Optimum Belt Truss Locations for High-Rise Structures

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INNOVATIVE STRUCTURAL schemes are continuously being sought in the design of high-rise structures with the intention of limiting the wind drift to acceptable limits without paying a high premium in steel tonnage. The savings in steel tonnage and cost can be dramatic in high-rise buildings if certain techniques are employed to utilize the full capacities of the structural elements. Various wind bracing techniques have been developed to this end; this paper deals with one such system, namely, the belt truss system. A method of analysis is presented for obtaining optimum combination of belt truss location which minimizes the wind drift of the building.

BELT TRUSS SYSTEM AND ITS PHYSICAL BEHAVIOR

A traditional approach to wind bracing for medium high-rise structures is to provide trussed bracing at the core or around stair wells or other convenient plan locations. But when buildings are higher than 500 ft or so, the core, if kept consistent with the vertical transportation and mechanical requirements, does not have adequate stiffness to keep the wind drift down to acceptable limits. A method of obtaining additional stiffness is to mobilize two or three floor levels to provide stiff points of resistance. At these levels, stiff outrigger arms are used to activate a perimeter truss which in turn enforces the participation of the axial capacity of the exterior columns in wind resistance. The structural system is shown schematically in Fig. 1.

Assuming that there are no surrounding structures, the behavior of a braced core would be similar to that of a free cantilever. But when the core is coupled to the exterior columns, it is no longer free to deform as a free cantilever; the outrigger arms trying to rotate with the core are restrained from doing so by the exterior columns. While providing the rotational constraint, the columns themselves are subjected to compressive forces on the leeward side of the building and a tensile force on the windward side. The net effect of the coupling action is to reduce the bending moments of the core and thereby reduce deflections.

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METHOD OF ANALYSIS

It is generally recognized that a three-dimensional analysis is necessary if full advantage is to be taken of the spacial interaction between the elements of the complete structure. Although such an analysis has come within reach as a normal structural design procedure, its use as an optimization tool may not be desirable in view of the expense and time required for such procedures. Herein, a method based on simplifying assumptions is presented and is believed to provide acceptable results.

For purposes of illustration, consider a high-rise structure in which the perimeter columns are tied to the core at two levels. The typical floor plan of the building is shown in Fig. 2. The assumed plan dimensions of the building and the arrangement of the core, outrigger and belt trusses are shown in Fig. 2. A wind load of an intensity increasing linearly with the height of the building is assumed to act on the long face. The building is assumed to be 50 stories high.

