# Plastic Design Applied to Trusses

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PLASTIC DESIGN is increasing in popularity as a method of designing one- and two-story unbraced frames and braced multistory frames. The method of design discussed herein allows an extension of plastic design concepts to the design of trusses. The method is not proposed for the design of individual members of the truss system, because truss compression members will generally buckle prior to deforming enough for load redistribution. However, using the method will allow continuous and fixed ended trusses to be designed for lower resisting moments at a section than would normally be determined using an indeterminate analysis approach. For example, the method presented allows a continuous truss to be designed to support gravity moments equal to $1/16wL^2$ as opposed to those higher moments determined from an elastic analysis. Consequently, the truss chord members are designed for smaller forces, thereby resulting in a direct savings in truss cost.

### GENERAL CONCEPT AND ROTATION CAPACITY

If the analogy between a continuous truss and a continuous beam is made, then it is obvious that before the continuous truss can collapse a collapse mechanism must form. This mechanism would be the same as that of a continuous beam. Plastic hinges would first develop at the ends and additional loads would be redistributed to the less heavily stressed center portion of the truss. In a beam, the required rotation at the point of maximum moments is accomplished through plastic hinge development. The tension and compression fibers of the beam reach the fully yielded condition, thereby allowing rotation at the cross section. In addition to the steel having the required ductility, it is necessary to prevent all forms of instability during this rotation. The rotation needed for redistribution of moment to occur in a truss can be provided by allowing the tension chord at plastic hinge locations to yield and elongate to strain-hardening. For example, for the uniformly loaded truss shown in Fig. 1(a), members $U_0U_1$ and $U_7U_8$ would yield and elongate. All other members are designed so that they will not yield or buckle at factored loads thus allowing the rotation to occur.

The angle of rotation through which the yielded portion of a beam must sustain its plastic moment value is called the *hinge angle*. For a fixed-end beam with a uniformly distributed load, the hinge angle is equal to $M_pL/6EI$.[2]

The rotational capacity of a truss must be greater than its hinge angle. This is not difficult to obtain. In fact, as shown in the appendix, the rotational capacity will be adequate as long as the end chord length is greater than the total truss length between supports divided by 72.

### DESIGN PROCEDURE

Assume that the fixed ended truss shown in Fig. 1(a) is to be designed. The design moments are determined

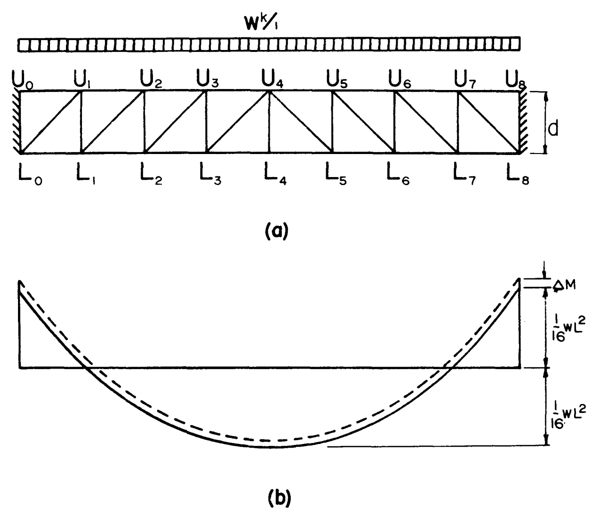


*Fig. 1. (a) Uniformly loaded truss of length L, and (b) ultimate moment diagrams for a fixed-ended truss*

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using plastic design techniques, i.e.,

$$M_p = \frac{1}{16} wL^2 \quad (1)$$

where $w$ is the factored load per linear foot of the truss. If the horizontal forces due to axial shortening of the truss are disregarded, the horizontal reactions at the top and bottom truss chords can be determined from:

$$R_H = \pm\frac{M_p}{d} \quad (2)$$

where $d$ is the depth of the truss. These reactions are then applied as tension and compression forces at the truss ends. The remainder of the truss is statically determinate and the factored bar forces can easily be obtained.

Once these factored bar forces are determined, the top chord member is designed. Assuming that the same size top chord is used throughout the entire length of the truss, the critical section for the design of the member will generally be at midspan where the member is in compression.

Unless this member is continuously laterally braced about both axes, the end result will be a member which is overdesigned for the ends of the truss where it acts as a tension member. That is, a larger area than required will be used at the fixed ends. A greater force would be required to yield this member; thus, the truss would attempt to support the larger moments [shown as the dashed lines in Fig. 1(b)]. This shift in the base line of the moment diagram does not affect the ultimate forces in the diagonals or uprights. It does affect the design of the lower chord. The lower chord must be designed for the increased moment capacity at the end of the truss, so that the top chord can reach yield and provide the required rotation. The difference between the force required to yield the top chord member and the force calculated from Eq. (2) must be added as a compressive force to all bottom chord members to account for the increased capacity of the top chord. The bottom chord, diagonals, and uprights may now be designed.

