

Direct Design of Columns Subjected to Combined Loading

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THE DESIGN OF wide-flange columns with combined loading according to the procedures presented in the AISC Manual¹ is very time consuming. Methods have been presented which tend to simplify this design procedure.^{2,3} All of these methods convert the applied bending moments to equivalent axial loads, through the use of bending factors. Once the equivalent axial loads are calculated, a trial section is chosen from the AISC Column Tables. Three modified AISC interaction formulas are then used to determine the required axial tabular load. From this load, another trial section is chosen. This procedure is repeated until a satisfactory section is obtained. Because fewer calculations are required, these methods are somewhat less time consuming than the method presented in the AISC Manual.

Due to the number of variables involved in designing a wide-flange steel column subjected to combined loading, a trial and error procedure or the common "guess" procedure is usually unavoidable. Once a trial section is chosen, the time is spent in determining the section properties and deciphering the relevant provisions of the AISC Specification for that section. This process must then be repeated for each trial section. Much time is also spent in calculating the factors required for AISC Formulas (1.6-1a), (1.6-1b), and (1.6-2) and finally solving the formulas themselves. Although the trial and error procedure is unavoidable, the excessive time spent can be shortened by the use of a graphical solution.

In the graphical analysis presented in this paper, the member properties and all the provisions of the AISC Specification are incorporated. All the information necessary for designing wide-flange steel columns subjected to either axial or combined loads is contained in the various graphs and tables, eliminating the need for referring to the AISC Specification or Manual. Although this paper is applied to A36 grade, W14 steel columns only, similar graphs and tables may be developed for designing other types and series of columns.

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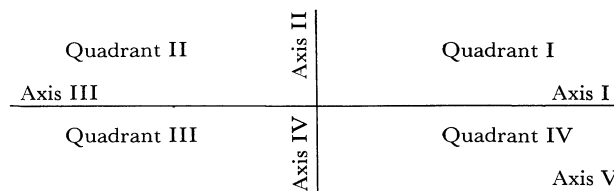
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NOMENCLATURE

Terms not defined below are found in the AISC *Manual of Steel Construction, Seventh Edition*.

- M_3 = Moment between supports, larger than M_1 or M_2 , used in Category C of beam-column.
- x = Subscript for $X-X$ axis
- y = Subscript for $Y-Y$ axis
- ψ = Factor used in determining C_m for Category C

Labeling of Axes and Quadrants:



DEVELOPMENT OF DESIGN GRAPHS

AISC Formulas—In designing wide-flange steel columns with combined stresses, according to the AISC requirements, the following three interaction formulas must be satisfied:

$$\frac{f_a}{F_a} + \frac{C_{mx}f_{bx}}{\left(1 - \frac{f_a}{F_{ex'}}\right)F_{bx}} + \frac{C_{my}f_{by}}{\left(1 - \frac{f_a}{F_{ey'}}\right)F_{by}} \leq 1.0$$

AISC Formula (1.6-1a)

$$\frac{f_a}{0.60F_y} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0$$

AISC Formula (1.6-1b)

If $f_a/F_a \leq 0.15$, AISC allows the use of Formula (1.6-2) in place of Formulas (1.6-1a) and (1.6-1b):

$$\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} \leq 1.0$$

AISC Formula (1.6-2)

All three formulas involve three stress ratios: axial, bending about the $X-X$ axis, and bending about the $Y-Y$ axis. The design graphs determine these three ratios, which are then added and checked to be less than unity.

Table 1. Properties—W14 Sections

Section	r_x/r_y	L_c	L_u	Section	r_x/r_y	L_c	L_u
730	1.74	18.9	181.6	176	1.60	16.6	62.4
665	1.73	18.7	170.3	167	1.60	16.5	59.6
605	1.72	18.4	160.2	158	1.60	16.5	57.1
550	1.70	18.2	150.4	150	1.60	16.4	54.5
500	1.69	18.0	140.3	142	1.59	16.4	51.8
455	1.68	17.8	131.6	136	1.67	15.6	49.2
426	1.67	17.7	125.5	127	1.67	15.6	46.5
398	1.66	17.6	119.4	119	1.67	15.5	43.7
370	1.66	17.4	113.0	111	1.67	15.5	41.0
342	1.65	17.3	106.5	103 ^a	1.67	15.4	38.6
314	1.64	17.2	99.8	95 ^a	1.66	15.4	35.7
287	1.63	17.1	93.0	87 ^a	1.66	15.4	33.1
264	1.63	17.0	87.2	84	2.03	12.7	30.5
246	1.62	16.9	82.4	78 ^a	2.03	12.7	28.5
237	1.62	16.8	79.9	74	2.44	10.7	25.8
228	1.61	16.8	77.6	68 ^a	2.45	10.6	23.8
219	1.62	16.8	75.0	61 ^a	2.44	10.6	21.5
211	1.61	16.7	72.6	53 ^a	3.07	8.6	17.7
202	1.61	16.7	70.2	48 ^a	3.07	8.5	16.0
193	1.61	16.6	67.5	43 ^a	3.08	8.5	14.3
184	1.61	16.6	65.0				

^a See Table 2.

Axial Load Ratio—The axial load ratio in AISC Formulas (1.6-1a) and (1.6-2) is f_a/F_a , which is equivalent to P/AF_a , where

$$F_a = \frac{\left[1 - \frac{(Kl/r)^2}{2C_c^2}\right] F_y}{\frac{5}{3} + \frac{3(Kl/r)}{8C_c} - \frac{(Kl/r)^3}{8C_c^3}} \quad \text{AISC Formula (1.5-1)}$$

When Kl/r exceeds C_c : $F_a = \frac{12\pi^2 E}{23(Kl/r)^2}$

The terms F_y , C_c , and E are all constants for a specific type of steel; therefore the axial load ratio is dependent on the variables P , A , Kl , and r . The ratio of Kl/r , as used in AISC Formula (1.5-1), should always be the largest of the ratios about the $X-X$ and $Y-Y$ axes. The proper axis giving the maximum ratio for W14 sections may be determined from Table 1.

Figure 1 consists of W14 sections, in order of their radius of gyration for both the $X-X$ and $Y-Y$ axes, located on Axis I, and Kl lines plotted in Quadrant I. These two variables are used in the determination of F_a , whose values are the ordinates of Axis II. Quadrant II relates P and F_a to determine the ratio P/F_a . The values of this ratio are located on Axis III. Finally, the areas of W14 sections, plotted in Quadrant III, are used to determine the axial load ratio of f_a/F_a , which is read on Axis IV of Fig. 1.