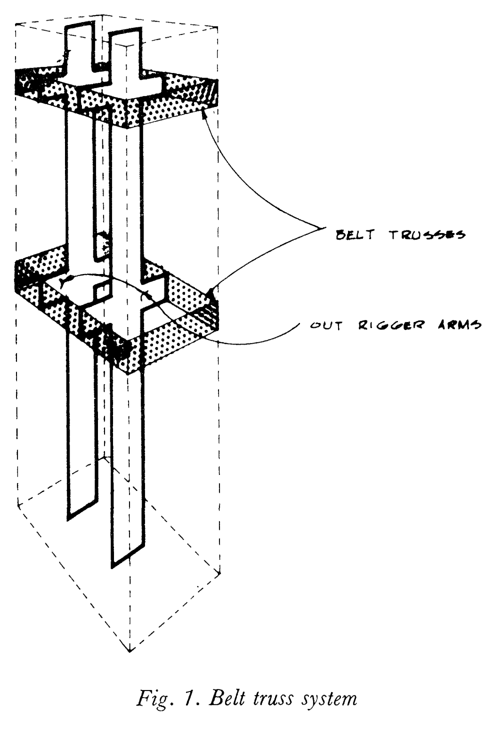

Fig. 1. Belt truss system

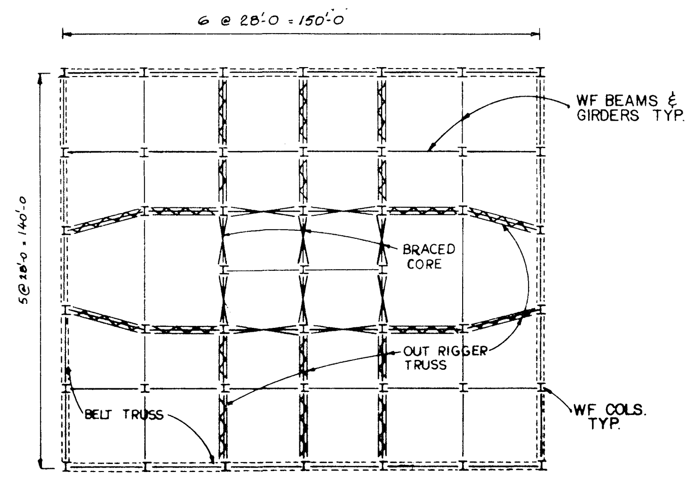

Fig. 2. Framing plan of belt truss structure

The following assumptions are made in the analysis:

1. The outrigger arms are connected to the columns in such a way that only axial forces are induced in the exterior columns.
2. The core walls in line with the outriggers are heavily braced, so that the rotation of the core due to bracing deformations is negligible.
3. The girder-to-column connections in all the frames are pinned; thus, the braced core acting in conjunction with the perimeter columns resists all the wind load.
4. The perimeter truss is infinitely rigid.
5. The axial stiffness of the perimeter columns and the moment of inertia of the cores decrease linearly with the height of the structure.

With these assumptions, the analytical model for the example problem reduces to a doubly tied cantilever as shown in Fig. 3. Here, the core and outrigger deformations are dependent primarily on the flexural energy changes, while the columns can only store direct force energy and their deformation will be dependent only upon this energy form. Either of the two classical methods (stiffness or flexibility) may be employed to obtain the solution. Before this is attempted, let us consider qualitatively how the location of the belt truss influences the magnitude of the wind drift. For simplicity, let us assume that there is only a single level of restraint located anywhere along the height of the structure and that the wind load and the member properties of the perimeter columns and the core remain constant for the full height. Figure 4 shows the analytical model incorporating the aforementioned assumptions. Consider the deflection of the tied cantilever, which is the algebraic sum of the deflections of the free cantilever under external load and the deflection due to the restraint of the

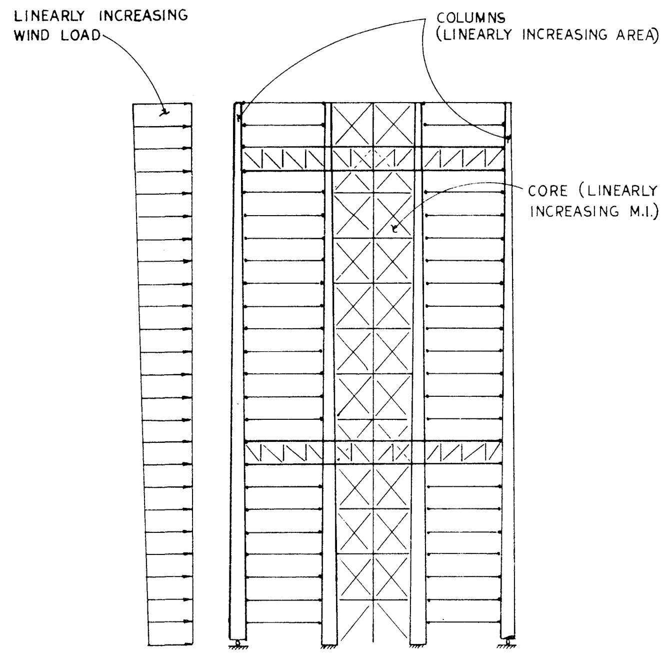

Fig. 3. Analytical model for the example problem

outriggers and columns. The effect of the outrigger and columns may be looked upon as being similar to that of a moment resisting spring whose stiffness depends on its location. Its stiffness, for example, is minimum when located at the top and maximum when at the bottom. The strain energy that can be stored in the spring is a function of stiffness and the rotation of the cantilever