If the top chord is not to be of the same size throughout, it should be designed for the end condition first. Since it is generally impossible to select a member of exactly the same area as needed, the yield force of the member selected will be larger than the force calculated from Eq. (2). This additional force is treated as described above, and the remaining members are designed.

The procedure described above assumes the end diagonal on each end of the truss is a compression diagonal. This is generally the case because it reduces (and sometimes eliminates completely) the compressive forces in the bottom chord members and usually reduces the cost of the truss.

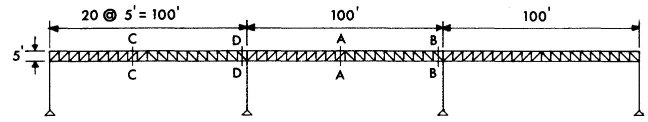


*Fig. 2. Continuous truss (example)*

**DESIGN EXAMPLE**

*Given:*
Design the continuous truss system shown in Fig. 2. $F_y = 36$ ksi.

Ultimate uniform load $= w = 1.7w_{LL} + 1.7w_{DL}$
$= 3.4$ kips/ft

For the center span, $M_p = \dfrac{(3.4)(100)^2}{16} = 2125.0$ kip-ft

The maximum chord reaction $= R_H = \pm\dfrac{2125.0}{5}$

$= \pm 425$ kips

The maximum vertical shear $= R_v = 170$ kips

The top and bottom chords are assumed to be continuous members and will be designed as structural tees.

*Solution:*
The first step is to design the top chord. The critical section is at section A-A. $M_p = 2125.0$ kip-ft.

From statics at section A-A:

$P$ (top chord) $= 416.5$ kips compression
$P$ (diagonal) $= 12.02$ kips compression
$P$ (bottom chord) $= 425$ kips tension

Try WT9×42.5:
$A = 12.5$ in.$^2$; $r_x = 2.60$ in.; $r_y = 2.05$ in.

$\dfrac{KL}{r_y} = 29.3$; $F_a = 19.99$ ksi

$\therefore P_{cr} = 1.7F_aA = 424.5 > 416.5$ **ok**

Use WT9×42.5 for top chord center span.

At section B-B, from statics:

$P$ (top chord) $= 425$ kips tension
$P$ (diagonal) $= 228.5$ kips compression
$P$ (bottom chord) $= 263.5$ kips compression

A force of 450 kips is required to yield the WT9×42.5 section. Thus, the additional force of 25 kips (450 kips − 425 kips) must be added as a tensile force at the top chord and as a compressive force at the bottom chord. The diagonals and uprights remain unchanged. The design of the top chord remains adequate; however, the bottom chord member must now be designed for 288.5 kips compression at B-B and 400 kips tension at A-A:

Area (bottom chord section A-A) $= \frac{400}{36} = 11.11 \text{ in.}^2$

Try WT5×38.5:

$A = 11.3 \text{ in.}^2; \quad r_x = 1.25 \text{ in.}; \quad r_y = 2.60 \text{ in.}$

Check section BB:

$\frac{KL}{r_x} = 48 \qquad F_a = 18.53 \text{ ksi}$

$P_{cr} = 1.7(11.3)(18.53) = 356 > 288.5 \text{ kips}$ **ok**

The diagonals, uprights, and connections can now be designed based on the ultimate loads. Their design is eliminated here for brevity.

If the exterior columns are designed to carry the full plastic moment of the truss, the two exterior bay trusses are identical to the center truss designed above. It is more likely that the connection between the exterior bay trusses and the end columns would be assumed to be a simple support. In this case the plastic moment to be carried by the end trusses is 2920.0 kip-ft. This moment can be calculated using either the virtual work method or statical method of plastic design. Based on this moment and using the design procedure as outlined above, one obtains a WT12×60 for the top chord and a WT6×49.5 for the bottom chord.

The truss shown in Fig. 2 was also analyzed elastically and the bars were designed (simple supports assumed at the end columns). The plastic design procedure resulted in a total weight savings of 8100 pounds of steel.

## CONCLUSIONS

The procedure as described herein is a simple straightforward method of obtaining moments for the design of continuous or fixed-ended trusses. A truss can provide the required rotational capacity for load redistribution to occur provided the design method as outlined is followed. Design time is reduced significantly and a considerable savings in truss weight will be realized by using plastic design for trusses.

