A Rapid Solution of Vierendeel Frames

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VIERENDEEL TYPE FRAMES are frequently used in steel structures and their analysis is of interest to writers and engineers.[1,2,3] The method described herein provides a quick solution to these frames by longhand computation. It is a non-iterative moment distribution process that is simple and involves a limited amount of labor. It also has the advantage of being close to the conventional process in its ease of application. The frames considered here are of the type shown in Figs. 1 and 2. These frames are symmetrical about the center line and they can be analyzed by considering only one-half of the frame. In order to avoid the shear correction problem that is usually associated with the lateral sway in these frames, use is made of the cantilever stiffness and carry-over[1] for chord members. Thus for any chord member **ij** the stiffness is determined by Eq. (1),

$$K_{ij} = \frac{EI}{h} \tag{1}$$

in which h is the length of chord member (panel length). The cantilever carryover factor, for all chord members, is -1. In the frames considered, web members (cross members) experience antisymmetrical deformation under the action of lateral loads. Thus the conventional stiffness of these members is modified by a factor of 3/2. Hence, for any web member **ii′** the stiffness is determined by Eq. (2),

$$K_{ii'} = \frac{4EI}{L} \times \frac{3}{2} = \frac{6EI}{L} \tag{2}$$

in which L is the length of web member.

DISTRIBUTION FACTORS

The distribution factors according to the process described here incorporate the function of relaxation factors utilized, in a different manner, in another solution (see pgs. 11–14 of Ref. 3). Thus, at any joint **i** it can be shown that the distribution factors are determined by Eqs. (3),

$$\left.\begin{aligned} D'_{ij} &= f_i D_{ij} \\ \text{where,} \quad f_i &= 1/(1 - D'_{i+1,i} D_{i,\,i+1}) \end{aligned}\right] \tag{3}$$

in which D_{ij} is the conventional distribution factor (based on stiffnesses defined by Eqs. (1) and (2)) at end **i** of member **ij** and D'_{ij} is the distribution factor according to the new method.

FIXED END MOMENTS

The fixed end moments are determined by Eq. (4),

$$\text{F.E.M.} = Fh/4 \tag{4}$$

in which F is the total shear force acting on the panel and h is the panel length.

EXAMPLE NO. 1

Figure 1a shows a single bay 3-story frame subject to the lateral loads indicated. Figure 1b shows the conventional distribution factors based on stiffnesses defined by Eqs. (1) and (2). From Fig. 1b and Eqs. (3), the distribution factors according to the new process are

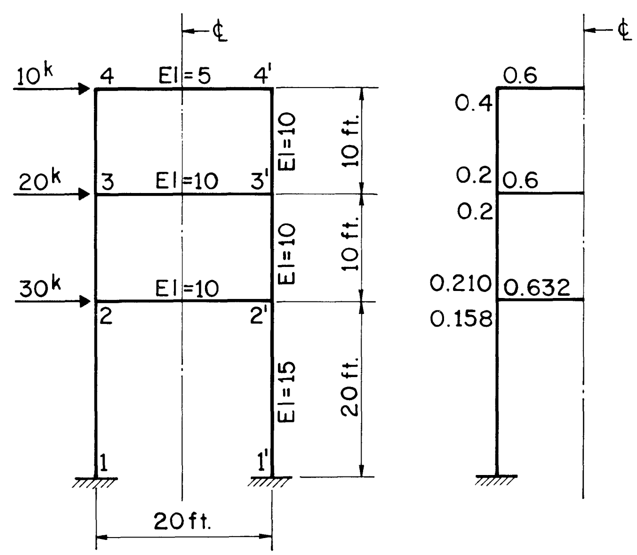

Fig. 1. Example No. 1

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Table 1. Solution of Example No. 1

Joint	1		2			3			4
Member	1–2	2–1	2–2′	2–3	3–2	3–3′	3–4	4–3	4–4′
Dist. Factors		0.166	0.662	0.220	0.217	0.652	0.217	0.4	0.6
F.E.M.	−300.0	−300.0		−75.0	− 75.0		−25.0	−25.0	
							−10.0 ←	10.0	15.0
				23.9 ←	23.9	71.7	23.9 →	−23.9	
	− 66.2 ←	66.2	264.1	87.8 →	− 87.8				
				−19.1 ←	19.1	57.2	19.1 →	−19.1	
							−17.2 ←	17.2	25.8
Final Moments	−366.2	−233.8	264.1	−30.2	−119.8	128.9	− 9.2	−40.8	40.8

Note: All moments are in kip-ft.

determined by starting at joint **4** and proceeding towards joint **2**. In the computation for f_4, below, it should be noted that the second term in the denominator is zero because joint **5** does not exist in the given frame. Hence, a zero substitution is made for D'_{54}.

