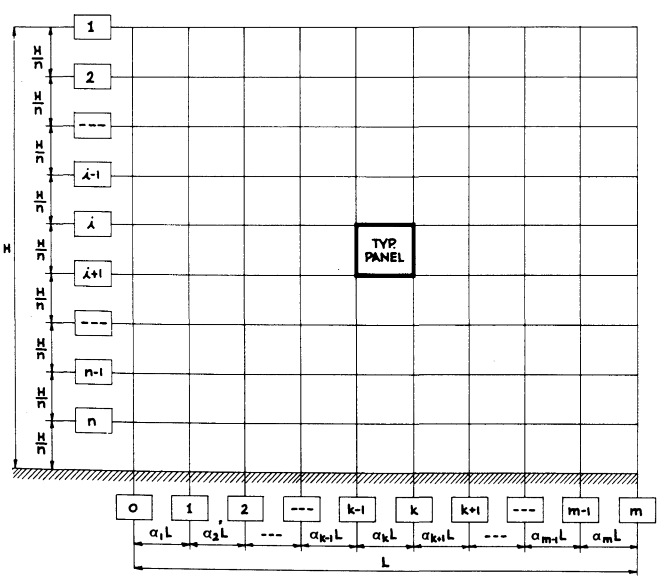

Figure 2

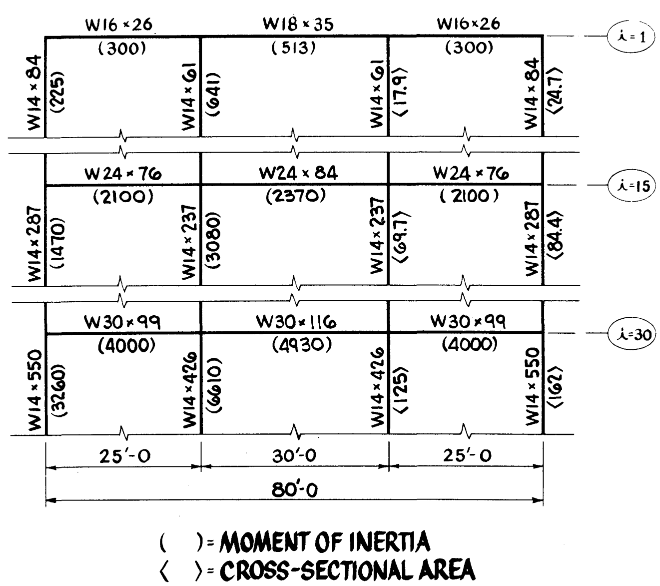

Figure 3

b. Sum of column stiffnesses:

$$T_{C(i)} = \sum_{k=0}^{k=m} \frac{nI_{C(k,i)}}{H}$$

$$T_{C(1)} = \frac{30}{360}(225 + 641 + 641 + 225)$$

$$= 144 \text{ in.}^4/\text{ft}$$

$$T_{C(15)} = \frac{30}{360}(1470 + 3080 + 3080 + 1470)$$

$$= 758 \text{ in.}^4/\text{ft}$$

$$T_{C(30)} = \frac{30}{360}(3260 + 6610 + 6610 + 3260)$$

$$= 1650 \text{ in.}^4/\text{ft}$$

c. Moment of inertia of bent immediately below level:

$$I_{B(i)} = \sum_{k=0}^{k=m} A_{C(k,i)} e_k^2$$

$$I_{B(1)} = \frac{2}{144}(24.7 \times 40^2 + 17.9 \times 15^2)$$

$$= 605 \text{ ft}^4$$

$$I_{B(15)} = \frac{2}{144}(84.4 \times 40^2 + 69.7 \times 15^2)$$

$$= 2090 \text{ ft}^4$$

$$I_{B(30)} = \frac{2}{144}(162 \times 40^2 + 125 \times 15^2)$$

$$= 3990 \text{ ft}^4$$

Step 3—Establish the values for the material property distribution factors b for all applicable material properties in Table 2 from Eq. (7) in Table 1:

a. For girder moments:

$$b_G = \frac{nT_{G(1)}}{T_{G(30)}} = \frac{30 \times 41.1}{484} = 2.55$$

b. For column moments:

$$b_{C'} = \frac{nT_{C(1)}}{T_{C(30)}} = \frac{30 \times 144}{1650} = 2.62$$

c. For column axial forces:

$$b_{C''} = \frac{nI_{B(1)}}{I_{B(30)}} = \frac{30 \times 605}{3990} = 4.55$$

Step 4—Establish the values for the material property distribution exponent c for all applicable material properties in Table 2 from Eq. (9) in Table 1:

a. For girder moments:

$$c_G = \frac{\log\left[\dfrac{T_{G(15)} - T_{G(1)}}{T_{G(30)} - T_{G(1)}}\right]}{\log\left[\dfrac{15-1}{30-1}\right]} = \frac{\log\left[\dfrac{247 - 41.1}{484 - 41.1}\right]}{\log\left[\dfrac{15-1}{30-1}\right]} = 1.05$$

b. For column moments:

$$c_{C'} = \frac{\log\left[\dfrac{T_{C(15)} - T_{C(1)}}{T_{C(30)} - T_{C(1)}}\right]}{\log\left[\dfrac{15-1}{30-1}\right]} = \frac{\log\left[\dfrac{758 - 144}{1650 - 144}\right]}{\log\left[\dfrac{15-1}{30-1}\right]} = 1.23$$

c. For column axial forces:

$$c_{C''} = \frac{\log\left[\dfrac{I_{B(15)} - I_{B(1)}}{I_{B(30)} - I_{B(1)}}\right]}{\log\left[\dfrac{15-1}{30-1}\right]} = \frac{\log\left[\dfrac{2090 - 605}{3990 - 605}\right]}{\log\left[\dfrac{15-1}{30-1}\right]} = 1.13$$

Step 5—Obtain the corresponding drift correction factors by proper interpolation from Tables 3, 4, and 5 or by use of Eqs. (16), (19), and (22) for the pertinent aspect ratio of the frame (frame height divided by frame depth) of $H/L = 360/80 = 4.50$:

a. For girder moments (Table 3): $\phi_G = 0.908$
b. For column moments (Table 4): $\phi_{C'} = 1.071$
c. For column axial forces (Table 5): $\phi_{C''} = 1.055$

