Inelastic K-factor for Column Design

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THE INELASTIC K-FACTOR for column design, suggested by Yura (*Engineering Journal*, April, 1971) has deservedly been well received by practicing structural engineers. Certain questions regarding its application have been asked (*Engineering Journal*, January, 1972). Yura's response was published (*Engineering Journal*, January, 1972 and October, 1972).

The purpose of this paper is to discuss further some of the questions which have been posed and also to show how the Yura Method may be applied without the iteration required in the original paper. Special tables are included to further simplify design.

BEAM END INELASTICITY

One of the questions which has arisen is that if there is any inelasticity at the end of the girders, the rotational restraint would be less than that assumed in the nomograph (AISC Commentary, Fig. C1.8.2), resulting in an unconservative column design. Of course, if this were a valid question it would apply not only to inelastic columns, but also to elastic columns where the Yura Method is not applicable.

In elastic design allowable stresses are such that the beams remain elastic at a specified overload (usually about 1.7 times working load). Since the columns are designed essentially for the same overload factor they are restrained by completely elastic beams at this overload.* How the structure behaves beyond the specified overload is academic and has no practical significance. For this reason inelasticity in the girders has been traditionally and safely ignored in column design using elastic analysis. There is nothing in the Yura Method which would change this. In fact, any effect of beam end inelasticity is the same for an inelastic column as it is for an elastic column.

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* *It should be emphasized that the girders which restrain the columns should not be designed with a bending stress at working load exceeding 0.6 F_y, whether columns are elastic or inelastic.*

STIFFNESS REDUCTION FACTOR

A second question that has been posed is whether a conservative AISC column formula may lead to an unconservative K-factor when using the Yura Method. This can be seen by Yura's formula, where he applies a stiffness reduction factor, F_a/F'_e, to obtain $G_{inelastic}$:

$$G_{inelastic} = \frac{F_a}{F'_e} G_{elastic} \tag{1}$$

where F_a = allowable axial stress and F'_e = Euler stress divided by a factor of safety.

Equation (1) shows that a conservative (low) value of F_a would result in a smaller and less conservative value of K when obtained from the nomograph. It is inconceivable to the author that the net effect, when combined with the conservative allowable stress, would be unconservative. However, for those who prefer the additional conservatism, the author suggests that F_a in Eq. (1) be replaced by $0.6F_y$, the theoretical maximum possible allowable stress in the inelastic range:

$$G_{inelastic} = \frac{0.6F_y}{F'_e} G_{elastic} \tag{2}$$

Using the stiffness reduction factor, $0.6F_y/F'_e$, there should be no question regarding the conservatism of the inelastic K-factor method.

SUGGESTED DESIGN PROCEDURE

Yura's original paper used an iterative procedure for determining K. However, by utilizing the actual stress in the stiffness reduction factor instead of the allowable stress (f_a/F'_e instead of F_a/F'_e), the iteration is eliminated and a direct solution results.**

Tables 1 and 2 relate the stress f_a to the stiffness reduction factor, $0.6F_y/F'_e$, used in Eq. (2), for 36 ksi and 50 ksi steel, respectively. Similar tables could be developed for those who prefer to use the more realistic stiffness reduction factor, f_a/F'_e.

** *This procedure was suggested to the author by Yura.*

The step-by-step procedure follows:

1. Assume a column size.
2. Calculate stress f_a.
3. Determine stiffness reduction factor:

$$\frac{0.6F_y}{F'_e} \quad \text{or} \quad \frac{f_a}{F'_e}$$

4. Calculate $G_{inelastic} = \dfrac{0.6F_y}{F'_e} G_{elastic}$

or $G_{inelastic} = \dfrac{f_a}{F'_e} G_{elastic}$

5. Determine K from nomograph, using $G_{inelastic}$.
6. Calculate Kl/r and determine F_a.
7. If $F_a > f_a$, column is **o.k.**

DESIGN EXAMPLE 1 (See Fig. 1):

Use: Stiffness Reduction Factor $= \dfrac{0.6F_y}{F'_e}$; $F_y = 36$ ksi

Step 1: Assume W12×120.

$A = 35.3$ in.^{2}; $I = 1070$ in.^{4};
$r_x = 5.51$ in.

Step 2: $f_a = 560/35.3 = 15.86$ ksi

Step 3: Table 1 yields reduction factor $0.6F_y/F'_e = 0.822$

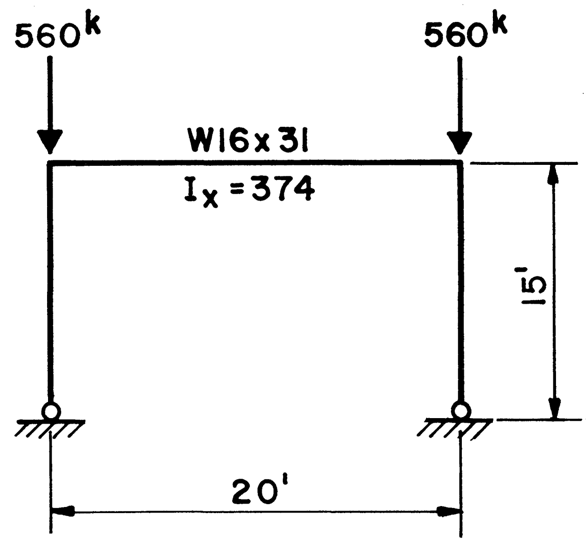

Figure 1

Table 1. Stiffness Reduction Factors (36 ksi Steel)