Table 2. Allowable Bending Stresses, W14 Sections

Axis	Section	F_b (ksi)	Requirements
X-X	All sections except: 103, 95, 87, 43	24.0	$l_x \leq L_c$
	103	23.7	$l_x \leq L_c = 15.4$ ft
	95	23.5	$l_x \leq L_c = 15.4$ ft
	87	23.2	$l_x \leq L_c = 15.4$ ft
	43	24.0	$l_x \leq L_c = 8.5$ ft $P \leq 69.0$ kips
		22.0	If the above requirements are not satisfied.
	All sections:	22.0	$L_c \leq l_x \leq L_u$
	All sections: $F_b = \frac{(12 \times 10^3)C_b}{ld/A_f}$, but not more than 22.0 ksi when $l_x > L_u$		
Y-Y	All sections except: 103, 95, 87	27.0	
	103	26.8	
	95	26.2	
	87	25.7	

Bending Ratios—Figures 2 and 3 are used to determine the bending ratio about the $X-X$ and $Y-Y$ axes respectively.

The bending ratio in AISC Formulas (1.6-1b) and (1.6-2), f_b/F_b , equivalent to the ratio M_b/S_bF_b . The applied bending moment is located on Axis I and the section modulus of the column being considered is plotted in Quadrant I. These two values determine the ratio of M_b/S_b on Axis II.

The allowable values of F_{bx} and F_{by} , as determined by AISC Specification Sect. 1.5.1.4 for W14 sections using A36 steel, are given in Table 2.

Figure 4 is used for the solution of F_{bx} when l_x is greater than L_u . The maximum unbraced length, l , about either the $X-X$ or $Y-Y$ axis is located on Axis I and d/A_f for each section in Quadrant I. C_b curves, which were determined by the ratio M_{1x}/M_{2x} , are plotted in Quadrant II. For Category C, based on the AISC Specification, C_b is taken to be equal to one, ignoring the ratio of M_{1x}/M_{2x} . The value of F_{bx} may now be read on Axis III.

Knowing M_b , S_b , and F_b , the value of the bending ratio for AISC Formulas (1.6-1b) and (1.6-2) may be read on Axis III of Figs. 2 and 3.

The axial load ratio in AISC Formula (1.6-1b), $f_a/0.60F_y$, which is equivalent to $P/0.60AF_y$, involves only the two variables P and A . The values of this ratio may be obtained from Axis II of Fig. 6.

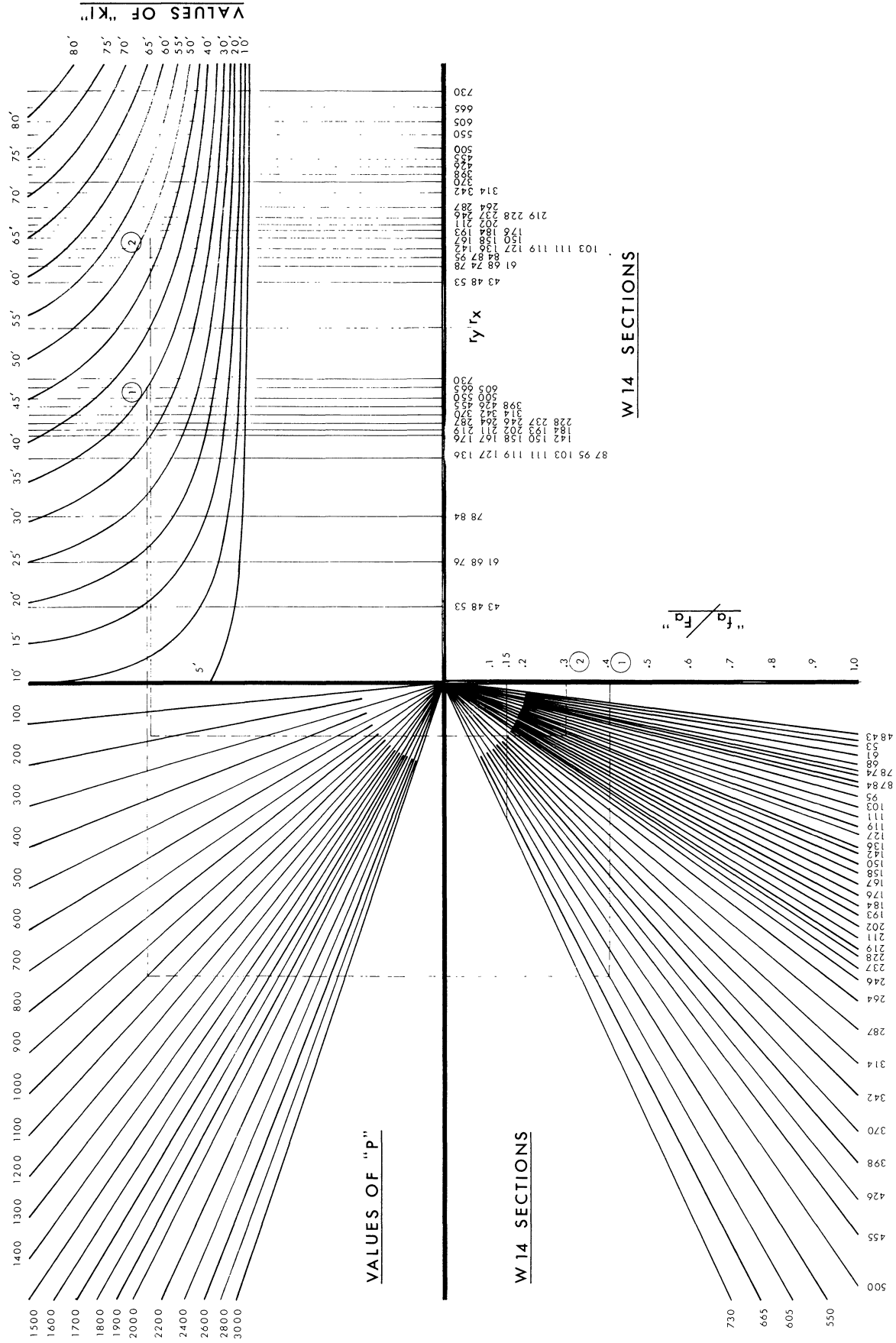


Fig. 1. Axial load ratio, Formulas (1.6-1a) and (1.6-2)