Step 6—Calculate the magnitude of the drift components from Eqs. (15), (18), and (21) in Table 1 [for rigid bents only; for braced bents use Eqs. (27) or (29) and (21)]:

a. Girder moment drift:

$$D_G = \frac{VH^2}{134\, nT_{G(n)}}(\phi_G) = \frac{252 \times 360^2 \times 0.908}{134 \times 30 \times 484}$$

$$= 15.2 \text{ in.}$$

b. Column moment drift:

$$D_{C'} = \frac{VH^2}{134\, nT_{C(n)}}(\phi_{C'}) = \frac{252 \times 360^2 \times 1.071}{134 \times 30 \times 1650}$$

$$= 5.3 \text{ in.}$$

c. Column chord drift:

$$D_{C''} = \frac{V}{I_{B(n)}}\left(\frac{H}{112}\right)^3(\phi_{C''}) = \frac{252 \times 360^3 \times 1.055}{3990 \times 112^3}$$

$$= 2.2 \text{ in.}$$

Table 3

H/L	b	1					2					3					4					5				
	c	1.0	1.1	1.2	1.3	1.4	1.0	1.1	1.2	1.3	1.4	1.0	1.1	1.2	1.3	1.4	1.0	1.1	1.2	1.3	1.4	1.0	1.1	1.2	1.3	1.4
	n																									
0	10	.774	.817	.861	.905	.950	.678	.704	.730	.756	.781	.618	.635	.652	.669	.685	.573	.585	.597	.608	.619	.538	.547	.555	.563	.570
	15	.832	.887	.944	1.004	1.065	.746	.783	.820	.857	.895	.689	.716	.743	.769	.795	.646	.667	.687	.706	.726	.612	.628	.643	.658	.673
	20	.864	.928	.995	1.065	1.138	.787	.832	.878	.925	.972	.734	.769	.803	.838	.872	.694	.721	.748	.775	.801	.660	.683	.705	.726	.747
	25	.885	.955	1.030	1.108	1.191	.815	.866	.919	.974	1.029	.766	.807	.848	.889	.930	.728	.761	.794	.827	.860	.696	.724	.751	.778	.805
	30	.900	.975	1.055	1.141	1.231	.836	.892	.951	1.011	1.073	.790	.835	.882	.929	.976	.754	.792	.830	.868	.906	.724	.756	.788	.820	.852
	35	.912	.990	1.075	1.166	1.263	.852	.912	.976	1.042	1.110	.809	.858	.909	.962	1.015	.774	.816	.859	.902	.945	.746	.782	.818	.855	.891
	40	.921	1.002	1.091	1.187	1.290	.865	.929	.996	1.067	1.140	.824	.877	.932	.989	1.047	.791	.837	.883	.931	.978	.764	.803	.843	.884	.924
3	10	.799	.843	.889	.935	.982	.696	.723	.750	.776	.802	.632	.650	.668	.684	.701	.585	.598	.610	.621	.632	.549	.558	.566	.574	.581
	15	.863	.922	.982	1.045	1.109	.769	.808	.847	.886	.924	.708	.736	.764	.791	.818	.663	.684	.705	.725	.745	.626	.643	.659	.674	.689
	20	.901	.969	1.041	1.115	1.192	.815	.862	.911	.959	1.009	.757	.793	.830	.865	.901	.714	.742	.770	.798	.825	.678	.701	.724	.746	.768
	25	.927	1.002	1.082	1.166	1.254	.847	.901	.957	1.014	1.072	.793	.835	.878	.921	.964	.751	.785	.820	.854	.888	.717	.746	.774	.802	.829
	30	.946	1.027	1.113	1.205	1.302	.871	.931	.993	1.057	1.123	.819	.867	.916	.966	1.015	.780	.819	.859	.899	.939	.747	.780	.814	.847	.880
	35	.960	1.046	1.138	1.237	1.341	.890	.955	1.022	1.092	1.164	.841	.893	.947	1.002	1.058	.803	.847	.891	.937	.982	.771	.809	.847	.885	.922
	40	.972	1.062	1.159	1.263	1.374	.905	.974	1.046	1.122	1.200	.858	.915	.973	1.033	1.094	.821	.870	.919	.968	1.018	.791	.833	.875	.917	.959
4	10	.818	.864	.911	.958	1.007	.710	.737	.765	.792	.818	.643	.662	.679	.697	.713	.595	.608	.620	.631	.643	.557	.566	.574	.582	.590
	15	.888	.949	1.012	1.077	1.143	.787	.827	.868	.908	.948	.723	.752	.781	.809	.836	.675	.697	.719	.739	.759	.637	.654	.671	.686	.702
	20	.930	1.001	1.076	1.154	1.235	.837	.886	.936	.987	1.037	.776	.813	.850	.887	.923	.729	.759	.788	.816	.844	.692	.716	.739	.762	.784
	25	.960	1.039	1.123	1.211	1.303	.872	.928	.987	1.046	1.106	.813	.857	.902	.946	.991	.769	.805	.840	.875	.910	.733	.762	.792	.820	.849
	30	.981	1.067	1.159	1.256	1.357	.898	.961	1.026	1.093	1.161	.842	.892	.943	.994	1.046	.800	.841	.882	.923	.964	.765	.799	.834	.868	.902
	35	.998	1.089	1.187	1.292	1.402	.919	.987	1.058	1.132	1.207	.866	.921	.977	1.034	1.092	.824	.870	.917	.963	1.010	.791	.830	.869	.908	.947
	40	1.012	1.108	1.212	1.322	1.440	.936	1.009	1.085	1.164	1.246	.885	.944	1.005	1.068	1.131	.845	.895	.946	.998	1.049	.812	.855	.899	.943	.986
5	10	.842	.890	.939	.988	1.038	.727	.756	.784	.812	.839	.657	.676	.695	.713	.730	.607	.620	.633	.645	.656	.568	.577	.585	.594	.601
	15	.920	.983	1.050	1.118	1.187	.811	.853	.894	.936	.977	.742	.772	.802	.831	.859	.692	.714	.736	.758	.778	.652	.669	.686	.702	.718
	20	.968	1.043	1.122	1.204	1.289	.865	.916	.969	1.022	1.075	.799	.838	.876	.915	.952	.749	.780	.810	.839	.868	.710	.735	.759	.782	.805
	25	1.001	1.086	1.175	1.269	1.366	.904	.963	1.024	1.087	1.149	.840	.886	.933	.979	1.025	.792	.829	.866	.903	.939	.753	.784	.814	.844	.873
	30	1.027	1.118	1.216	1.320	1.428	.933	1.000	1.068	1.139	1.210	.872	.924	.977	1.031	1.085	.825	.868	.912	.954	.997	.788	.824	.860	.895	.930
	35	1.047	1.145	1.250	1.362	1.480	.957	1.029	1.105	1.182	1.261	.898	.956	1.015	1.075	1.135	.853	.901	.949	.998	1.046	.816	.857	.898	.938	.979
	40	1.064	1.167	1.279	1.398	1.525	.977	1.054	1.135	1.219	1.305	.919	.982	1.046	1.112	1.178	.875	.928	.981	1.035	1.089	.839	.885	.930	.976	1.021
6	10	.872	.922	.973	1.025	1.077	.749	.779	.808	.837	.865	.675	.694	.713	.732	.750	.622	.635	.648	.660	.672	.581	.590	.599	.607	.615
	15	.958	1.026	1.096	1.168	1.241	.840	.883	.927	.970	1.014	.766	.797	.828	.858	.887	.712	.736	.758	.780	.802	.670	.688	.705	.722	.738
	20	1.013	1.093	1.178	1.265	1.355	.899	.954	1.009	1.064	1.120	.827	.868	.908	.948	.988	.774	.806	.837	.868	.898	.732	.757	.782	.806	.830
	25	1.052	1.143	1.239	1.339	1.443	.942	1.006	1.071	1.136	1.203	.873	.921	.970	1.019	1.067	.820	.859	.898	.936	.974	.779	.811	.842	.873	.903
	30	1.082	1.181	1.287	1.399	1.515	.976	1.047	1.120	1.195	1.270	.908	.963	1.020	1.076	1.132	.857	.902	.947	.992	1.037	.816	.854	.891	.928	.965
	35	1.107	1.213	1.327	1.449	1.576	1.003	1.081	1.161	1.244	1.328	.937	.999	1.061	1.124	1.188	.887	.938	.989	1.040	1.091	.847	.890	.933	.975	1.017
	40	1.127	1.240	1.362	1.491	1.628	1.026	1.110	1.196	1.286	1.378	.961	1.028	1.097	1.166	1.236	.912	.968	1.025	1.081	1.138	.873	.921	.968	1.016	1.063
≥ 6.12	10	.876	.926	.978	1.030	1.083	.752	.782	.811	.840	.869	.677	.697	.716	.734	.752	.624	.637	.650	.663	.674	.583	.592	.601	.609	.617
	15	.964	1.032	1.102	1.175	1.248	.844	.887	.931	.975	1.018	.769	.801	.831	.862	.891	.715	.738	.761	.783	.805	.672	.690	.708	.724	.741
	20	1.019	1.100	1.185	1.274	1.365	.904	.959	1.014	1.070	1.126	.831	.872	.913	.953	.993	.777	.809	.841	.872	.902	.735	.760	.785	.810	.833
	25	1.059	1.151	1.248	1.349	1.454	.948	1.012	1.077	1.143	1.210	.877	.926	.975	1.024	1.073	.824	.863	.902	.941	.978	.782	.814	.846	.877	.908
	30	1.090	1.190	1.297	1.410	1.527	.982	1.054	1.127	1.202	1.279	.913	.969	1.025	1.082	1.139	.861	.907	.952	.998	1.042	.820	.858	.896	.933	.969
	35	1.115	1.222	1.338	1.460	1.589	1.010	1.088	1.169	1.252	1.337	.942	1.004	1.067	1.131	1.195	.892	.943	.994	1.046	1.097	.851	.894	.938	.980	1.023
	40	1.135	1.250	1.373	1.504	1.642	1.033	1.117	1.205	1.295	1.388	.967	1.035	1.103	1.173	1.244	.917	.974	1.031	1.088	1.145	.877	.925	.974	1.022	1.069

