# Simplified Approach to AISC Bending Formulas

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In Section 1.5.1.4.6a of the AISC Specification[1] there are three equations for determining the allowable bending stress in a beam, and the equations to be used depend on the $l/r_T$ limits of the beam in question. Where $C_b = 1$ and $F_y = 36$ or 50 ksi, these equations are plotted for W and M shapes on pages 2–87 through 2–105 of the AISC Manual.[2] However, where $C_b$ is greater than one, or for other steel strengths, these equations must be used to determine $F_b$.

In looking at Formulas (1.5-6a) and (1.5-6b) and at their corresponding $l/r_T$ limits, it can be noted that all numerical constants are $510 \times 10^3$, or multiples thereof. Further, these same expressions contain the variables $F_y$ and $C_b$, except for Formula (1.5-6b) which is independent of steel strength. Thus, with all of these common terms it would appear that there exists a procedure wherein the mathematics involved could be simplified. To develop that procedure is the intent of this paper.

The first step will be to introduce a new term which includes the common variables. This term, which is noted as $Q$, is defined as follows:

$$Q = \frac{F_y(l/r_T)^2}{510 \times 10^3 C_b} \qquad (1)$$

The $l/r_T$ limits of Eq. (1.5-6a) are:

$$\sqrt{\frac{102 \times 10^3 C_b}{F_y}} \leq \frac{l}{r_T} \leq \sqrt{\frac{510 \times 10^3 C_b}{F_y}} \qquad (2)$$

If all terms in Eq. (2) are squared and combined with the expression for $Q$, Eq. (2) can be rewritten as

$$\frac{(l/r_T)^2}{5Q} \leq (l/r_T)^2 \leq \frac{(l/r_T)^2}{Q} \qquad (3)$$

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Equation (3) can be further simplified by multiplying all terms by $Q/(l/r_T)^2$, which gives

$$0.2 \leq Q \leq 1.0 \qquad (4)$$

The usefulness of the $Q$-term now becomes obvious in simplifying the determination of $F_b$.

| | |
|---|---|
| If $Q \leq 0.2$: | $F_b = 0.6F_y$ |
| If $0.2 < Q \leq 1.0$ | Use Formula (1.5-6a) for $F_b$ |
| If $Q > 1.0$ | Use Formula (1.5-6b) for $F_b$ |

To this point, only one calculation has been made, that of determining $Q$. It still remains that $F_b$ be found using either Formula (1.5-6a) or Formula (1.5-6b); these equations can also be revised to a simplified form containing the quantity $Q$.

Formula (1.5-6a) is

$$F_b = \left[\frac{2}{3} - \frac{F_y(l/r_T)^2}{1{,}530 \times 10^3 C_b}\right] F_y \qquad (5)$$

The second term within the brackets is equivalent to $Q/3$; therefore, Eq. (5) can be rewritten as follows:

$$F_b = \frac{(2 - Q)}{3} F_y \qquad (6)$$

Eq. (6) is the simplified form of AISC Formula (1.5-6a).

Formula (1.5-6b) can be similarly simplified by the introduction of $Q$ as follows:

$$F_b = \frac{F_y}{3Q} \qquad (7)$$

All of the above ignores Formula (1.5-7), which should be checked when the compression flange is solid and approximately rectangular in cross-section and its area is not less than that of the tension flange.

In summary, with the introduction of a new term, $Q$, the simplified four-step procedure for finding $F_b$ by the AISC Specification, Sect. 1.5.1.4.6a, is as follows:

Step 1: $Q = \dfrac{F_y(l/r_T)^2}{510 \times 10^3 C_b}$

Step 2: (a) If $Q \leq 0.2$: $F_b = 0.6F_y$

(b) If $0.2 < Q \leq 1.0$: $F_b = \dfrac{(2 - Q)F_y}{3}$

(c) If $Q > 1$: $F_b = \dfrac{F_y}{3Q}$

Step 3: If applicable, solve Formula (1.5-7):

$$F_b = \frac{12 \times 10^3 C_b}{ld/A_f}$$

Step 4: $F_b$ = larger value of steps 2 and 3, but not to exceed $0.6\ F_y$.