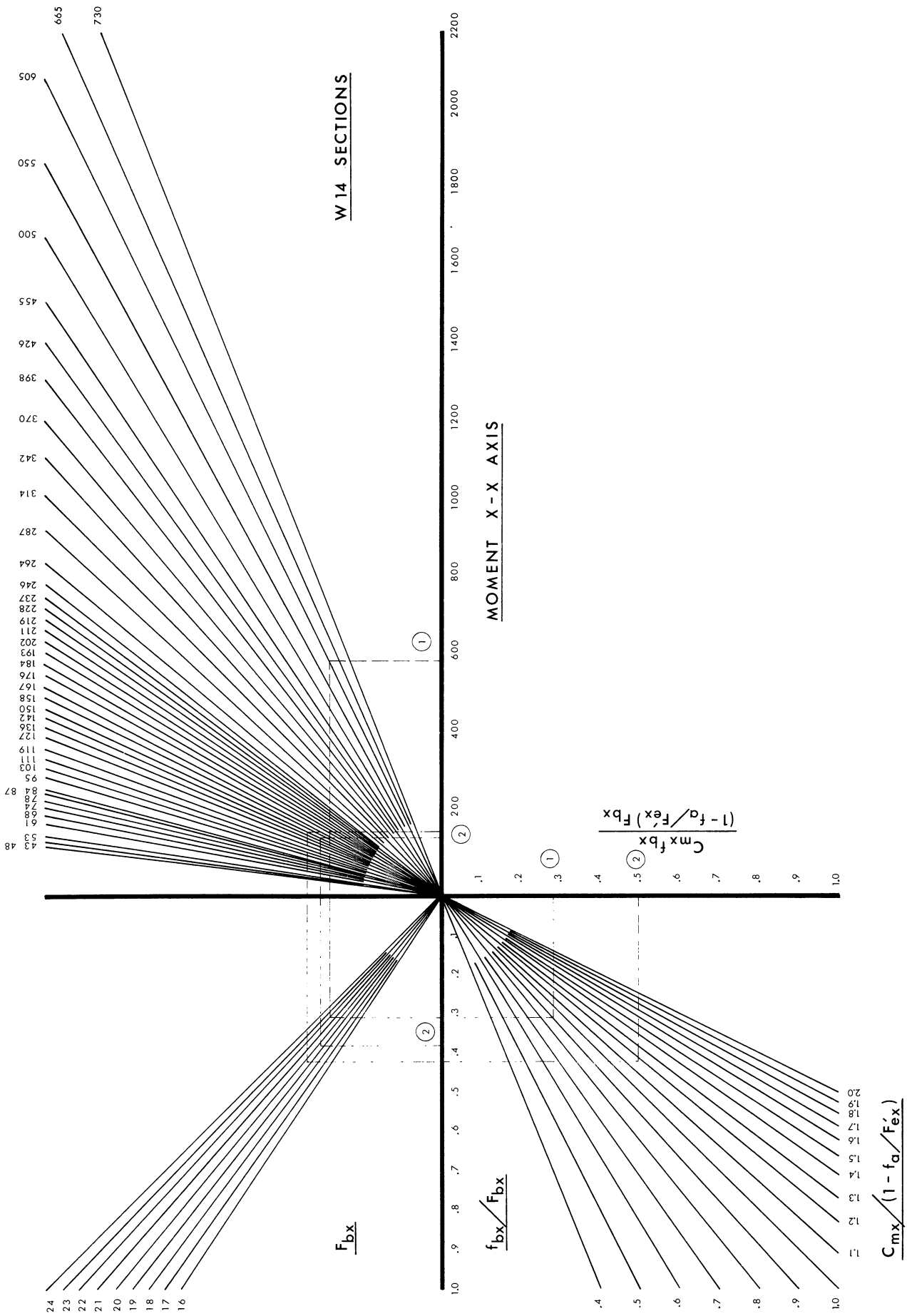


Fig. 2. Bending ratio about X-X axis

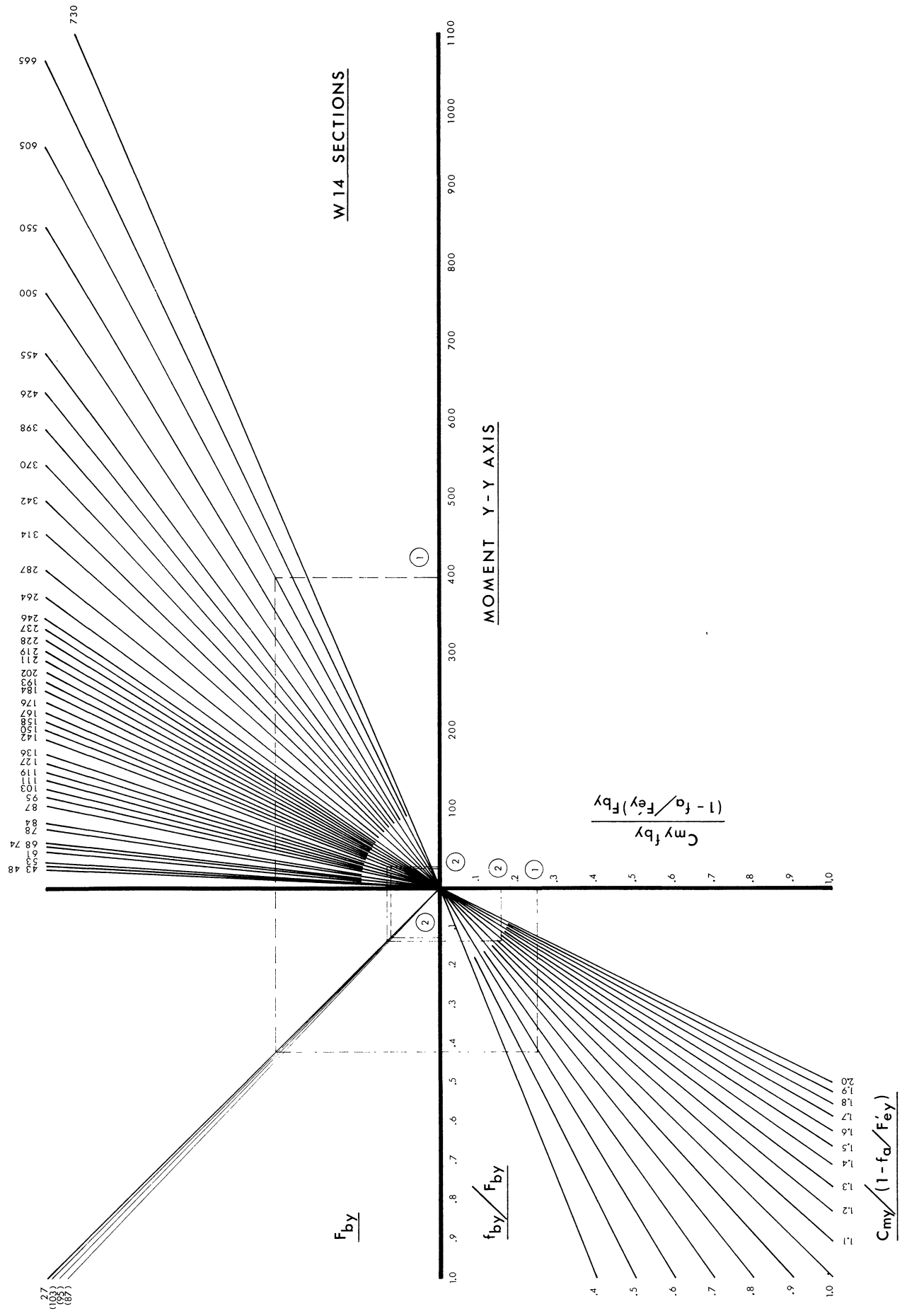


Fig. 3. Bending ratio about Y-Y axis

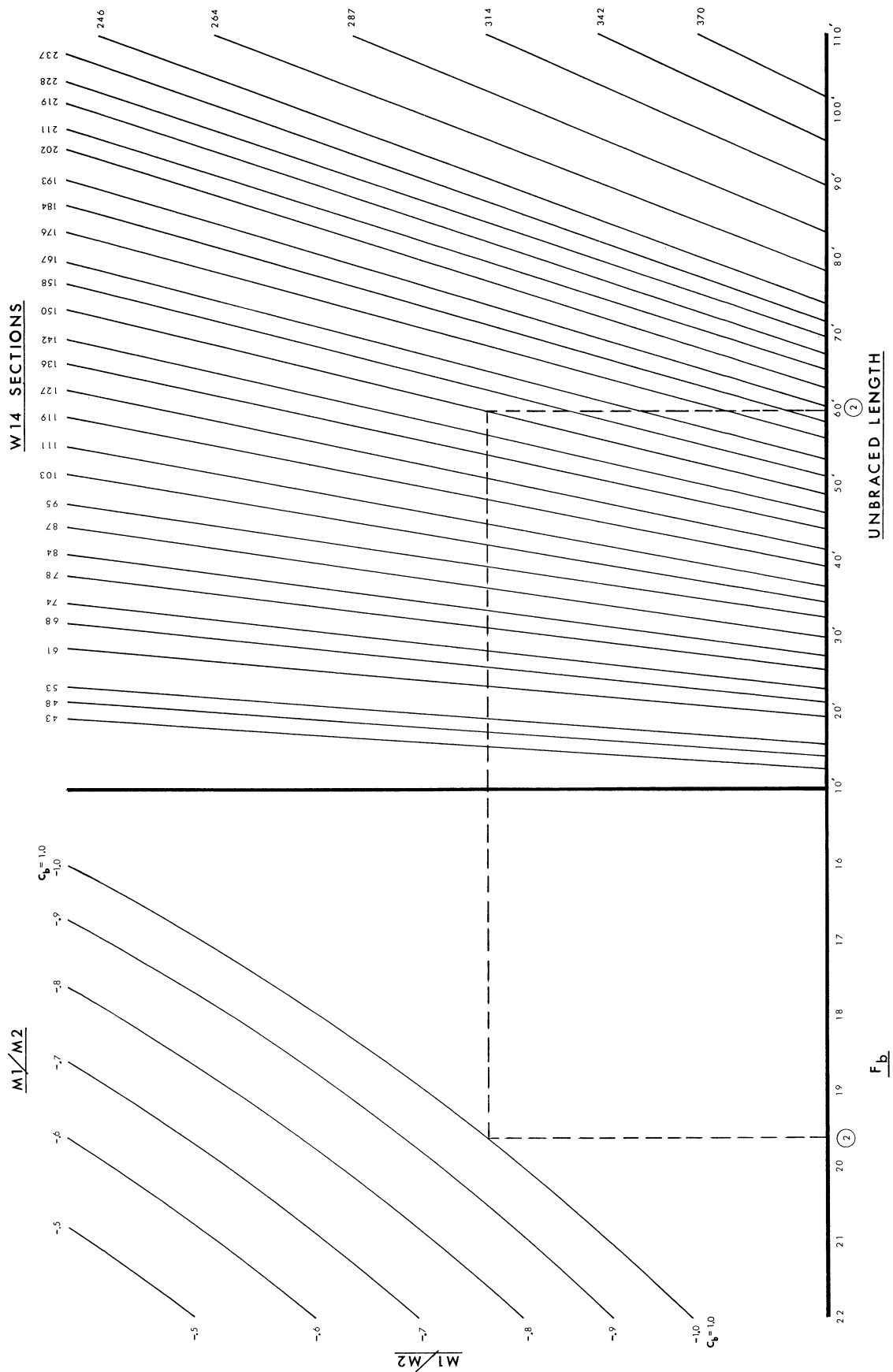


Fig. 4. F_{bx} for $l > l_u$

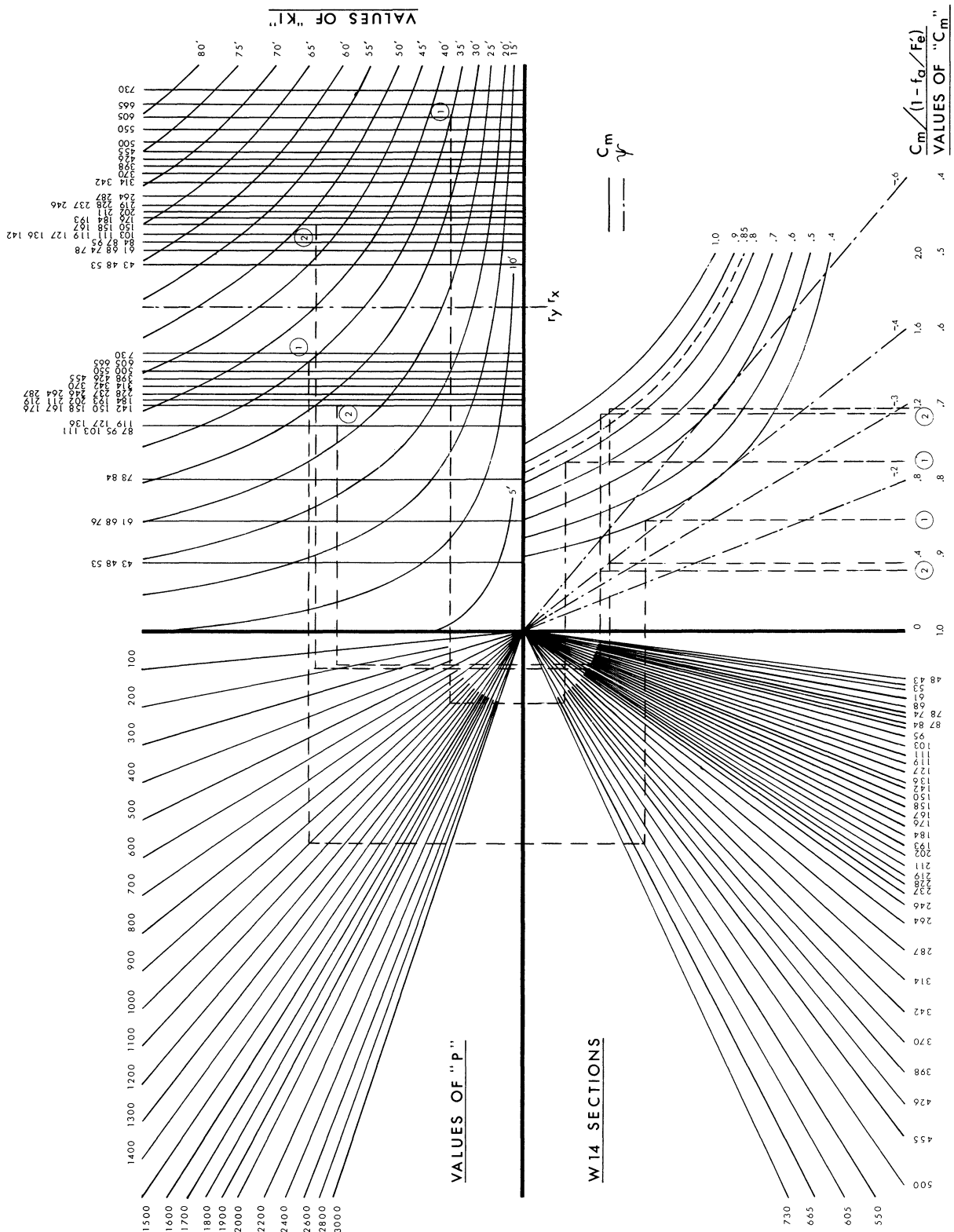


Fig. 5. $C_{mb}/(1 - f_a/F_{eb})$

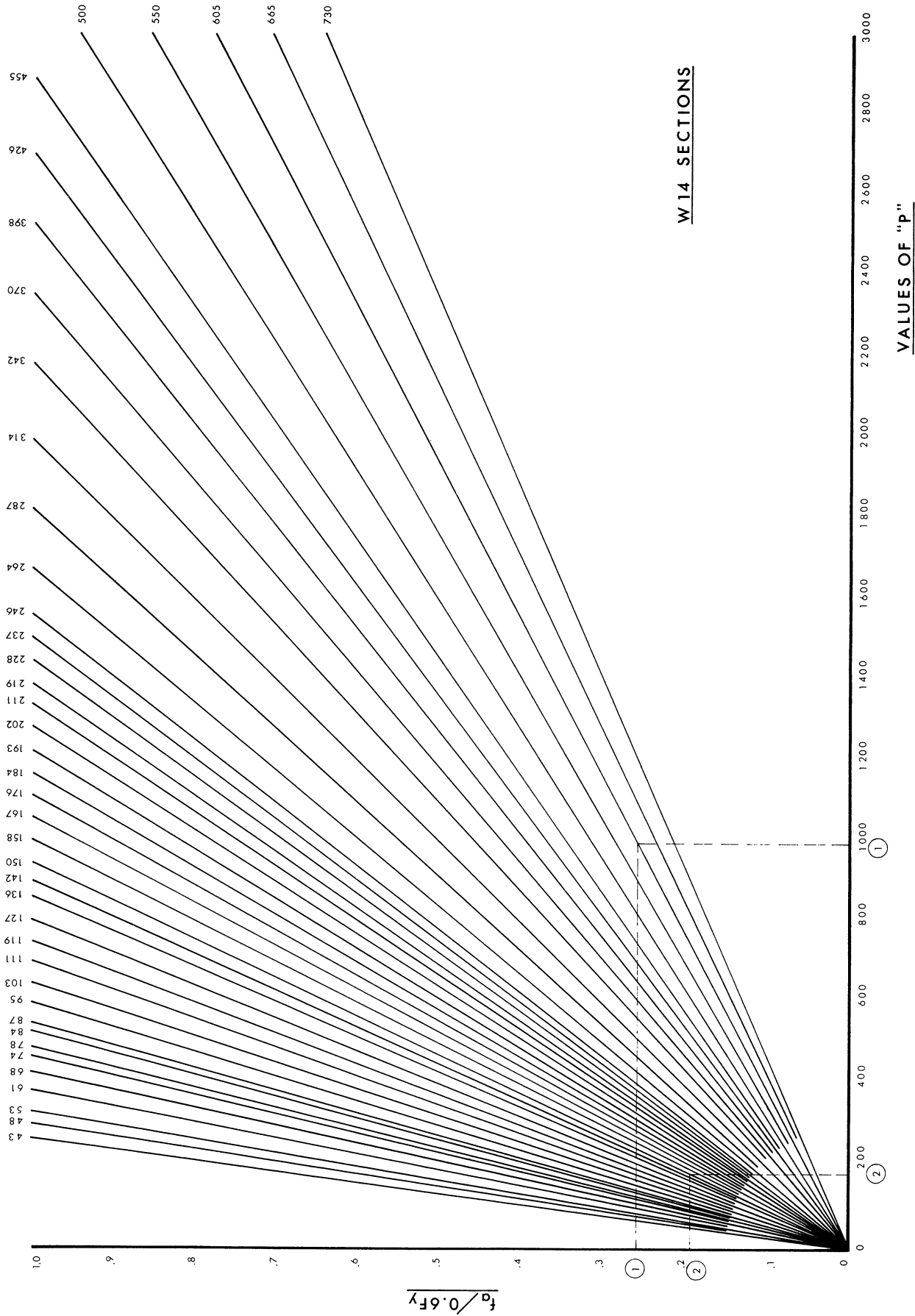


Fig. 6. Axial load ratio, Formula (1.6-1b)

When using AISC Formula (1.6-1a), the amplification factor, $C_m/(1 - f_a/F_e')$, must first be determined for each axis of bending, where

$$F_e' = \frac{12\pi^2 E}{23(Kl_b/r_b)^2}$$

F_e' is determined in a similar manner as was F_a , and may be obtained from Axis II of Fig. 5. The ratio of f_a/F_e' can also be treated as was f_a/F_a , with the values of P and A plotted in Quadrant II and III of Fig. 5. The resulting values of $(1 - f_a/F_e')$ are located on Axis IV.

The value of C_m may be determined according to AISC Table C1.6.1.1. As can be seen from this table, C_m has a constant value of 0.85 for Category A. For Category B, $C_m = 0.6 - 0.4(\pm M_1/M_2)$. This C_m value can be determined from the values of the applied moments and remains constant for any section. For Category C, the value of C_m is equal to $1 + \psi(f_a/F_e')$ and is dependent on the section being used.