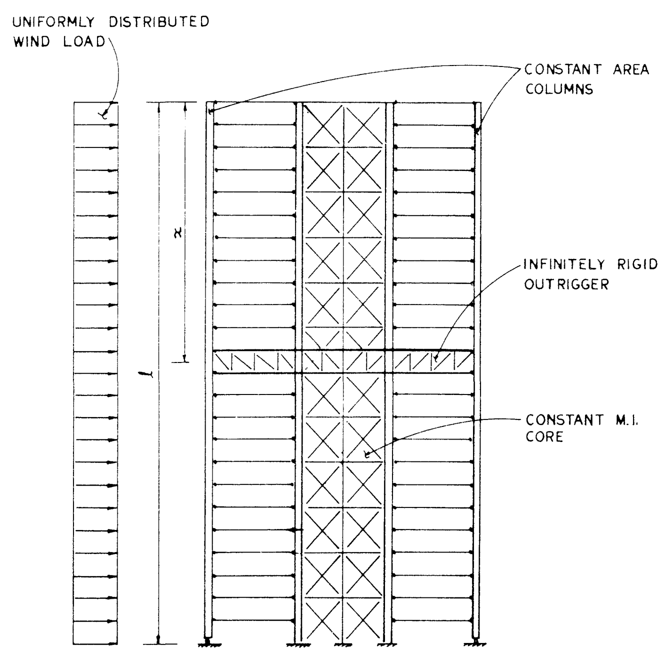

Fig. 4. Analytical model for the simplified structure

at its location. The rotation of the free cantilever for the assumed wind load varies parabolically from a maximum value at the top to zero at the bottom. Therefore, from the point of view of spring stiffness alone it is desirable to locate the outrigger at the bottom, whereas from a consideration of rotation the converse is true. It is obvious that the optimum location is somewhere in between.

For the simplified structure shown in Fig. 4, assuming the outrigger to be infinitely rigid, a closed form solution for the optimum location may be derived by the principle of calculus. First, we write the compatibility equation for the rotation at x, which is the location of the outrigger on the cantilever (Fig. 4.).

$$\frac{W}{GEI}(x^3 - l^3) - \frac{M_x}{EI}(l - x) = \frac{M_x}{K_x} \tag{1}$$

where

- W = intensity of the wind load per height of the structure
- M_x = moment at x, representing the outrigger and columns restraint
- K_x = spring stiffness at x equal to $\dfrac{AE}{(l - x)}$
- E = modulus of elasticity of the core
- I = moment of inertia of the core
- A = area of the perimeter columns
- l = height of the building

Next, obtain the deflection at the top of the structure due to M_x:

$$Y_M = \frac{M_x(l - x)}{2EI}(l + x) \tag{2}$$

From our definition, the optimum location of the belt truss is that location for which the deflection Y_M is a maximum. This is obtained by differentiating Eq. (2) with respect to x and equating to zero. Thus,

$$\frac{d}{dx}\left[\frac{W(x^3 - l^3)(l + x)}{12(EI)^2\,(1/AE + 1/EI)}\right] = 0 \tag{3}$$

$$4x^3 + 3x^2 l - l^3 = 0 \tag{3a}$$

giving the optimum location at $x = 0.455l$. If the flexibility of the outrigger is taken into account, even for the overly simplified model, the corresponding equation for the solution of x becomes too involved to be solved by hand. Extension of the solution to two or more outrigger trusses further complicates the solution, thus necessitating a formulation suitable for computer solution. This is considered next.

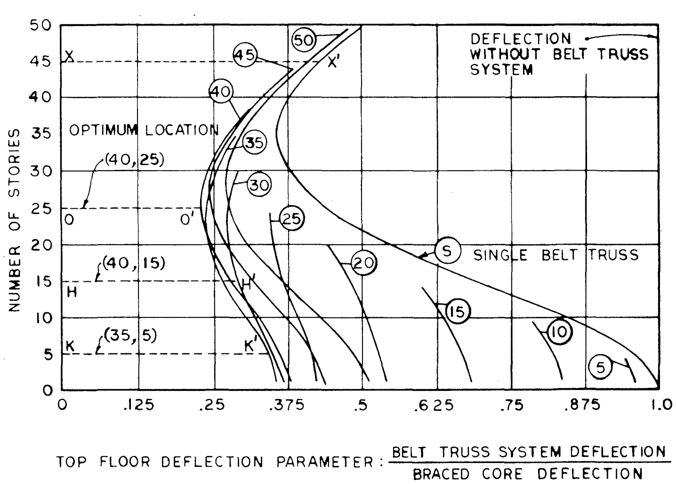

Fig. 5. Results of computer analysis

COMPUTER SOLUTION

A flexibility approach has been employed for the solution. The method is briefly explained with reference to the example problem. The moments at the outrigger locations are chosen as the unknown arbitrary constants M_1 and M_2 and the structure is released by removing the rotational restraints, making it statically determinate, so that the effects of any loading can be easily calculated. The flexibility coefficients $f11$, $f12$, $f22$ are calculated by using integrals of the form

