Effective Length of Columns with Intermediate Axial Load

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THE EFFECTIVE LENGTH factor K for columns loaded at ends with various end conditions can be determined easily using the method outlined in Section 1.8 of Reference 2. There are some cases of column design in which there is an axial load at an intermediate point between ends in addition to load at the ends. An example of this type is the column of a two-story building with floor beams at an intermediate floor level framed to the column flanges and no beams framing to the web. The column is free to buckle in the weak direction at the intermediate level. Similarly a column supporting a crane runway in an industrial building falls into this category if orientation of column is such that it can buckle in the y-direction at the crane runway level. The crane load is the intermediate load in this case. To use the full length of the column as the actual unbraced length and multiply by the K factor determined from Table C1.8.1 or Fig. 1.8.2, AISC *Manual of Steel Construction*, is conservative. However, some economy can be achieved by using a modified effective length factor.

Consider the pin ended column loaded as in Fig. 1. The column is free to buckle about its weak axis and is in a state of unstable equilibrium. If the stability equation is derived by solving the differential equations of the deflection curves for the upper portion and lower portion of column separately, taking into account compatability at the intermediate load point, the following equation will result:

$$kl \frac{(1 + a' - a)}{\sqrt{a'}} \cot \sqrt{a'}\, k(1 - a)l + \frac{(1 + a' - a)}{\sqrt{1 + a'}} kl \cot \sqrt{1 + a'}\, akl = \frac{1}{a'(1 + a')} \quad (1)$$

which is the stability equation for the column with intermediate axial load.

Equation (1) can be solved for kl for different values of a and a' and effective length factor evaluated as below.

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Example—Determine the effective length factor of a column for

$$a = 0.5, \qquad a' = 1.0$$

Substituting these values in stability Eq. (1) gives

$$\frac{3}{2} kl \cot \frac{kl}{2} + 1.06kl \cot \frac{1}{\sqrt{2}} kl = \frac{1}{2} \quad (2)$$

By trial it is found that $kl = 2.54$ satisfies Eq. (2), which is therefore the solution of the stability equation.

Squaring $k^2l^2 = 6.45$ or $\left(\frac{P}{EI}\right) l_2 = 6.45$,

$$P = 6.45 \frac{EI}{l^2}$$

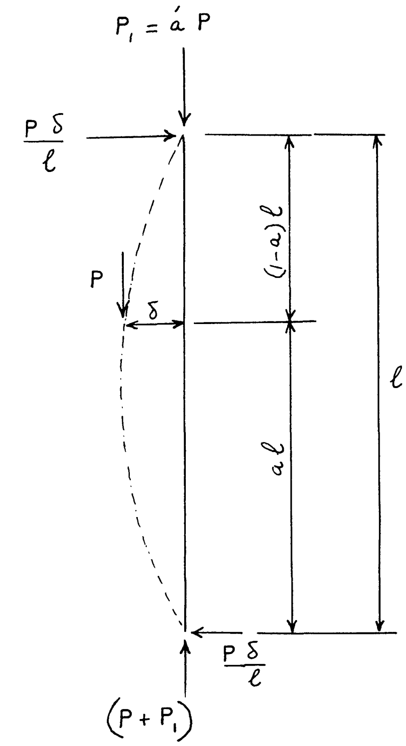

Fig. 1. Pin-ended column loaded at ends and an axial load at intermediate point

$$\text{Total column load} = P + P_1 = 2P = 12.9\,\frac{EI}{l^2}$$

The Euler critical load for a pin-ended column loaded at ends only is $P_{cr} = \frac{\pi^2 EI}{l^2}$. The effective length factor K for a column with intermediate axial load is given by

$$12.9\,\frac{EI}{l^2} = \frac{\pi^2 EI}{(Kl)^2}$$

or

$$K^2 = \frac{\pi^2}{12.9} = 0.765$$

$$K = 0.875$$

The writer has solved the stability equation for a = 0.25, 0.50, 0.75, 0.85, 0.95 and a' = 0.1, 0.25, 0.50, 0.75, 1.0, 2.0, 3.0, 4.0. The results are tabulated in Table 1 and plotted in Fig. 2.

Since these curves of K values are for a pin-ended column, the value of K found from these curves is to be multiplied by the effective length factor found from Table C1.8.1 or Fig. C1.8.2 of AISC *Manual of Steel Construction*. The use of the curves in Fig. 2 is illustrated in the numerical example.

