Curvilinear Grid Frames

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IN DEALING with a structural system for the first time, especially when the system is new, one finds himself consciously involved in many facets of design which would ordinarily be intuitive in the more familiar systems of construction. One must first satisfy himself about the limits and the possibilities of the systems *per se*, before he can even begin to think about developing an architecture out of the constructional method. The use of curved surfaces is a case in point.

From a structural point of view, there are two problems which face a designer. The first deals with a succinct knowledge of the stress distribution within the framework of the design project, and the second with the assurance that the materials applied to the resolution of the internal stress pattern will react in a predictable manner. The first is usually a paper problem using a more or less idealized situation. The second is a laboratory or field problem. Not all structures are capable of a rational analysis, particularly shells, and those structures, if built, must undergo experimental analysis to a lesser or greater degree. This is where the applied researcher renders his valuable service. And yet his role also involves the examination of those units for which there is a rational approach, if for nothing else than to instill confidence in a particular analytical method.

It is essentially to this latter end that I have been working with the Inland Steel Company for the past year and one-half on the steel hyperbolic paraboloid. As a result of these investigations, I have been charged to discuss this structural form.

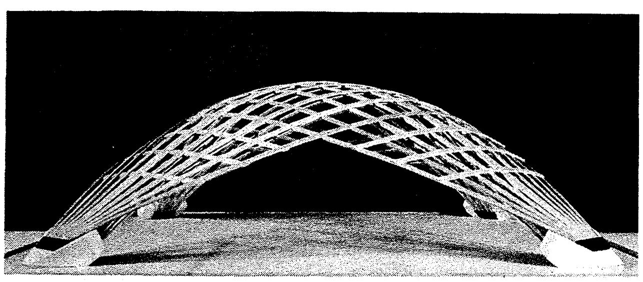

Fig. 1. Elevation of a ribbed dome on four supports

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TYPES OF RIBBED SURFACES

It should be noted that the ribbed surface is only one of a general class of structures. There are at least three important curved surfaces which lend themselves to the concepts of a ribbed structure. They are the hyperbolic paraboloid, the monkey saddle and the dome. The mechanics of these surfaces will depend upon the method of rib layout. In the case of the dome, the ribs act as a series of arches forming a grid pattern. They will sustain load both by direct stress from the arch action and by the interchange of load at the joints, similar to a flat grid. Although there are several methods of organizing the ribs in a dome, the design will differ only in detail. The monkey saddle may be viewed as a kind of suspension system where the superimposed loads on the ribs are

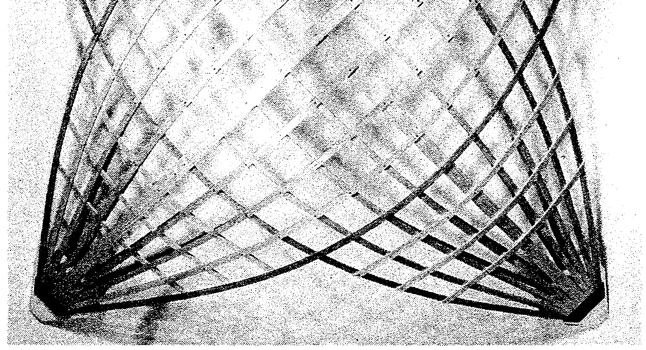

Fig. 2. Plan view of a ribbed dome resting on four supports. This system of framing is remarkably stable. The structure sustains load by the combined arch action and grid characteristics of the surface.