To determine the value of C_m for Category C, the same steps are followed as in determining $(1 - f_a/F_e')$. In this case, the values of f_a/F_e' are the ordinates of Axis IV of Fig. 5. The appropriate value of ψ , according to AISC Table C1.6.1.2, is chosen in Quadrant IV and the value of C_m is read on Axis V. The value of C_m is used in Quadrant IV of Fig. 5, and the value of the amplification factor, $C_m/(1 - f_a/F_e')$, is read on Axis V.

The amplification factor can now be used in Quadrant III of Figs. 2 and 3 to determine the bending ratios for AISC Formula (1.6-1a). These ratios are the ordinates of Axis IV.

The trial and error procedure is now repeated until the sum of ratios is less than or equal to one. Procedural steps for designing a W14 column are presented and illustrative design examples have been worked out to show the use of steps described in this paper.

DESIGN PROCEDURES

The following steps provide a systematic procedure for designing a W14 column by using the design graphs and tables:

Category A: Select a trial section.

1. Determine the value of f_a/F_a :
 - a. Determine which axis has the maximum Kl/r ratio according to Table 1.
 - b. Refer to Fig. 1. Select the trial section on Axis I for the axis giving the maximum Kl/r ratio. Proceed counterclockwise, selecting proper values of Kl , P , and the section number. Read the value of f_a/F_a on Axis IV. If $f_a/F_a \leq 0.15$, only AISC For-

mula (1.6-2) may be used and values of f_b/F_b must be determined according to Steps 3 and 4.

2. Determine the value of $f_a/0.60F_y$:
 - a. Refer to Fig. 6. Select the value of P on Axis I. Proceed counterclockwise, selecting the column section number and read the value of $f_a/0.60F_y$ on Axis II.
3. Determine the value of f_{bx}/F_{bx} :
 - a. Determine the value of F_{bx} , according to Table 2. If l_x is less than L_u , use Fig. 4 by following Step 3b. If l_x is less than L_u , continue from Step 3c.
 - b. Refer to Fig. 4. Select the longest unbraced length about the $X-X$ and $Y-Y$ axes on Axis I. Continue counterclockwise, selecting the values of the section number and the ratio of M_{1x}/M_{2x} . Read the value of F_{bx} on Axis III.
 - c. Refer to Fig. 2. Select M_{2x} on Axis I. Proceed counterclockwise, selecting the section number and the value of F_{bx} . Read the value of f_{bx}/F_{bx} on Axis III.
4. Determine the value of f_{by}/F_{by} :
 - a. Refer to Fig. 3. Select the value of M_{2y} on Axis I. Proceed counterclockwise, selecting the section number and F_{by} . Read the value of f_{by}/F_{by} on Axis III.
5. Check AISC Formula (1.6-2), when $f_a/F_a \leq 0.15$:
 - a. Add the values from Steps 1b, 3c, and 4a. If the sum is greater than unity, choose another trial section.
6. Check AISC Formula (1.6-1b):
 - a. Add the values from Steps 2a, 3c, and 4a. If the sum is greater than unity, choose another trial section.
7. Determine the bending ratio about the $X-X$ axis for AISC Formula (1.6-1a):
 - a. Refer to Fig. 5. Select the section number to the right of the r_y-r_x line on Axis I. Proceed counterclockwise, selecting the values of Kl_x , P , the section number, and 0.85 for C_{mx} . Read the value of the amplification factor for the $X-X$ axis on Axis V.
 - b. Refer to Fig. 2. Select the value of F_{bx}/F_{bx} , from Step 3c, on Axis III, and the value of the amplification factor for the $X-X$ axis in Quadrant III. Read the value of the bending ratio on Axis IV.
8. Determine the bending ratio about the $Y-Y$ axis for AISC Formula (1.6-1a):
 - a. Refer to Fig. 5. Select the section number to the left of the r_y-r_x line on Axis I. Proceed counterclockwise, selecting the values of Kl_y , P , the section number, and 0.85 for C_{my} . Read the value of the amplification factor for the $Y-Y$ axis on Axis V.
 - b. Refer to Fig. 3. Select the value of f_{by}/F_{by} , from Step 4a, on Axis III, and the value of the amplification factor for the $Y-Y$ axis in Quadrant III. Read the value of the bending ratio on Axis IV.

9. Check AISC Formula (1.6-1a):
 - a. Add the values from Steps 1b, 7b, and 8b. If the sum is greater than one, select another trial section.

Category B: Select a trial section.

1. f_a/F_a : Follow Step 1 of Category A.
2. $f_a/0.60F_y$: Follow Step 2 of Category A.
3. f_{bx}/F_{bx} : Follow Step 3 of Category A.
4. f_{by}/F_{by} : Follow Step 4 of Category A.
5. AISC Formula (1.6-2), when $f_a/F_a \leq 0.15$: Follow Step 5 of Category A.
6. AISC Formula (1.6-1b): Follow Step 6 of Category A.
7. Bending ratio about the $X-X$ axis for AISC Formula (1.6-1a):
 - a. Follow Step 7a of Category A, except the value of C_{mx} must be determined according to AISC Table C1.6.1.1.
 - b. Follow step 7b of Category A.
8. Bending ratio about the $Y-Y$ axis for AISC Formula (1.6-1a):
 - a. Follow Step 8a of Category A, except the value of C_{my} must be determined according to AISC Table C1.6.1.1.
 - b. Follow Step 8b of Category A.
9. AISC Formula (1.6-1a): Follow Step 9 of Category A.

Category C: Select a trial section.

1. f_a/F_a : Follow Step 1 of Category A.
2. $f_a/0.60F_y$: Follow Step 2 of Category A.
3. $f_{bx}/F_{bx}(M_{2x})$: Follow Step 3 of Category A. $C_b = 1.0$ when $M_{3x} > M_{2x}$.
4. f_{bx}/F_{bx} for M_{3x} : Follow Step 3 of Category A, replacing M_{2x} with M_{3x} . $C_b = 1.0$ when $M_{3x} > M_{2x}$.
5. f_{by}/F_{by} for M_{2y} : Follow Step 4 of Category A.
6. F_{by}/F_{by} for M_{3y} : Follow Step 4 of Category A, replacing M_{2y} with M_{3y} .
7. AISC Formula (1.6-2), when $f_a/F_a \leq 0.15$:
 - a. Add the value from Step 1, larger value from Steps 3 and 4 and larger value from Steps 5 and 6. If the sum is greater than one, choose another trial section.
8. AISC Formula (1.6-1b):
 - a. Add the values from Steps 2, 3, and 5. If the sum is greater than one, choose another trial section.
9. Bending ratio about the $X-X$ axis for AISC Formula (1.6-1a):
 - a. Refer to Fig. 5. Select the section number to the right of the r_y-r_x line on Axis I. Proceed counterclockwise, selecting the values of Kl_x , P , the section number, and the value of ψ according to AISC Table C1.6.1.2. Read the value of C_{mx} on Axis V.

b. Continue from Axis IV of Step 9a of Category C, by selecting the value of C_{mx} in Quadrant IV. Read the value of the amplification factor for the $X-X$ axis on Axis V.