ϕ_G-FACTORS

Table 4

H/L	b	1					2					3					4					5				
	c	1.0	1.1	1.2	1.3	1.4	1.0	1.1	1.2	1.3	1.4	1.0	1.1	1.2	1.3	1.4	1.0	1.1	1.2	1.3	1.4	1.0	1.1	1.2	1.3	1.4
	n																									
0	10	.939	.988	1.039	1.090	1.141	.783	.813	.842	.870	.898	.699	.718	.737	.755	.773	.641	.654	.667	.679	.691	.596	.606	.615	.623	.631
	15	.958	1.020	1.085	1.151	1.220	.829	.870	.911	.952	.993	.754	.784	.813	.842	.870	.701	.724	.745	.766	.787	.660	.677	.694	.710	.725
	20	.968	1.039	1.113	1.191	1.271	.857	.906	.956	1.007	1.057	.790	.827	.864	.901	.938	.741	.770	.799	.828	.856	.702	.726	.749	.772	.794
	25	.974	1.051	1.133	1.218	1.308	.876	.931	.988	1.046	1.106	.815	.858	.902	.946	.990	.770	.805	.840	.875	.909	.733	.762	.791	.820	.848
	30	.978	1.060	1.147	1.239	1.337	.890	.950	1.013	1.078	1.144	.834	.882	.931	.981	1.031	.792	.832	.872	.912	.952	.757	.791	.825	.858	.891
	35	.981	1.066	1.158	1.256	1.360	.901	.965	1.033	1.103	1.175	.849	.901	.955	1.010	1.066	.809	.853	.898	.943	.988	.776	.814	.852	.890	.928
	40	.984	1.072	1.167	1.270	1.379	.909	.977	1.049	1.124	1.201	.861	.917	.975	1.034	1.095	.823	.871	.920	.969	1.019	.792	.834	.876	.918	.960
3	10	.976	1.026	1.078	1.131	1.184	.806	.836	.866	.896	.924	.716	.736	.755	.774	.792	.655	.668	.682	.694	.706	.609	.618	.627	.636	.644
	15	1.003	1.068	1.136	1.205	1.276	.859	.901	.943	.986	1.028	.777	.808	.838	.867	.896	.720	.743	.766	.787	.808	.676	.694	.711	.727	.743
	20	1.020	1.094	1.173	1.254	1.338	.891	.943	.995	1.047	1.100	.817	.856	.894	.933	.971	.764	.794	.824	.854	.883	.722	.747	.771	.794	.817
	25	1.031	1.112	1.199	1.290	1.385	.915	.973	1.033	1.094	1.155	.846	.891	.937	.982	1.028	.796	.832	.869	.905	.941	.756	.786	.816	.846	.875
	30	1.039	1.126	1.220	1.318	1.422	.932	.996	1.062	1.130	1.199	.868	.919	.970	1.022	1.074	.821	.862	.904	.946	.988	.783	.818	.853	.888	.922
	35	1.046	1.137	1.236	1.341	1.452	.946	1.015	1.086	1.160	1.236	.885	.941	.997	1.055	1.113	.840	.887	.933	.980	1.027	.804	.844	.883	.923	.962
	40	1.051	1.147	1.250	1.361	1.479	.957	1.030	1.106	1.185	1.267	.900	.959	1.020	1.083	1.146	.857	.907	.958	1.010	1.062	.822	.865	.909	.953	.997
4	10	1.004	1.056	1.109	1.163	1.217	.824	.855	.885	.915	.944	.730	.750	.770	.789	.807	.666	.680	.693	.706	.718	.618	.628	.637	.645	.654
	15	1.039	1.106	1.175	1.247	1.320	.881	.925	.969	1.012	1.055	.795	.826	.857	.887	.917	.735	.758	.781	.803	.825	.689	.707	.724	.741	.757
	20	1.060	1.137	1.219	1.303	1.391	.918	.971	1.025	1.079	1.134	.838	.878	.918	.957	.996	.782	.813	.844	.874	.903	.738	.763	.787	.811	.834
	25	1.075	1.160	1.251	1.346	1.445	.945	1.005	1.067	1.130	1.194	.870	.917	.964	1.011	1.058	.816	.854	.891	.928	.965	.774	.805	.836	.866	.895
	30	1.086	1.178	1.276	1.380	1.488	.965	1.031	1.100	1.171	1.243	.894	.947	1.000	1.054	1.108	.843	.886	.929	.973	1.015	.802	.839	.875	.911	.946
	35	1.096	1.193	1.297	1.408	1.525	.981	1.053	1.128	1.205	1.283	.914	.971	1.030	1.090	1.150	.865	.913	.961	1.010	1.058	.826	.867	.908	.948	.988
	40	1.103	1.205	1.315	1.432	1.556	.994	1.071	1.151	1.233	1.319	.930	.992	1.056	1.121	1.186	.883	.935	.988	1.041	1.095	.845	.890	.935	.981	1.026
5	10	1.041	1.094	1.149	1.204	1.260	.847	.879	.910	.940	.970	.747	.768	.788	.807	.826	.680	.694	.708	.721	.733	.630	.640	.649	.658	.666
	15	1.084	1.154	1.226	1.301	1.377	.911	.956	1.001	1.046	1.091	.818	.850	.882	.913	.943	.754	.778	.802	.824	.846	.705	.724	.742	.759	.775
	20	1.111	1.193	1.278	1.367	1.458	.953	1.008	1.064	1.120	1.177	.866	.907	.948	.988	1.029	.805	.837	.869	.900	.930	.758	.783	.809	.833	.857
	25	1.131	1.222	1.317	1.418	1.522	.983	1.047	1.111	1.177	1.244	.901	.949	.998	1.047	1.096	.842	.881	.920	.958	.996	.797	.829	.861	.892	.922
	30	1.147	1.245	1.349	1.459	1.573	1.007	1.077	1.149	1.223	1.298	.928	.983	1.039	1.095	1.151	.872	.917	.962	1.007	1.051	.828	.866	.903	.940	.976
	35	1.160	1.263	1.375	1.493	1.617	1.026	1.102	1.181	1.262	1.345	.950	1.011	1.072	1.135	1.197	.896	.946	.997	1.047	1.097	.854	.896	.939	.981	1.022
	40	1.171	1.280	1.397	1.523	1.655	1.042	1.123	1.208	1.295	1.385	.969	1.034	1.101	1.169	1.238	.917	.971	1.027	1.082	1.138	.875	.922	.969	1.016	1.063
6	10	1.085	1.141	1.197	1.255	1.313	.875	.907	.940	.971	1.002	.768	.790	.810	.830	.849	.697	.712	.726	.739	.752	.645	.655	.664	.673	.682
	15	1.139	1.212	1.288	1.366	1.446	.947	.994	1.041	1.087	1.134	.846	.879	.912	.944	.975	.777	.802	.826	.850	.872	.725	.744	.763	.780	.797
	20	1.174	1.260	1.351	1.445	1.541	.995	1.053	1.112	1.170	1.229	.899	.942	.985	1.027	1.068	.833	.866	.899	.931	.962	.782	.809	.835	.860	.885
	25	1.200	1.297	1.399	1.505	1.616	1.031	1.097	1.166	1.235	1.304	.938	.989	1.041	1.092	1.142	.874	.915	.955	.995	1.034	.825	.858	.891	.923	.955
	30	1.221	1.326	1.438	1.555	1.678	1.059	1.133	1.209	1.287	1.366	.970	1.028	1.086	1.145	1.203	.907	.954	1.001	1.048	1.094	.859	.899	.937	.976	1.013
	35	1.238	1.350	1.470	1.597	1.730	1.081	1.162	1.246	1.332	1.419	.995	1.059	1.124	1.190	1.255	.935	.987	1.040	1.093	1.146	.888	.932	.977	1.020	1.064
	40	1.253	1.371	1.498	1.634	1.777	1.101	1.187	1.277	1.371	1.466	1.017	1.086	1.157	1.229	1.301	.958	1.015	1.074	1.132	1.191	.912	.961	1.010	1.059	1.108
≥ 6.12	10	1.091	1.147	1.204	1.262	1.320	.879	.911	.944	.975	1.006	.771	.792	.813	.833	.852	.700	.714	.728	.742	.754	.647	.657	.667	.676	.684
	15	1.146	1.220	1.297	1.375	1.455	.952	.999	1.046	1.093	1.139	.850	.883	.916	.948	.980	.780	.805	.830	.853	.876	.728	.747	.766	.783	.800
	20	1.183	1.270	1.361	1.455	1.552	1.001	1.059	1.118	1.177	1.236	.904	.947	.990	1.032	1.074	.836	.870	.903	.935	.967	.785	.812	.838	.864	.888
	25	1.210	1.307	1.410	1.517	1.629	1.037	1.104	1.173	1.243	1.313	.944	.995	1.046	1.098	1.148	.879	.920	.960	1.000	1.039	.829	.862	.895	.928	.959
	30	1.231	1.337	1.450	1.568	1.692	1.066	1.141	1.218	1.296	1.376	.975	1.034	1.092	1.151	1.210	.912	.960	1.007	1.054	1.100	.863	.903	.942	.981	1.018
	35	1.249	1.362	1.483	1.612	1.746	1.089	1.170	1.255	1.341	1.430	1.001	1.066	1.131	1.197	1.263	.940	.993	1.046	1.099	1.152	.892	.937	.982	1.026	1.069
	40	1.264	1.383	1.512	1.649	1.793	1.109	1.196	1.287	1.381	1.477	1.023	1.093	1.164	1.237	1.310	.963	1.021	1.080	1.139	1.198	.917	.966	1.016	1.065	1.114