**EXAMPLE 1**

*Given:* W16×36, $L = 20$ ft, $F_y = 50$ ksi, simply supported and braced at ends only.

*Problem:* Find $F_b$ for design.

*Solution:*
From AISC Manual, p. 1–33:
$r_T = 1.81$ and $d/A_f = 5.30$

$M_1 = M_2 = 0 \quad \therefore C_b = 1.75$

Step 1: $Q = \dfrac{50(20 \times 12/1.81)^2}{510 \times 10^3 \times 1.75} = 0.985$

Step 2: $0.2 < Q < 1.0$

$$\therefore F_b = \frac{(2 - 0.985)50}{3} = 16.9 \text{ ksi}$$

Step 3: $F_b = \dfrac{12 \times 10^3 \times 1.75}{20 \times 12 \times 5.30} = 16.5$ ksi

Step 4: $F_b$ for design = larger value = 16.9 ksi

**EXAMPLE 2**

*Given:* 30-ft continuous beam, $F_y = 36$ ksi braced at center and at ends, with $w = 2.0$ kips/ft

*Problem:* Design the beam

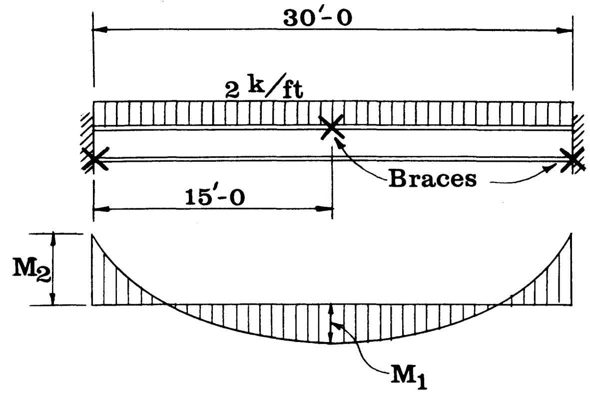


*Figure 1*

*Solution:* (see Fig. 1.)

$$M_1 = \frac{wl^2}{24} = \frac{(2)(30)^2}{24} = 75 \text{ kip-ft}$$

$$M_2 = \frac{wL^2}{12} = \frac{(2)(30)^2}{12} = 150 \text{ kip-ft}$$

$M_1/M_2 = 0.5; \quad C_b = 2.35 > 2.3$ max

$\therefore C_b = 2.3$ for design

Assume $F_b = 0.6F_y = 21.6$ ksi

$$S_{req'd} = \frac{150 \times 12}{21.6} = 83.3 \text{ in.}^3$$

Try W21×44: $S = 93.3$; $r_T = 1.49$; $d/A_f = 7.05$

$$f_b = \frac{150 \times 12}{93.3} = 19.29 \text{ ksi}$$

Step 1: $Q = \dfrac{36(15 \times 12/1.49)^2}{510 \times 10^3 \times 2.3} = 0.448$

Step 2: $0.2 < Q < 1.0$

$$\therefore F_b = \frac{(2 - 0.448)}{3}(36) = 18.62 \text{ ksi}$$

Step 3: $F_b = \dfrac{12000 \times 2.3}{(15 \times 12)(7.05)} = 21.75$ ksi

Step 4: $F_b = 21.75 > 21.6 \quad \therefore$ use $F_b = 21.6$ ksi

$f_b = 19.29 \text{ ksi} < F_b = 21.6$ ksi

$\therefore$ W21×44 o.k.

**REFERENCES**


1. Specification for the Design, Fabrication and Erection of Structural Steel for Buildings *February 12, 1969, American Institute of Steel Construction, New York, N. Y.*
2. Manual of Steel Construction Seventh Edition, *1970, American Institute of Steel Construction, New York, N. Y.*