- c. Refer to Fig. 2. Select the value of f_{bx}/F_{bx} from Step 4 of Category C, on Axis III, and the value of the amplification factor for the $X-X$ axis from Step 9b, in Quadrant III. Read the value of the bending ratio for the $X-X$ axis on Axis IV.
10. Bending ratio about the $Y-Y$ axis for AISC Formula (1.6-1a):
 - a. Refer to Fig. 5. Select the section number to the left of the r_y-r_x line on Axis I. Proceed counterclockwise, selecting the values of Kl_y , P , the section number, and the value of ψ according to AISC Table C1.6.1.2. Read the value of C_{my} on Axis V.
 - b. Continue from Axis IV of step 10a of Category C, by selecting the value of C_{my} in Quadrant IV. Read the value of the amplification factor for the $Y-Y$ axis on Axis V.
 - c. Refer to 3. Select the value of f_{by}/F_{by} from Step 6 of Category C, on Axis III, and the value of the amplification factor for the $Y-Y$ axis from step 10b in Quadrant III. Read the value of the bending ratio for the $Y-Y$ axis on Axis IV.
 11. AISC Formula (1.6-1a):
 - a. Add the values from Steps 1, 9c, and 10c. If the sum exceeds one, choose another trial section.

DESIGN EXAMPLES

The following examples will help to illustrate the steps listed in this paper. The mathematical as well as the graphical solution has been provided, to give the reader an idea of the accuracy which can be obtained.

Examples 1 and 2 correspond to dashed lines one and two drawn in the figures.

Example 1—Design the column with given numerical values shown in Fig. 7 by using the AISC Specification, Category B, A36 steel, and a W14 section. Assume that $Kl_x = Kl_y = 35$ ft.

Mathematical Solution:

Choose a W14 \times 605 as a trial section.

$$A = 178 \text{ in.}^2$$

$$S_x = 1,040 \text{ in.}^3; \quad S_y = 423 \text{ in.}^3$$

$$r_x = 7.81 \text{ in.}; \quad r_y = 4.55 \text{ in.}$$

$$L_c = 18.4 \text{ ft}; \quad L_u = 160.2 \text{ ft} \quad (\text{AISC Spec. Column Tables})$$

$$Kl_x/r_x = 53.8; \quad Kl_y/r_y = 92.3 \quad \therefore (Kl/r)_{max} = 92.3$$

$$F_a = 13.93 \quad (\text{AISC Spec. Table 1-36, Appendix A})$$

$$F_{ex}' = 51.64; \quad F_{ey}' = 17.53 \quad (\text{AISC Spec. Table 2, Appendix A})$$

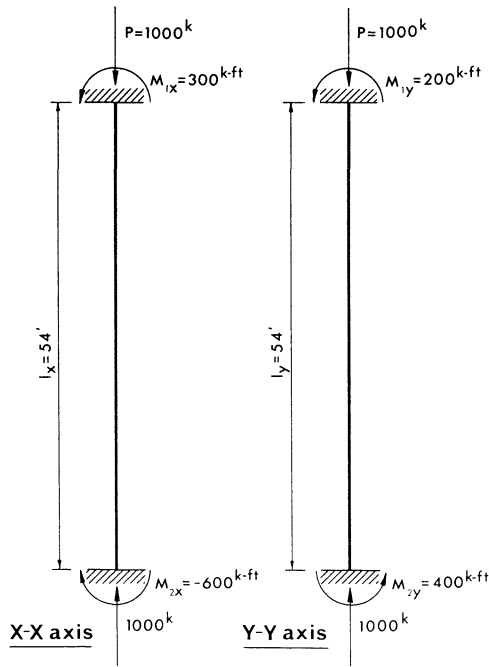


Fig. 7. Design Example 1

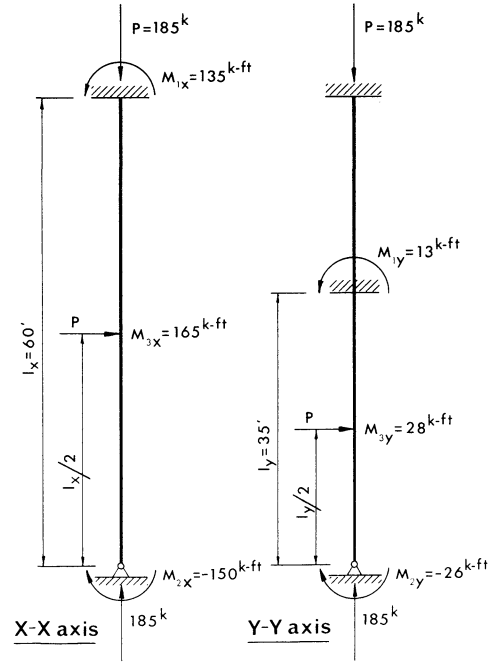


Fig. 8. Design Example 2

$$F_{bx} = 22 \text{ ksi} \quad (\text{AISC Spec. 1.5.1.4.6a})$$

$$F_{by} = 27 \text{ ksi} \quad (\text{AISC Spec. 1.5.1.4.3})$$

$$f_{bx}/F_{bx} = 0.31; \quad f_{by}/F_{by} = 0.42$$

$$f_a/F_a = 0.40; \quad f_a/0.6F_y = 0.26$$

AISC Formula (1.6-1b):

$$\frac{f_a}{0.6F_y} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} = 0.26 + 0.31 + 0.42 = 0.99 < 1.0 \text{ o.k.}$$

$$C_{mx} = 0.8; \quad C_{my} = 0.4$$

$$f_a/F_{ex'} = 0.109; \quad f_a/F_{ey'} = 0.320$$

$$\frac{C_{mx}f_{bx}}{(1 - f_a/F_{ex'}) F_{bx}} = 0.28$$

$$\frac{C_{my}f_{by}}{(1 - f_a/F_{ey'}) F_{by}} = 0.25$$

AISC Formula (1.6-1a):

$$\frac{f_a}{F_a} + \frac{C_{mx}f_{bx}}{(1 - f_a/F_{ex'}) F_{bx}} + \frac{C_{my}f_{by}}{(1 - f_a/F_{ey'}) F_{by}} \leq 1.0$$

$$0.40 + 0.28 + 0.25 = 0.93 < 1.0 \text{ o.k.}$$

Graphical Solution: Trial section is W14×605 and $(Kl/r)_{max}$ is known.