$\phi_{c'}$ - FACTORS

Table 5

H/L	b	1					2					3					4					5				
	C	1.0	1.1	1.2	1.3	1.4	1.0	1.1	1.2	1.3	1.4	1.0	1.1	1.2	1.3	1.4	1.0	1.1	1.2	1.3	1.4	1.0	1.1	1.2	1.3	1.4
	n																									
0	10	1.019	1.047	1.076	1.106	1.137	.975	.996	1.016	1.037	1.059	.940	.956	.971	.986	1.001	.911	.922	.934	.945	.956	.885	.894	.902	.910	.918
	15	1.013	1.044	1.077	1.111	1.146	.982	1.007	1.033	1.059	1.085	.955	.976	.997	1.018	1.039	.932	.949	.966	.983	1.000	.911	.925	.939	.953	.967
	20	1.010	1.043	1.077	1.113	1.151	.986	1.013	1.042	1.071	1.101	.964	.988	1.012	1.037	1.062	.945	.966	.987	1.007	1.029	.928	.946	.964	.982	.999
	25	1.008	1.042	1.077	1.115	1.154	.988	1.018	1.048	1.079	1.112	.970	.996	1.023	1.050	1.078	.954	.977	1.000	1.024	1.048	.939	.960	.980	1.001	1.022
	30	1.007	1.041	1.077	1.116	1.156	.990	1.021	1.052	1.085	1.120	.975	1.002	1.030	1.059	1.089	.961	.985	1.011	1.036	1.063	.947	.970	.993	1.016	1.039
	35	1.006	1.041	1.077	1.116	1.158	.991	1.023	1.055	1.090	1.125	.978	1.006	1.036	1.067	1.098	.965	.991	1.018	1.046	1.074	.954	.978	1.002	1.027	1.053
	40	1.005	1.040	1.078	1.117	1.159	.992	1.024	1.058	1.093	1.130	.980	1.010	1.040	1.072	1.105	.969	.996	1.024	1.053	1.083	.959	.984	1.010	1.036	1.063
3	10	1.048	1.077	1.108	1.139	1.172	1.002	1.023	1.045	1.067	1.089	.965	.981	.997	1.013	1.029	.935	.947	.958	.970	.982	.908	.917	.926	.934	.942
	15	1.044	1.077	1.111	1.147	1.184	1.011	1.037	1.064	1.092	1.120	.983	1.005	1.026	1.048	1.070	.959	.977	.994	1.012	1.030	.937	.952	.966	.981	.995
	20	1.043	1.077	1.113	1.151	1.191	1.017	1.046	1.075	1.106	1.138	.994	1.019	1.044	1.070	1.096	.974	.995	1.017	1.039	1.061	.956	.974	.993	1.011	1.030
	25	1.041	1.077	1.114	1.154	1.195	1.020	1.051	1.083	1.116	1.150	1.001	1.028	1.056	1.084	1.113	.984	1.008	1.032	1.057	1.082	.968	.989	1.011	1.033	1.055
	30	1.040	1.077	1.115	1.155	1.198	1.022	1.054	1.088	1.123	1.159	1.006	1.035	1.064	1.095	1.126	.991	1.017	1.044	1.071	1.098	.977	1.001	1.025	1.049	1.073
	35	1.040	1.077	1.115	1.157	1.200	1.024	1.057	1.092	1.128	1.165	1.010	1.040	1.071	1.103	1.136	.997	1.024	1.052	1.081	1.111	.984	1.009	1.035	1.061	1.088
	40	1.039	1.076	1.116	1.158	1.202	1.026	1.059	1.095	1.132	1.171	1.013	1.044	1.076	1.109	1.144	1.001	1.029	1.059	1.089	1.121	.990	1.016	1.043	1.071	1.099
4	10	1.071	1.101	1.133	1.165	1.198	1.023	1.045	1.067	1.090	1.112	.985	1.002	1.018	1.034	1.050	.953	.966	.978	.990	1.002	.926	.935	.944	.953	.961
	15	1.069	1.103	1.138	1.175	1.213	1.034	1.061	1.089	1.117	1.146	1.005	1.027	1.050	1.072	1.095	.980	.998	1.016	1.035	1.053	.957	.972	.987	1.002	1.017
	20	1.068	1.103	1.141	1.180	1.222	1.041	1.071	1.101	1.133	1.166	1.017	1.043	1.069	1.095	1.122	.996	1.018	1.041	1.063	1.086	.977	.996	1.015	1.035	1.054
	25	1.067	1.104	1.143	1.184	1.227	1.045	1.077	1.110	1.144	1.179	1.025	1.053	1.082	1.111	1.141	1.007	1.032	1.057	1.083	1.109	.991	1.013	1.035	1.057	1.080
	30	1.067	1.104	1.144	1.186	1.231	1.048	1.081	1.115	1.152	1.189	1.031	1.060	1.091	1.123	1.155	1.015	1.042	1.069	1.097	1.126	1.000	1.025	1.049	1.074	1.100
	35	1.066	1.104	1.145	1.188	1.233	1.050	1.084	1.120	1.157	1.196	1.035	1.066	1.098	1.131	1.166	1.021	1.049	1.078	1.108	1.139	1.008	1.034	1.060	1.088	1.115
	40	1.066	1.105	1.146	1.189	1.235	1.051	1.086	1.123	1.162	1.202	1.038	1.070	1.103	1.138	1.174	1.025	1.055	1.086	1.117	1.150	1.014	1.041	1.069	1.098	1.128
