Discussion

Applied Plastic Design of Unbraced Multistory Frames

Paper presented by ROBERT O. DISQUE (October 1971 issue)

Discussion by **KENNETH B. WIESNER**

Mr. Disque's article did not state the assumptions upon which Eqs. (1) and (2) are based. The writer has assumed the following:

1. All joint rotations (θ) are equal at a given floor level.
2. All girders and columns have points of contraflexure at mid-length.
3. The effects of axial deformation and shear deformation are neglected.
4. Story heights and bay sizes are relatively uniform.
5. Story shear magnitudes vary gradually in adjacent stories.

Based upon the above assumptions, and the definitions below, I suggest these alternative formulations of the author's Eqs. (1) and (2):

$$\Delta = h\theta\left[\frac{\Sigma G_{girders}}{\Sigma G_{columns}} + 1\right] \qquad \text{(1-Rev.)}$$

$$\Delta = \frac{hM_{sway}}{12E}\left[\frac{1}{\Sigma G_{columns}} + \frac{1}{\Sigma G_{girders}}\right] \qquad \text{(2-Rev.)}$$

where

$\Sigma G_{columns}$ = $\Sigma(I/h)$ for all columns in the story considered

$\Sigma G_{girders}$ = The average $\Sigma(I/L)$ for Σ girders at top and Σ girders at bottom of the story

M_{sway} = Qh for a top or intermediate story

Subsequent Eqs. (10) and (21) should be changed similarly.

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The writer also disagrees with the author's statement following Eq. (10) that the total M_{sway} for a particular story is the M_{sway} computed for that story by Eq. (9) *plus* the sum of all the sway moments above the story.

The total story sway moment, including $P\Delta$ effect, should be calculated from

$$M_{sway} = h(Q + Q') \qquad (23)$$

in which Q = horizontal story shear due to external loads, and Q' = horizontal story shear due to the $P\Delta$ effect. The value of Q' is computed from

$$Q' = \frac{P\Delta'}{h} = PR \qquad (24)$$

where

P = Sum of all vertical loads above the given story on columns of the frame plus columns braced by the frame

Δ' = Total relative horizontal displacement of the top versus the bottom of the story, including $P\Delta$ effect. It could also include the effect of column axial deformation, although this may not be significant for only one story.

R = Story rotation angle due to sway

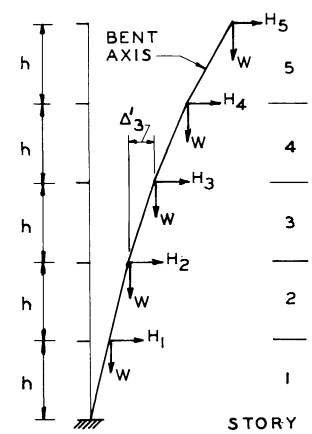

Fig. 6. Actual loads on deflected bent

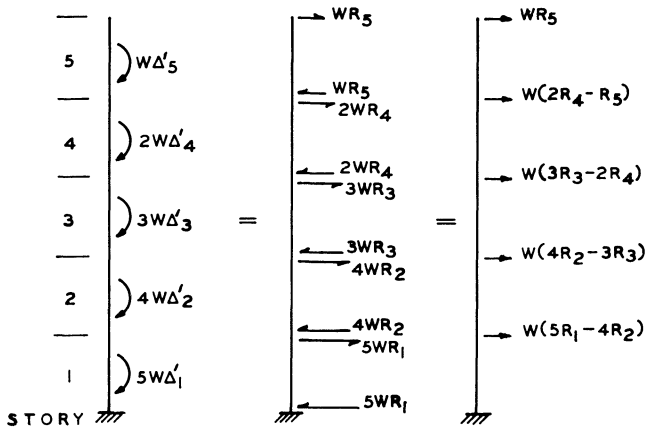

Fig. 7. PΔ equivalent loading on bent

It is true that the absolute deflection of the frame is affected by the sum of the sway moments for a given story plus all sway moments above it, because such a moment sum corresponds to the total overturning moment at the story. However, total overturning moment affects only column axial forces and "chord drift". The "web drift" computed from Eq. (2) and frame column and girder moments are affected only by the total story horizontal shear. Figures 6 and 7 illustrate how the $P\Delta$ shears can be calculated for a multistory frame. A simple 5-story example bent is shown, schematically. For the general case of a non-linear deflected shape, with unequal story R's, the total $P\Delta$ story shear at the third level is computed as follows:

$$Q'_3 = W(R_5 + 2R_4 - R_5 + 3R_3 - 2R_4)$$

Therefore, $Q'_3 = 3WR_3$, showing that the P-Δ shear at the story is the story sway angle, R, multiplied by the sum of vertical loads above the story, $\Sigma W = P$. From either Fig. 6 or Fig. 7, the total $P\Delta$ overturning moment at the bottom of story 3 is $M'_3 = W\Delta'_5 + 2W\Delta'_4 + 3W\Delta'_3 = Wh\,(R_5 + 2R_4 + 3R_3)$.

The story drift Δ' can be directly calculated as follows, from Eqs. (2-Rev.), (23), and (24), if axial and shear deformations are neglected:

$$\Delta = \frac{Qh^2}{12E}\left(\frac{1}{\Sigma G_c} + \frac{1}{\Sigma G_g}\right)$$

$$\Delta' = \frac{h(Qh + Q'h)}{12E}\left[\frac{1}{\Sigma G_c} + \frac{1}{\Sigma G_g}\right] = \frac{h^2(Q + (P\Delta'/h))}{12E}\left[\frac{1}{\Sigma G_c} + \frac{1}{\Sigma G_g}\right]$$

Solving for Δ',

$$\Delta' = \frac{(Qh^2/12E)}{\left[\dfrac{1}{(1/\Sigma G_c) + (1/\Sigma G_g)}\right] - \dfrac{Ph}{12E}} \qquad (25)$$

This equation may be simplified using the following relationship:

Therefore,

$$\Delta' = \frac{1}{(1/\Delta) - (P/Qh)} = \frac{\Delta}{1 - (P/Qh)} \qquad (26)$$