$f_4 = 1/(1 - 0) = 1$
$D'_{43} = 1 \times 0.4 = 0.4$
$D'_{44'} = 1 \times 0.6 = 0.6$

$f_3 = 1/[(1 - (0.4 \times 0.2)] = 1.087$
$D'_{32} = 1.087 \times 0.2 = 0.217$
$D'_{34} = 1.087 \times 0.2 = 0.217$
$D'_{33'} = 1.087 \times 0.6 = 0.652$

$f_2 = 1/[1 - (0.217 \times 0.210)] = 1.048$
$D'_{21} = 1.048 \times 0.158 = 0.166$
$D'_{23} = 1.048 \times 0.210 = 0.220$
$D'_{22'} = 1.048 \times 0.632 = 0.662$

From Fig. 1a, the total shear forces acting on panels **4-**,**3** **3-2** and **2-1** are 10, 30 and 60 kips, respectively. The fixed end moments in chord members are determined by Eq. (4) and they are −25, −75 and −300 kip-ft for members **4-3**, **3-2**, and **2-1**, respectively. The sign convention adopted for the moments is plus if the moment acts clockwise at end of member. Table 1 shows the solution of the example considered. Starting with joint **4** and proceeding towards joint **2** then back to **4**, the exact solution is obtained through a round-trip of moment distribution and carry over.

According to this method of solution it is noted from Table 1 that the sum of the distribution factors at any joint is not generally equal to 1. Thus, the balance of final joint moments is an indication that all computations have been carried out according to the established relationships. In the conventional process, joint moment balance is obtained at the end of every distribution cycle regardless of any errors that could be made in the moments carried over. In the new process, a final

Table 2. Solution of Example No. 2

Joint	1			2			3			4			5
Member	1–1′	1–2	2–1	2–2′	2–3	3–2	3–3′	3–4	4–3	4–4′	4–5	5–4	5–5′
Dist. Factors	0.878	0.147	0.169	0.675	0.224	0.305	0.456	0.305	0.215	0.647	0.162	0.143	0.857
F.E.M.		−27.5	−27.5		−7.5	− 7.5		17.5	17.5		17.5	17.5	
											2.5 ←	−2.5	−15.0
								8.1 ←	−8.1	−24.3	−6.1 →	6.1	
					5.5 ←	− 5.5	−8.3	−5.5 →	5.5				
		− 5.0 ←	5.0	19.9	6.6 →	− 6.6							
	28.5	4.8 →	− 4.8										
		− 0.8 ←	0.8	3.2	1.1 →	− 1.1							
					−2.3 ←	2.3	3.5	2.3 →	−2.3				
								0.7 ←	−0.7	− 2.1	− 0.5 →	0.5	
											0.9 ←	−0.9	− 5.7
Final Moments	28.5	−28.5	−26.5	23.1	3.4	−18.4	−4.8	23.1	11.9	−26.4	14.3	20.7	−20.7

Note: All moments are in kip-ft.

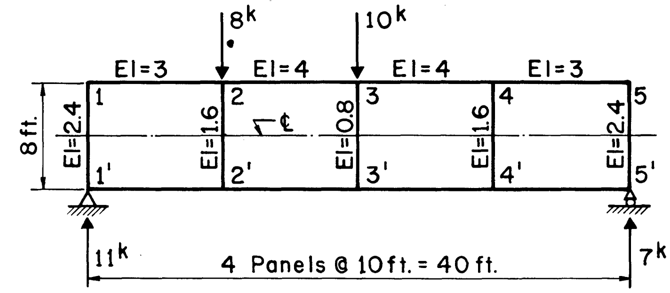

Fig. 2. Example No. 2

joint moment balance is achieved only if all computations were correct. Thus the final joint moment balance, according to the new method, provides a better check on the accuracy of computation.

EXAMPLE NO. 2

In this example a vierendeel truss is loaded as shown in Fig. 2. The example is analyzed by considering joints along the top chord members. The stiffness of chord and web members are determined by Eqs. (1) and (2) and the conventional distribution factors are determined as was done in Example No. 1 (Fig. 1b). Starting with joint **5** and proceeding towards joint **1** the distribution factors, according to the new method, are determined by Eqs. (3). Starting, arbitrarily, at the left reaction and proceeding towards the right of the frame in Fig. 2, the total shear force acting on each panel is determined and the fixed end moments are computed in the same fashion as in Example No. 1. Table 2 shows the solution of the example. The moment distribution process is started at joint **5** and continued through joint **1** then back to **5**. The steps followed are similar to those in Table 1 and are self explanatory.

CONCLUSION

The method provides a quick and exact solution. It is self-checking and it has the advantage of being close to the conventional process in its simplicity and ease of application.

REFERENCES

1. *Grinter, L., and Tsao, C. H.* Joint Translation by Cantilever Moment Distribution *Transactions, ASCE, Vol. 119, 1954, pp. 1195–1211.*
2. *Gurfinkel, German* Simple Method of Analysis of Vierendeel Structures *Journal of the Structural Division, ASCE, Vol. 93, No. ST3, Proc. Paper 5296, June, 1967, pp. 273–285.*
3. *Abdul-Shafi, A.* Analysis of Coupled Shear Walls and Vierendeel Frames *Bulletin No. 21, Engineering Experiment Station, South Dakota State University, Brookings, South Dakota, October, 1972.*