5	10	1.100	1.131	1.164	1.198	1.233	1.050	1.073	1.096	1.119	1.143	1.010	1.027	1.044	1.061	1.078	.977	.990	1.003	1.015	1.027	.949	.958	.967	.976	.985
	15	1.100	1.135	1.173	1.211	1.251	1.064	1.092	1.121	1.150	1.180	1.033	1.056	1.079	1.103	1.126	1.007	1.026	1.045	1.064	1.082	.983	.999	1.014	1.030	1.045
	20	1.100	1.137	1.177	1.218	1.261	1.071	1.103	1.135	1.168	1.202	1.047	1.073	1.101	1.128	1.156	1.025	1.048	1.071	1.094	1.118	1.005	1.025	1.045	1.065	1.085
	25	1.100	1.139	1.180	1.223	1.268	1.076	1.110	1.144	1.180	1.217	1.056	1.085	1.115	1.146	1.177	1.037	1.063	1.089	1.116	1.143	1.019	1.042	1.066	1.089	1.113
	30	1.100	1.140	1.181	1.226	1.272	1.080	1.115	1.151	1.189	1.228	1.062	1.093	1.125	1.158	1.192	1.045	1.073	1.102	1.131	1.161	1.030	1.055	1.081	1.107	1.134
	35	1.100	1.140	1.183	1.228	1.276	1.082	1.118	1.156	1.195	1.236	1.067	1.099	1.133	1.168	1.204	1.052	1.082	1.112	1.143	1.176	1.038	1.065	1.093	1.122	1.150
	40	1.100	1.141	1.184	1.230	1.279	1.084	1.121	1.160	1.200	1.243	1.070	1.104	1.139	1.175	1.213	1.057	1.088	1.120	1.153	1.187	1.045	1.073	1.103	1.133	1.164
6	10	1.135	1.169	1.203	1.238	1.275	1.083	1.107	1.131	1.155	1.180	1.041	1.059	1.077	1.094	1.112	1.007	1.020	1.033	1.046	1.059	.977	.987	.996	1.006	1.015
	15	1.138	1.176	1.215	1.255	1.297	1.100	1.129	1.159	1.190	1.222	1.067	1.092	1.116	1.140	1.165	1.040	1.059	1.079	1.099	1.119	1.015	1.031	1.047	1.064	1.080
	20	1.140	1.179	1.221	1.264	1.310	1.109	1.142	1.176	1.211	1.247	1.083	1.111	1.139	1.168	1.198	1.060	1.084	1.108	1.133	1.157	1.039	1.059	1.080	1.101	1.122
	25	1.140	1.181	1.225	1.270	1.318	1.115	1.150	1.187	1.225	1.264	1.093	1.124	1.155	1.188	1.221	1.073	1.100	1.128	1.156	1.184	1.055	1.079	1.103	1.128	1.153
	30	1.141	1.183	1.227	1.274	1.324	1.119	1.156	1.195	1.235	1.276	1.100	1.133	1.167	1.202	1.237	1.083	1.112	1.142	1.173	1.204	1.067	1.093	1.120	1.148	1.175
	35	1.141	1.184	1.229	1.277	1.328	1.123	1.161	1.200	1.242	1.285	1.106	1.140	1.176	1.212	1.250	1.090	1.121	1.153	1.186	1.220	1.076	1.104	1.133	1.163	1.193
	40	1.142	1.185	1.231	1.279	1.331	1.125	1.164	1.205	1.248	1.293	1.110	1.145	1.182	1.221	1.261	1.096	1.128	1.162	1.197	1.233	1.083	1.113	1.144	1.175	1.208
≥ 6.12	10	1.140	1.174	1.208	1.244	1.281	1.087	1.111	1.136	1.160	1.185	1.046	1.063	1.081	1.099	1.116	1.011	1.024	1.037	1.050	1.063	.981	.991	1.000	1.010	1.019
	15	1.143	1.181	1.220	1.261	1.304	1.104	1.134	1.165	1.196	1.227	1.072	1.096	1.121	1.145	1.170	1.044	1.064	1.084	1.104	1.124	1.019	1.036	1.052	1.068	1.084
	20	1.145	1.185	1.227	1.271	1.317	1.114	1.147	1.182	1.217	1.253	1.088	1.116	1.145	1.174	1.204	1.064	1.089	1.113	1.138	1.163	1.043	1.064	1.085	1.106	1.127
	25	1.146	1.187	1.231	1.277	1.325	1.120	1.156	1.193	1.231	1.270	1.098	1.129	1.161	1.193	1.227	1.078	1.105	1.133	1.161	1.190	1.060	1.084	1.108	1.133	1.158
	30	1.147	1.189	1.233	1.281	1.331	1.125	1.162	1.200	1.241	1.283	1.105	1.138	1.172	1.208	1.244	1.088	1.117	1.148	1.179	1.210	1.072	1.098	1.125	1.153	1.181
	35	1.147	1.190	1.235	1.284	1.335	1.128	1.166	1.206	1.248	1.292	1.111	1.145	1.181	1.218	1.257	1.095	1.127	1.159	1.192	1.226	1.081	1.109	1.139	1.169	1.199
	40	1.147	1.191	1.237	1.286	1.338	1.131	1.170	1.211	1.254	1.299	1.115	1.151	1.188	1.227	1.267	1.101	1.134	1.168	1.203	1.239	1.088	1.118	1.149	1.181	1.214