$$\text{Step 1: } f_a/F_a = 0.40$$

$$\text{Step 2: } f_a/0.6F_y = 0.26$$

$$\text{Step 3: } l_x = 54 \text{ ft} < L_u = 160.2 \text{ ft} \quad (\text{Table 1})$$

$$F_{bx} = 22 \text{ ksi} \quad (\text{Table 2})$$

$$f_{bx}/F_{bx} = 0.31$$

$$\text{Step 4: } F_{by} = 27 \text{ ksi} \quad (\text{Table 2})$$

$$f_{by}/F_{by} = 0.42$$

$$\text{Step 5: } f_a/F_a > 0.15 \quad \therefore \text{Does not apply}$$

$$\text{Step 6: } 0.26 + 0.31 + 0.42 = 0.99 < 1.0 \text{ o.k.}$$

$$\text{Step 7: } C_{mx} = 0.8$$

$$\frac{C_{mx}}{(1 - f_a/F_{ex'})} = 0.90$$

$$0.90(f_{bx}/F_{bx}) = 0.28$$

$$\text{Step 8: } C_{my} = 0.4$$

$$\frac{C_{my}}{(1 - f_a/F_{ey'})} = 0.59$$

$$0.59(f_{by}/F_{by}) = 0.25$$

$$\text{Step 9: } 0.40 + 0.28 + 0.25 = 0.93 < 1.0 \text{ o.k.}$$

Example 2—Design the column with given numerical values shown in Fig. 8 by using the AISC Specification, Category C, A36 Steel, and a W14 section. Assume that $Kl_y = 28$ ft.

Mathematical Solution: Choose W14×150 as a trial section.

$$A = 44.1 \text{ in.}^2; \quad d/A_f = 0.85 \text{ in.}^{-1}$$

$$S_x = 240 \text{ in.}^3; \quad S_y = 90.6 \text{ in.}^3$$

$$r_x = 6.37 \text{ in.} \quad r_y = 3.99 \text{ in.}$$

$$L_c = 16.4 \text{ ft}; \quad L_u = 54.5 \text{ ft} \quad (\text{AISC Spec. Column Tables})$$

$\psi_x = \psi_y = -0.4$ (AISC Spec. Table C1.6.1.2)
 $Kl_x/r_x = 90.4$; $Kl_y/r_y = 84.2 \therefore (Kl/r)_{max} = 90.4$
 $F_a = 14.16$ (AISC Spec. Table 1-36, Appendix A)
 $F_{ex'} = 18.28$; $F_{ey'} = 21.06$ (AISC Spec. Table 2, Appendix A)

$f_a/F_a = 0.296$; $f_a/0.6F_y = 0.194$
 $f_a/F_{ex'} = 0.229$; $f_a/F_{ey'} = 0.199$

$$C_{mx} = 1 + \psi \frac{f_a}{F_{ex'}} = 0.91$$

$$C_{my} = 1 + \psi \frac{f_a}{F_{ey'}} = 0.92$$

$C_b = 1.0$ (AISC Spec. 1.5.1.4.6a)

$$F_{bx} = \frac{(12 \times 10^3)C_b}{l(d/A_f)} = 19.6 \text{ ksi}$$

$F_{by} = 27.0 \text{ ksi}$ (AISC Spec. 1.5.1.4.3)

$f_{bx}/F_{bx} = 0.38$, using M_{2x}

$f_{by}/F_{by} = 0.13$, using M_{2y}

AISC Formula (1.6-1b), using M_2 :

$$f_a/0.6F_y + f_{bx}/F_{bx} + f_{by}/F_{by} = 0.194 + 0.38 + 0.13 = 0.704 < 1.0 \quad \text{o.k.}$$

$f_{bx}/F_{bx} = 0.42$, using M_{3x}

$f_{by}/F_{by} = 0.137$, using M_{3y}

$$\frac{C_{mx}f_{bx}}{(1 - f_a/F_{ex'}) F_{bx}} = 0.496$$

$$\frac{C_{my}f_{by}}{(1 - f_a/F_{ey'}) F_{by}} = 0.157$$

AISC Formula (1.6-1a), using M_3 :

$$\frac{f_a}{F_a} + \frac{C_{mx}f_{bx}}{(1 - f_a/F_{ex'}) F_{bx}} + \frac{C_{my}f_{by}}{(1 - f_a/F_{ey'}) F_{by}} \leq 1.0$$

$$0.296 + 0.496 + 0.157 = 0.95 < 1.0 \quad \text{o.k.}$$

Graphical Solution: Trial section is W14×150 and $(Kl/r)_{max}$ is known.

Step 1: $f_a/F_a = 0.296$

Step 2: $f_a/0.6F_y = 0.193$

Step 3: $l_x = 60 \text{ ft} > L_u = 54.5 \text{ ft}$ (Table 1)

\therefore Use Fig. 4.

$C_b = 1.0$ and $F_{bx} = 19.61 \text{ ksi}$

$f_{bx}/F_{bx} = 0.38$, using M_{2x}

Step 4: $f_{bx}/F_{bx} = 0.42$, using M_{3x}

Step 5: $F_{by} = 27 \text{ ksi}$ (Table 2)

$f_{by}/F_{by} = 0.125$, using M_{2y}

Step 6: $f_{by}/F_{by} = 0.137$, using M_{3y}

Step 7: $f_a/F_a > 0.15 \therefore$ Does not apply

Step 8: $0.194 + 0.38 + 0.13 = 0.704 < 1.0 \quad \text{o.k.}$

Step 9: $C_{mx} = 0.91$

$$\frac{C_{mx}}{(1 - f_a/F_{ex'})} = 1.18$$

$$1.18(f_{bx}/F_{bx}) = 0.496$$

Step 10: $C_{my} = 0.92$

$$\frac{C_{my}}{(1 - f_a/F_{ey'})} = 1.15$$

$$1.15(f_{by}/F_{by}) = 0.158$$

Step 11: $0.297 + 0.496 + 0.157 = 0.95 < 1.0 \quad \text{o.k.}$

CONCLUSIONS

This paper provides a rapid graphical method for designing W14 steel columns subjected to combined loading, in accordance with the new AISC Specification.

Graphs, modeled after the ones presented in this paper, may now be developed for any series and type of steel. These graphs would be very helpful to the design engineer, giving him a rapid solution with practically no calculations to be performed.

The computer program written to accommodate this method was developed so that the input can be determined quickly and accurately. It contains all the provisions of the AISC Specification and it tries all possible sections, beginning with the smallest, until a satisfactory section has been obtained.

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