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## APPENDIX A: ROTATIONAL CAPACITY REQUIREMENTS

$I$ = Moment of inertia
$L$ = Total length of truss between supports
$A_T$ = Area of top chord member
$d$ = Height of truss
$l$ = Length of end panel chord
$\epsilon$ = Strain
$f$ = Stress
$\theta$ = Truss rotation

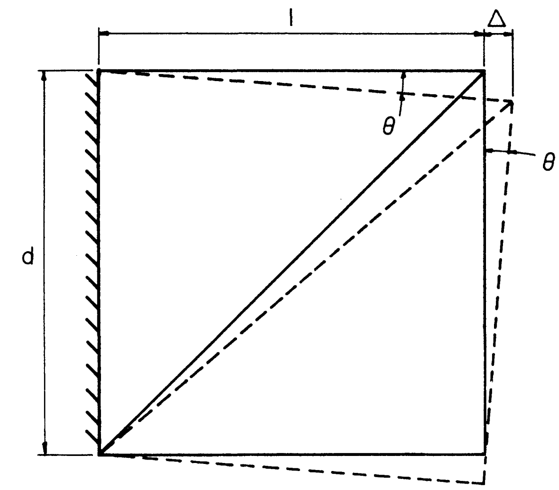


*Fig. 3. Truss rotation*

The subscript $y$ indicates conditions at yield and the subscript $s$ indicates strain-hardening.

$$\theta_y = \Delta_y/d \qquad \text{(see Fig. 3)}$$
$$\Delta_y = \epsilon_y l$$
$$\therefore \theta_y = \epsilon_y(l/d)$$
$$= (F_y/E)(l/d)$$
$$= F_y l/Ed$$
$$\theta_s = \Delta_s/d$$
$$\Delta_s = \epsilon_s l$$
$$\therefore \theta_s = \epsilon(l/d)$$
$$= (\epsilon_s/\epsilon_y)(\epsilon_y l/d)$$
$$= (\epsilon_s/\epsilon_y)(F_y l/Ed)$$

The value of $\theta_s$ must be greater than the required hinge angle, $M_pL/6EI$, or $(\epsilon_s/\epsilon_y)(F_y l/Ed) > M_pL/6EI$.

Rearranging terms,

$$(M_pL/EI)(Ed/P_y l) < 6(\epsilon_s/\epsilon_y)$$
$$\therefore M_pLd/IP_y l < 6(\epsilon_s/\epsilon_y)$$

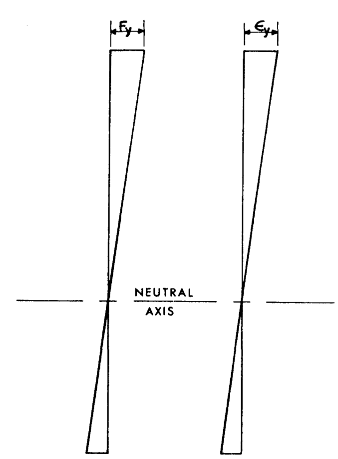


*Fig. 4. Stress-strain relationships versus depth at yield*

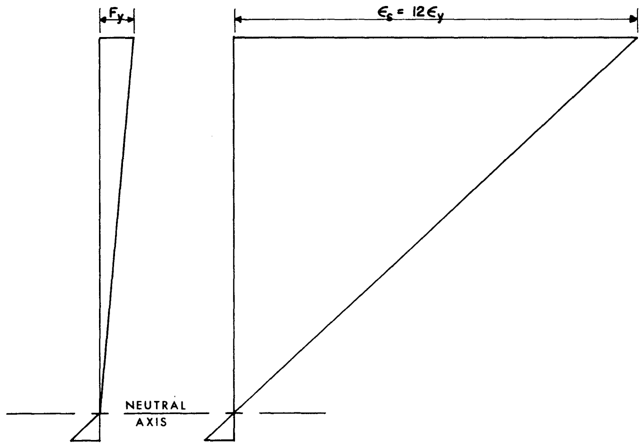


*Fig. 5. Stress-strain relationships versus depth at strain hardening*

For the grades of steel that can be used in plastic design, $(\epsilon_s/\epsilon_y) \simeq 12$. Therefore, $M_pLd/IF_yl < 72$.

Substituting $M_p = A_TF_yd$:

$A_TLd^2/Il < 72$.

This equation can be simplified even further by writing an approximate equation for $I$.

The stress and strain versus depth diagrams at yield appear as in Fig. 4, and at strain hardening as in Fig. 5.

This shows that the neutral axis drops at strain-hardening, which makes the distance from the top chord to the neutral axis approximately equal to $d$. The largest term in the expression for the moment of inertia of the combined section will be the transfer term for the top chord member, $A_Td^2$. By making the assumption that $I = A_Td^2$, one obtains:

$$A_Td^2L/A_Td^2l < 72$$

$\therefore L/l < 72$ for adequate rotational capacity.

**REFERENCES**


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