$\phi_{C''}$ - FACTORS

Step 7—Determine the total drift (sum of components from Step 6):

$$D_T = 15.2 + 5.3 + 2.2 = 22.7 \text{ in.}$$

This ends the procedure of drift establishment. If this amplitude should exceed the desired or permissible drift, the reader is advised to avail himself of previously published material[1] for material optimized drift adjustments and estimating material quantities.

DEVELOPMENT OF METHOD AND WORKING EQUATIONS

Seismic Loading—Currently accepted seismic design practices require that the total seismic base shear of the bent be distributed over the height of the bent in the following manner:[2]

$$F_{(i)} = \frac{VmH_{(i)}}{\sum_{i=1}^{i=n} mH_{(i)}}\left(1 - \frac{F_t}{V}\right) \qquad (1)$$

in which

$$\frac{F_t}{V} = 0.004\left(\frac{H}{L}\right)^2 \qquad (2)$$

Figure 4 shows the general arrangement of this loading requirement under the assumption of equal story heights and equal masses per level throughout the bent.

If the value for

$$H_{(i)} = H\left(1 - \frac{i-1}{n}\right) \qquad (3)$$

is introduced into Eq. (1), the following expression will result:

$$F_{(i)} = \frac{VmH\left(1 - \frac{i-1}{n}\right)}{mH\sum_{i=1}^{i=n}\left(1 - \frac{i-1}{n}\right)}\left(1 - \frac{F_t}{V}\right)$$

which will develop into the equation of the typical seismic panel load at any given level:

$$F_{(i)} = 2V\frac{(n+1-i)}{(n)(n+1)}\left(1 - \frac{F_t}{V}\right) \qquad (4)$$

At this stage, it should be pointed out that for aspect ratios of $H/L < 3$, no top loading F_t need be applied. Furthermore, in no case will it be required that the top loading F_t exceed 15% of the total base shear V (for aspect ratios greater than 6.12).

Using Eq. (4) as a base, other general statements can now be obtained (see Fig. 4):

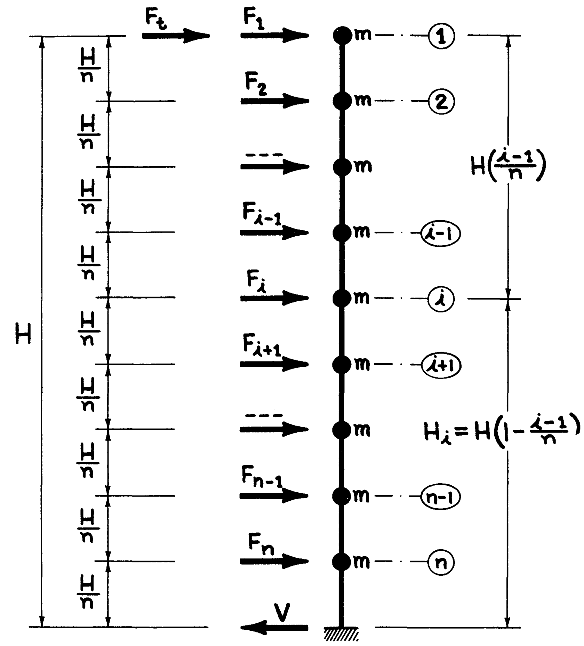

Figure 4

Seismic Shear Immediately below Level *i*:

$$V_{(i)} = F_t + \sum_{a=1}^{a=i} F_{(a)}$$

$$= F_t + \frac{2V}{(n)(n+1)}\left(1 - \frac{F_t}{V}\right)\sum_{a=1}^{a=i}(n+1-a)$$

Developed:

$$V_{(i)} = V\left[1 - \left(1 - \frac{F_t}{V}\right)\frac{(n-i)(n+1-i)}{(n)(n+1)}\right] \qquad (5)$$