Numerical Example

Given—Unbraced column length 20′–0″ in an industrial building, with intermediate load of 60 kips from a crane runway at a height of 10′–0″. The load at top of column is 15 kips. The end conditions are such that the effective length factor for a column loaded at ends as found from Table C.1.8.1 of the AISC *Manual of Steel Construction* is K = 2.0

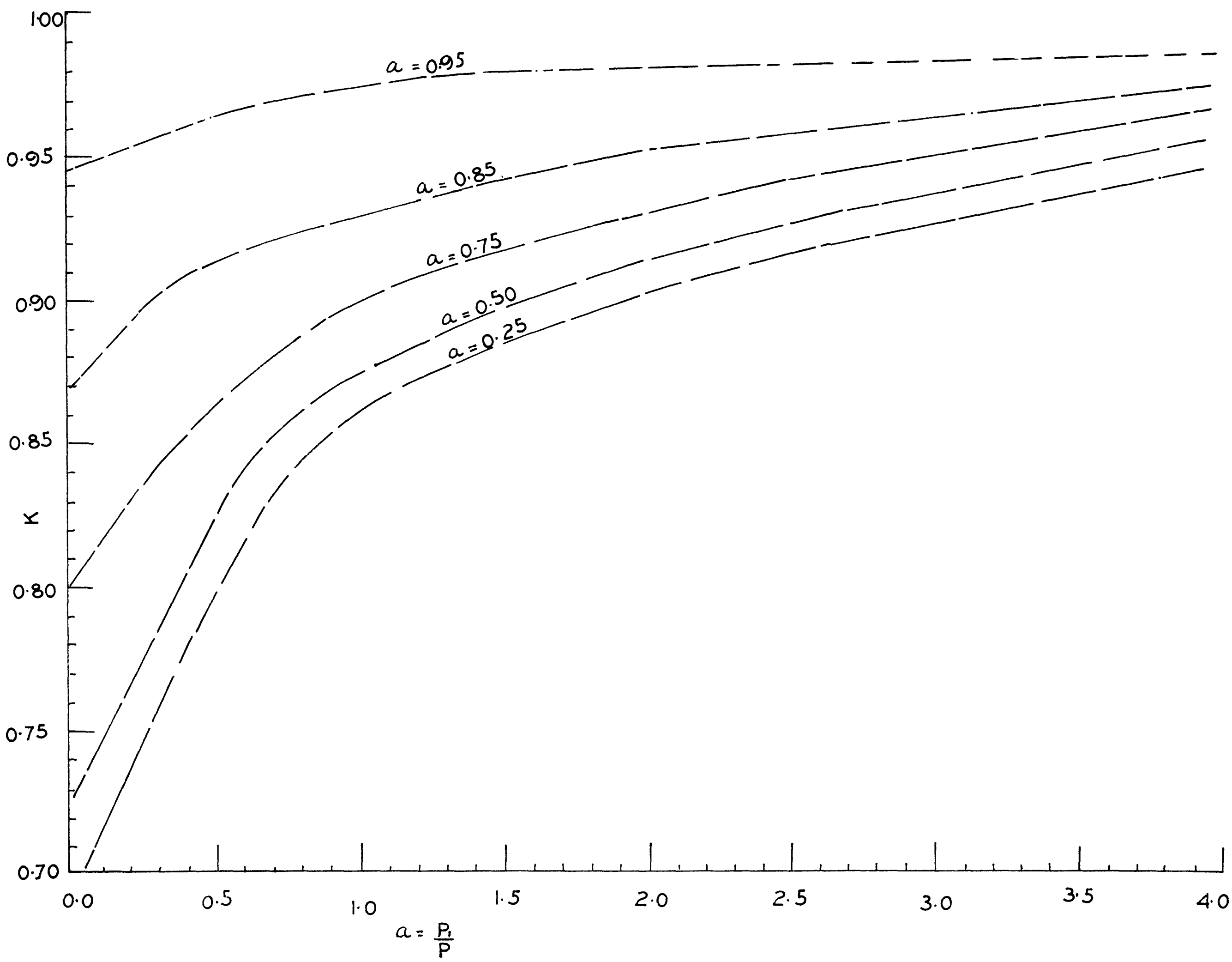

Fig. 2 Effective length factor K for pin-ended column with axial load at ends and at intermediate points

Table 1. Effective Length Factors K for a pin-ended column with an intermediate axial load P at height aL and a load P_1 at the top

	$\frac{P_1}{P} = a' =$	0.10	0.25	0.50	0.75	1.0	2.0	3.0	4.0
a	Effective Length Factor K								
0.25		0.715	0.750	0.795	0.840	0.863	0.903	0.927	0.940
0.50		0.745	0.775	0.825	0.860	0.875	0.915	0.935	0.955
0.75		0.815	0.835	0.860	0.890	0.895	0.930	0.950	0.965
0.85		0.880	0.905	0.910	0.920	0.930	0.955	0.965	0.975
0.95		0.950	0.955	0.965	0.970	0.975	0.980	0.985	0.985
1.00		1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

Determine the modified effective length for the column design.

Solution:

$$P = 60 \text{ kips}; \qquad P_1 = 15 \text{ kips}$$

$$a' = \frac{P_1}{P} = \frac{15}{60} = 0.25$$

$$a = \frac{10}{20} = 0.5$$

From Fig. 2, the effective length factor for pin-ended column = 0.775. The modified effective length factor for the actual column is 0.775 × 2.0 = 1.55. The column effective length for design is 1.55 × 20 = 31.0 ft.

CONCLUSIONS

A method of column design with an intermediate axial load in addition to load at the ends has been described. The saving in steel weight by using modified effective length factor is significant in the case of long columns with an intermediate load much larger than the load at the top end. The method can be extended to cases where there are more than one intermediate load. In such cases the modified K-factor can be found for each intermediate load separately and the average of these values can be taken as the K-factor for the design of the column.

REFERENCES

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