$$\int \frac{m_i m_j}{EI}\,ds + K \int \frac{\zeta_i \zeta_j}{GA}\,ds + \int \frac{n_i n_j}{EA}\,ds$$

where m, s, and n represent the moment, shear force, and the axial load distribution on the statically determinate system due to the application of a unit moment at the location and in the direction of the arbitrary constants E, G, I, and A, the familiar notations for the material and member properties of the element of the structure for which the integral is being calculated. It is to be noted that different forms of energy are significant in different members. Next, the compatibility equations for the rotations at the truss locations are set up and the magnitudes of the arbitrary constants M_1 and M_2 obtained. The tip deflection for the structure is obtained by superimposition of the solutions for the external load and for the moments M_1 and M_2. A single solution to the problem is trivial and may easily be carried out by hand calculations. A computer solution is necessary, however, since the object of the exercise is to seek an optimum combination of the truss locations to minimize the wind drift, requiring many solutions for different truss locations. A computer program was written for this purpose and computations were carried out for the example structure shown in Fig. 2. The results of the analysis are given in the form of graphs in Fig. 5.

EXPLANATION OF GRAPHS

The magnitudes of the top floor deflection of the structure for three assumed modes of resistance have been presented in a non-dimensional form in Fig. 5. The vertical ordinate with the value of the deflection parameter equal to 1 represents the top floor deflection obtained by assuming that there are no belt trusses; the resistance is provided by the cantilever action of the braced core alone. The curve designated as "S" represents the deflection assuming that a single belt truss located anywhere along the height of the structure is acting in conjunction with the braced core. The deflection for a particular location of the truss is obtained by the horizontal distance between the curve "S" and the vertical axis measured at the floor level (e.g., distance XX′ multiplied by the cantilever deflection gives the top floor deflection for the location of the belt truss at floor 45). It is seen that the wind drift is quite sensitive to the truss location. The most favorable location is at floor 35; the resulting deflection is reduced to less than a third of the pure cantilever deflection.

The set of curves designated as 5, 10, . . . 50 represent the top floor deflections obtained by assuming that there are two belt trusses located anywhere along the height of the structure. To obtain each curve, the location of the upper outrigger was considered fixed in relation to the building height, while the location of the lower outrigger was moved in single story increments, starting from the first floor to the floor immediately below the top outrigger.

The number designations of the curves represent the floor number at which the upper outrigger is located. The second outrigger location is given on the vertical axis. The horizontal distance between the curve and the vertical axis is the tip deflection parameter for the particular combination of truss locations given by the curve designation and the vertical ordinate. For example, let us assume that the tip deflection is desired for the combination (30, 15), the numbers 30 and 15 being the floors at which the upper and lower outriggers are located. The procedure is to select the curve with the designation 30 and to draw a horizontal line from the vertical ordinate at 15 to this curve. The required tip deflection parameter is the horizontal distance between the vertical axis and the curve 30 (distance *HH′* in Fig. 5). Similarly, distance *KK′* gives the deflection for the combination (35, 5). It is seen from Fig. 5 that the relative location of the trusses has a significant effect on controlling the wind drift. Furthermore, it is evident that a deflection very nearly equal to the optimum solution may be obtained for a number of combinations. For the example problem, a tip deflection parameter of 0.24, which differs negligibly from the optimum value of 0.23, is achieved by the combinations (40, 20), (35, 20) etc. The effectiveness of the belt truss system is self-evident from the figure.

CONCLUDING REMARKS

Although the analysis presented herein is based on certain simplifying assumptions, it is believed that the results do provide sufficiently accurate information for the location of the belt trusses in high-rise structures. Significant reductions in wind drift may be obtained by judiciously selecting the locations.

Furthermore, since solutions very nearly equal to the optimum solution are obtained for various combinations of truss locations, it should be relatively easy to pick a combination that satisfies simultaneously the structural, mechanical, and architectural requirements.