Discussion

A Unified Approach to the Elastic Lateral Buckling of Beams

Paper presented by NETHERCOT and ROCKEY
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Discussion by **Dwayne D. Piepenburg & James Chinn**

THE AUTHORS are to be commended for integrating a great number of loading and support conditions under a single formula. The paper made the writers curious as to whether it could be related to a formula used for doubly-symmetric I-beams in papers by de Vries[26] and Hechtman et al,[27] which were not included in the list of references. This formula is

$$f_{cr} = \frac{I_y}{I_x}\frac{d^2}{L^2}Ek \qquad (15)$$

in which k is a function of L/a_T, and a_T is a characteristic length in torsion of I-beams and is

$$\sqrt{\frac{EC_w}{GJ}} = \frac{d}{2}\sqrt{\frac{EI_y}{GJ}}$$

for doubly-symmetric I-beams.

The authors' general formula, Eq. (4), is

$$M_{cr} = \alpha(EI_yGJ)^{1/2}\left[\frac{\pi}{L}\left(1+\frac{\pi^2}{R^2}\right)^{1/2}\right] \qquad (4)$$

where R^2 is defined as $R^2 = \dfrac{L^2GJ}{EC_w}$, but $a_T = \sqrt{\dfrac{EC_w}{GJ}}$,

so $R^2 = \dfrac{L^2}{a_T{}^2} = \left(\dfrac{L}{a_T}\right)^2$.

Stress $f_{cr} = M_{cr}c/I_x$, in which c is the distance from neutral axis to extreme fiber and equals $d/2$ for the doubly-symmetric I-beam.

$$f_{cr} = \frac{\alpha d}{2I_x}(EI_yGJ)^{1/2}\left[\frac{\pi}{L}\left(1+\frac{\pi^2a_T{}^2}{L^2}\right)^{1/2}\right]$$

$$= \frac{\alpha d}{2I_x}\frac{\pi}{L}\left[(EI_yGJ)\left(\frac{L^2+\pi^2a_T{}^2}{L^2}\right)\right]^{1/2}$$

$$= \frac{\pi\alpha d}{2I_xL^2}\left[(EI_yGJ)\,a_T{}^2\left(\frac{L^2}{a_T{}^2}+\pi^2\right)\right]^{1/2}$$

However,

$$a_T = \frac{d}{2}\sqrt{\frac{EI_y}{GJ}}$$

Then,

$$f_{cr} = \frac{\pi\alpha d}{2I_xL^2}\left[(EI_yGJ)\left(\frac{d^2}{4}\frac{EI_y}{GJ}\right)\left(\frac{L^2}{a_T{}^2}+\pi^2\right)\right]^{1/2}$$

$$= \frac{\pi\alpha d}{2I_xL^2}EI_y\frac{d}{2}\left(\frac{L^2}{a_T{}^2}+\pi^2\right)^{1/2}$$

Finally,

$$f_{cr} = \frac{I_y}{I_x}\frac{d^2}{L^2}E\left\{\frac{\pi\alpha}{4}\left[\left(\frac{L}{a_T}\right)^2+\pi^2\right]^{1/2}\right\} \qquad (16)$$

When this is compared to Eq. (15), it is seen that

$$k = \frac{\pi\alpha}{4}\left[\left(\frac{L}{a_T}\right)^2+\pi^2\right]^{1/2} = \frac{\pi\alpha}{4}(R^2+\pi^2)^{1/2} \qquad (17)$$

Values for a_T for rolled sections are given in Ref. 28 under the symbol a, or R^2 may be calculated using values of C_w from the AISC Steel Construction Manual.[29]

For doubly-symmetric beams, Eqs. (15) and (17), used in conjunction with the authors' formulas for α, provide an alternate to Eq. (4a) used in conjunction with the flexure formula. In addition, this alternate, like the authors' equations, is considerably simpler than Clark and Hill's Equation (14).

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ADDITIONAL REFERENCES

26. *de Vries, Karl* Strength of Beams as Determined by Lateral Buckling *Transactions, ASCE, Vol. 112, 1947, pp. 1245–1320.*
27. *Hechtman, R. A., J. S. Hattrup, E. F. Styer, and J. L. Tiedemann* Lateral Buckling of Rolled Steel Beams *Transactions, ASCE, Vol. 122, 1957, pp. 823–843.*
28. Torsion Analysis of Rolled Steel Sections *Handbook 1963-B, Bethlehem Steel Corporation, Bethlehem, Pa.*
29. Manual of Steel Construction *Seventh Edition, 1970, American Institute of Steel Construction, New York, N. Y.*