Overturning Moment at Half-Story Point below Level *i*:

$$M_{(i)} = F_tH\left(\frac{i-0.5}{n}\right) + \sum_{a=1}^{a=i}\left[F_{(a)}H\frac{(i-a+0.5)}{(n)}\right]$$

Substituting:

$$M_{(i)} = F_tH\left(\frac{i-0.5}{n}\right) + 2VH\left(1 - \frac{F_t}{V}\right)\sum_{a=1}^{a=i}\frac{(n+1-a)(i+0.5-a)}{(n^2)(n+1)}$$

Developed:

$$M_{(i)} = VH\left\{\left(\frac{i-0.5}{n}\right) - \frac{1}{3}\left(1 - \frac{F_t}{V}\right)\left[\left(1 - \frac{1}{n}\right) - \left(1 - \frac{i}{n}\right)\left(1 - \frac{i}{n+1}\right)\left(1 - \frac{i-0.5}{n}\right)\right]\right\} \qquad (6)$$

Material Properties—In a normally proportioned steel bent, cross-sectional material properties (hereinafter in general called P), such as cross-sectional area, section modulus, and moment of inertia, usually increase from a minimum value at roof level, P_1, to a maximum value at the level above grade, P_n. In steel bents where drift investigations become advisable (starting at about 10 to 15 levels), it has been found to be justifiable to express the increase and distribution of cross-sectional properties as a continuous mathematical function, as shown in Fig. 5.

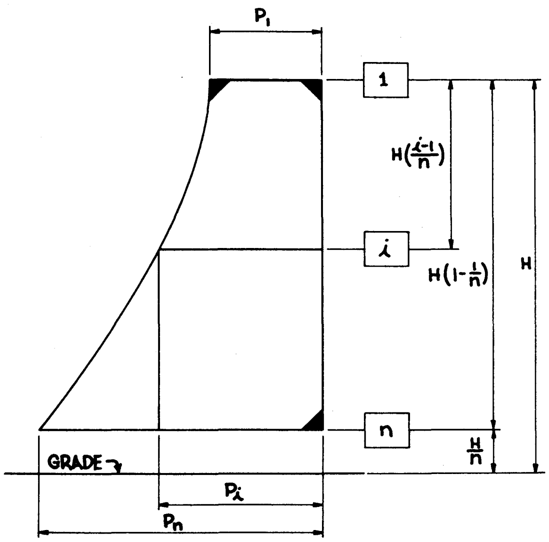

Figure 5

Introducing a "Material Property Distribution Factor,"

$$b = \frac{nP_1}{P_n} \tag{7}$$

where P_1 signifies the magnitude of a given material property at level 1 and P_n stands for the size of this material property at level n, it is possible to state the magnitude of any material property at any level throughout the bent as:

$$P_i = \frac{P_n}{n}\left[b + (n - b)\left(\frac{i-1}{n-1}\right)^c\right] \tag{8}$$

Rearranging Eq. (8) will then yield the "Material Property Distribution Exponent," c:

$$c = \frac{\log\left[\dfrac{P_i - P_1}{P_n - P_1}\right]}{\log\left[\dfrac{i-1}{n-1}\right]} \tag{9}$$

For drift computations, specific material property values for girder stiffnesses, column stiffnesses, bent moments of inertia, and bracing "stiffnesses," at levels 1, i, and n, will be substituted for the general expression P in Eqs. (7), (8), and (9). The mathematical expressions for these values are summarized in Table 2.

DRIFT IN PLANAR RIGID BENTS

In planar rigid bents, three different drift components will be investigated:

a. Girder Moment Drift (caused by the flexing of the girders)

b. Column Moment Drift (caused by the flexing of the columns)

c. Column Chord Drift (caused by the axial deformation of the columns)

Drift due to axial forces in the girders can be regarded as insignificant and will not be included in this investigation.

The establishment of these drift components can be facilitated by making certain simplifying assumptions, based on the widely used and, for this purpose, accepted portal-cantilever method of rigid multi-story frames:

a. Girders have points of contraflexure at mid-span.

b. Columns have points of contraflexure at mid-story height.

c. The plane of floor levels remains plane after deformation due to bending; therefore, columns deform axially in proportion to their distance to the common center of gravity of the cross-sectional areas of all columns in the same bent.

Using these assumptions, it can be demonstrated[1] that the contribution of a typical level to the total lateral bent motion will be as follows:

Girder Moment Drift:

$$d_{G(1)} = \frac{V_{(1)}H^2}{48n^2ET_{G(1)}} \tag{10}$$

$$d_{G(i)} = \frac{H^2}{24n^2ET_{G(i)}}\left(V_{(i)} + V_{(i-1)}\right) \tag{11}$$

Column Moment Drift:

$$d_{C'(i)} = \frac{V_{(i)}H^2}{12n^2ET_{C(i)}} \tag{12}$$

Column Chord Drift:

$$d_{C''(i)} = \frac{M_{(i)}H^2(i - 0.5)}{n^2EI_{B(i)}} \tag{13}$$

To obtain the total drift components for the whole bent, the following steps will now be taken:

1. Express the material properties $T_{G(i)}$, $T_{C(i)}$, and $I_{B(i)}$, as prescribed in Eq. (8).
2. Formulate building shears as per Eq. (5) as follows:

$$V_{(1)} = V\left[1 - \left(1 - \frac{F_t}{V}\right)\left(\frac{n-1}{n+1}\right)\right]$$

to be used in Eq. (10)

$$V_{(i)} = V\left[1 - \left(1 - \frac{F_t}{V}\right)\frac{(n-i)(n+1-i)}{(n)\ (n+1)}\right]$$

(note that $i \neq 1$); to be used in Eqs. (11) and (12)

$$V_{(i-1)} = V\left[1 - \left(1 - \frac{F_t}{V}\right)\frac{(n+1-i)(n+2-i)}{(n)\ (n+1)}\right]$$

to be used in Eq. (11)

3. Insert Eq. (6) into Eq. (13).
4. Form the summation of Eqs. (11), (12), and (13) (except as noted) from $i = 1$ to $i = n$.

The total drift components for the entire bent will then work out to the following working equations, which are listed for your convenience in Table 1:

Girder Moment Drift:

$$D_G = \frac{VH^2}{8nET_{G(n)}}\ (\phi_G) \tag{14}$$

or, expressed in customary design units (see Table 1)

$$D_G = \frac{VH^2}{134nT_{G(n)}}\ (\phi_G) \tag{15}$$

in which

$$\Phi_G = \frac{1 - \left(1 - \dfrac{F_t}{V}\right)\left(\dfrac{n-1}{n+1}\right)}{6b_G} + \frac{2}{3}\sum_{i=2}^{i=n}\left[\frac{1 - \left(1 - \dfrac{F_t}{V}\right)\dfrac{(n+1-i)^2}{(n)\ (n+1)}}{b_G + (n - b_G)\left(\dfrac{i-1}{n-1}\right)^{c_G}}\right] \tag{16}$$

Values of Φ_G are tabulated in Table 3 for various values of H/L, b_G and c_G. They represent frame constants.