f_a	$\frac{0.6F_y}{F'_e}$	f_a	$\frac{0.6F_y}{F'_e}$
20.5	0.068	17.7	0.473
20.4	0.078	17.6	0.491
20.3	0.088	17.5	0.509
20.2	0.099	17.4	0.527
20.1	0.110	17.3	0.545
20.0	0.122	17.2	0.563
19.9	0.135	17.1	0.581
19.8	0.148	17.0	0.600
19.7	0.161	16.9	0.619
19.6	0.174	16.8	0.638
19.5	0.187	16.7	0.657
19.4	0.200	16.6	0.676
19.3	0.214	16.5	0.695
19.2	0.228	16.4	0.714
19.1	0.243	16.3	0.734
19.0	0.258	16.2	0.754
18.9	0.274	16.1	0.774
18.8	0.290	16.0	0.794
18.7	0.306	15.9	0.814
18.6	0.322	15.8	0.834
18.5	0.338	15.7	0.854
18.4	0.354	15.6	0.874
18.3	0.370	15.5	0.894
18.2	0.387	15.4	0.915
18.1	0.404	15.3	0.936
18.0	0.421	15.2	0.957
17.9	0.438	15.1	0.978
17.8	0.455	15.0	1.000

Table 2. Stiffness Reduction Factors (50 ksi Steel)

f_a	$\frac{0.6F_y}{F'_e}$	f_a	$\frac{0.6F_y}{F'_e}$
28.0	0.104	24.4	0.496
27.9	0.112	24.3	0.509
27.8	0.121	24.2	0.522
27.7	0.130	24.1	0.535
27.6	0.139	24.0	0.548
27.5	0.148	23.9	0.561
27.4	0.157	23.8	0.574
27.3	0.168	23.7	0.587
27.2	0.177	23.6	0.601
27.1	0.186	23.5	0.614
27.0	0.196	23.4	0.628
26.9	0.206	23.3	0.642
26.8	0.216	23.2	0.656
26.7	0.226	23.1	0.670
26.6	0.236	23.0	0.684
26.5	0.247	22.9	0.698
26.4	0.258	22.8	0.712
26.3	0.269	22.7	0.726
26.2	0.280	22.6	0.740
26.1	0.291	22.5	0.754
26.0	0.302	22.4	0.768
25.9	0.314	22.3	0.782
25.8	0.325	22.2	0.796
25.7	0.336	22.1	0.811
25.6	0.347	22.0	0.826
25.5	0.359	21.9	0.841
25.4	0.371	21.8	0.856
25.3	0.373	21.7	0.871
25.2	0.395	21.6	0.886
25.1	0.407	21.5	0.901
25.0	0.419	21.4	0.916
24.9	0.431	21.3	0.931
24.8	0.444	21.2	0.946
24.7	0.457	21.1	0.961
24.6	0.470	21.0	0.986
24.5	0.483	20.9	1.000

Step 4:

$$G_{inelastic} = 0.822 \left[\frac{1070/15}{374/20}\right] = 3.14$$

Step 5: $G_{top} = 3.14; \qquad G_{bot} = 10; \qquad K = 2.3$

Step 6:

$$\frac{Kl}{r} = \frac{2.3\ (12 \times 15)}{5.51} = 75$$

$$F_a = 15.9 \text{ ksi}$$

Step 7: $15.9 > 15.86$ ksi

W12×120 is o.k.

DESIGN EXAMPLE 2

Rework Design Example 1 using Stiffness Reduction Factor $= f_a/F'_e$.

Step 1: Assume W12×106

$$A = 31.2 \text{ in.}^2; \quad I = 931 \text{ in.}^4; \quad r_x = 5.46 \text{ in.}$$

Step 2: $f_a = 560/31.2 = 17.95$ ksi

Step 3: From AISC column stress tables,
17.95 ksi corresponds to $Kl/r = 54.5$
$F'_e = 50.29$ ksi for $Kl/r = 54.5$

$$\text{Stiffness Reduction Factor} = \frac{17.95}{50.29}$$

Step 4:

$$G_{inelastic} = \frac{17.95}{50.29}\left[\frac{931/15}{374/20}\right] = 1.18$$

Step 5: $G_{top} = 1.18; \qquad G_{bot} = 10; \qquad K = 1.9$

Step 6:

$$\frac{Kl}{r} = \frac{1.9(180)}{5.46} = 62.6$$

$$F_a = 17.18$$

Step 7: $17.18 < 18.0$ ksi **n.g**

Use W12×120.