Column Moment Drift:

$$D_{C'} = \frac{VH^2}{8nET_{C(n)}}\ (\phi_{C'}) \tag{17}$$

or, again using customary design units

$$D_{C'} = \frac{VH^2}{134nT_{C(n)}}\ (\phi_{C'}) \tag{18}$$

in which

$$\Phi_{C'} = \frac{2}{3}\sum_{i=1}^{i=n}\left[\frac{1 - \left(1 - \dfrac{F_t}{V}\right)\dfrac{(n-i)(n+1-i)}{(n)\ (n+1)}}{b_{C'} + (n - b_{C'})\left(\dfrac{i-1}{n-1}\right)^{c_{C'}}}\right] \tag{19}$$

Values of $\phi_{C'}$ are tabulated in Table 4 for various values of H/L, $b_{C'}$, and $c_{C'}$. They represent frame constants.

Column Chord Drift:

$$D_{C''} = \frac{VH^3}{4EI_{B(n)}}\ (\phi_{C''}) \tag{20}$$

or, using design units as indicated in Table 1,

$$D_{C''} = \frac{V}{I_{B(n)}}\left(\frac{H}{112}\right)^3\ (\phi_{C''}) \tag{21}$$

in which

$$\phi_{C''} = 4\sum_{i=1}^{i=n}\left(\frac{i-0.5}{n}\right) \times \left\{\frac{\left(\dfrac{i-0.5}{n}\right) - \dfrac{1}{3}\left(1 - \dfrac{F_t}{V}\right) \times \left[\dfrac{(n-1)}{(n)} - \dfrac{(n-i)(n+1-i)(n+0.5-i)}{(n)\quad(n+1)\qquad(n)}\right]}{b_{C''} + (n - b_{C''})\left(\dfrac{i-1}{n-1}\right)^{c_{C''}}}\right\} \tag{22}$$

Values of $\phi_{C''}$ are tabulated in Table 5 for various values of H/L, $b_{C''}$ and $c_{C''}$. They represent frame constants.

The total drift of the planar rigid bent will be simply the sum of the three components: Girder Moment Drift, Column Moment Drift, and Column Chord Drift, or

$$D_T = D_G + D_{C'} + D_{C''} \tag{23}$$

DRIFT IN PLANAR BRACED BENTS

In planar braced bents, only two different drift components have to be established:

a. Bracing Drift (caused by the axial deformation of the bracing members)
b. Column Chord Drift (caused by the axial deformation of the columns)

The magnitude of the Column Chord Drift can be assumed to be the same as in planar rigid bents. Equations (20), (21), and (22), therefore, also apply for drift calculations in planar braced bents. Only one idealizing assumption has to be made to obtain the size of the bracing drift:

> Any point of intersection of frame components is a pinned joint, allowing unrestrained rotation of any joining frame component.

With this assumption, it has been shown[1] that the contribution of a typical level to the total bracing drift can be formulated in the following manner:

X-Bracing Drift:

$$d_D = \frac{V_{(i)}}{ET_{D(i)}} \tag{24}$$

Compression diagnonals of X-bracing are assumed to buckle and become ineffective.

K-Bracing Drift:

$$d_D = \frac{2V_{(i)}}{ET_{D(i)}} \tag{25}$$

To obtain the total bracing drift components for the whole bent, the following steps have to be taken:

1. Express the material property $T_{D(i)}$ as prescribed in Eq. (8).
2. Insert Eq. (5) into Eqs. (24) and (25).
3. Form the summation of Eqs. (24) and (25) from $i = 1$ to $i = n$.

This procedure will then result in a statement on the total bracing drift for the entire bent. These working equations are listed in Table 1:

X-Bracing Drift:

$$D_D = \frac{1.5nV}{ET_{D(n)}} (\phi_D) \tag{26}$$

or, using customary design units as indicated in Table 1

$$D_D = \frac{nV}{1610T_{D(n)}} (\phi_D) \tag{27}$$

K-Bracing Drift:

$$D_D = \frac{3nV}{ET_{D(n)}} (\phi_D) \tag{28}$$

or, again using the design unit notations from Table 1,

$$D_D = \frac{nV}{805T_{D(n)}} (\phi_D) \tag{29}$$

In Eqs. (28) and (29), the value for ϕ_D is identical to the value of $\phi_{C'}$ as listed in Table 4.

The total drift of the planar braced bent will be the sum of the Bracing Drift component and the Column Chord Drift component, or

$$D_T = D_D + D_{C''} \tag{30}$$

Under certain circumstances it will be desirable or required [Section 2313(j)b][2] to use a lateral force resisting system consisting of a braced frame working in conjunction with a moment resisting space frame.

In these cases, the shear drift components of the moment resisting subsystem (girder moment drift D_G plus column moment drift $D_{C'}$) have to be of the same magnitude as the shear drift component of the braced subsystem (bracing drift D_D).

The resulting shear induced motion amplitudes of this balanced system will be as follows:

$$D_G = \frac{VH^2(\phi_G)}{134nT_{G(n)}} \left(\frac{1}{\lambda}\right) \tag{31}$$

$$D_{C'} = \frac{VH^2(\phi_{C'})}{134nT_{C(n)}} \left(\frac{1}{\lambda}\right) \tag{32}$$

$$D_D = D_G + D_{C'} \tag{33}$$

in which

$$\lambda = 1 + 12\left(\frac{H}{n}\right)^2 \left[\frac{T_{D(n)}}{\varphi_D}\right]\left[\frac{\varphi_G}{T_{G(n)}} + \frac{\varphi_{C'}}{T_{C(n)}}\right] \tag{34}$$

for X-braced subsystems, and

$$\lambda = 1 + 12\left(\frac{H}{n}\right)^2 \left[\frac{T_{D(n)}}{2\varphi_D}\right]\left[\frac{\varphi_G}{T_{G(n)}} + \frac{\varphi_{C'}}{T_{C(n)}}\right] \tag{35}$$

for K-braced subsystems.

The total frame drift will be as per Eq. (23). In the above listed equations, the dimensional units of Table 1 apply.

CONCLUSIONS

The foregoing has demonstrated again, as have other technical papers in the past, that high-rise frames as an entity are predictable as to their material distribution and also as to their performance. Only a minimum of detailed information is necessary. By looking at a structure as a whole, rather than using the traditional "stick by stick" design approach, the designer of a structure will have a much better "reading" than with the traditional method without ever losing the "feel" of the structure.

It is to be hoped that the time will come when interpolative design methods, based on the analysis of only a few strategic features of the structure, will be fully acceptable. The practicing designer will be a grateful beneficiary.

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