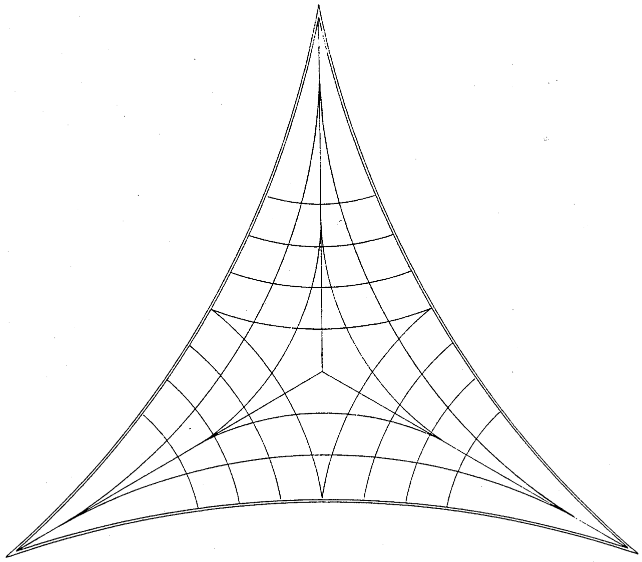

Fig. 3. One of several rib layouts of triangular domes

transferred to the ties. It is possible to conceive a rib layout as a grid, but probably the layout shown in Fig. 3 is the most direct way of forming the surface. The hyperbolic paraboloid lends itself easily to both of these interpretations. As a grid, the surface would be formed by two sets of arches, one positive and one negative, whose directions are diagonal with respect to the boundary of the surface. In the plan view this layout would take on the appearance of a diagonal grid system. The ribs may also be patterned orthogonally. The rib elements would then be straight rather than arched. They will be supported by a series of ties traversing the surface in the direction of the peaks.

THE HYPERBOLIC PARABOLOID

The hyperbolic paraboloid as a continuous surface is for all intents and purposes a funicular surface; except for the possibility of local disturbance along the connection of the edge member and the membrane, it will remain moment free, provided the surface loading is evenly distributed with respect to the horizontal projection. As a series of positive and negative arches (Fig. 6), an evenly distributed load will be equally divided between each set of curved ribs. In this way the ribbed frame is like the hyperbolic paraboloid as a continuous surface. Such a system is uniquely simple not only in concept, but in its design as well, provided we are not concerned with concentrated or partial loads. For uniform loading, the deflection at each joint for both the positive and negative ribs will be identical, so that there are no moments set up in the ribs. It is the property of this surface that the thrusts of all the arches will have the same absolute value. The forces in the edge member or fascia of the frame will balance the thrusts of the positive and negative arches, and will be equal to $H\sqrt{2}$ for each fascia joint. When concentrated or partial loads are applied to the frame, it becomes necessary to consider the grid characteristics of the surface.

When such loads are encountered, the designer must resort to deflection principles in a manner similar to the solution of a flat grid. Furthermore, such loads will produce moments of varying magnitudes within the ribs.

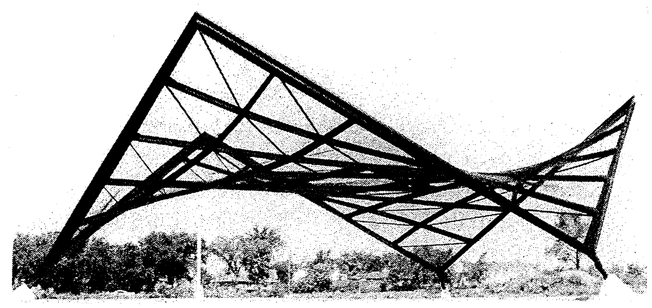

Fig. 4. Monkey saddle

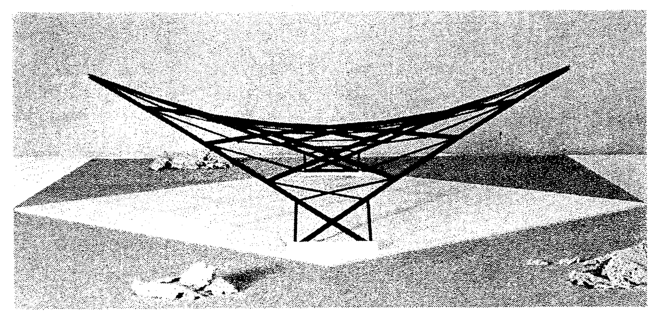

Fig. 5. Elevation of a hyperbolic paraboloid with an orthogonal rib layout

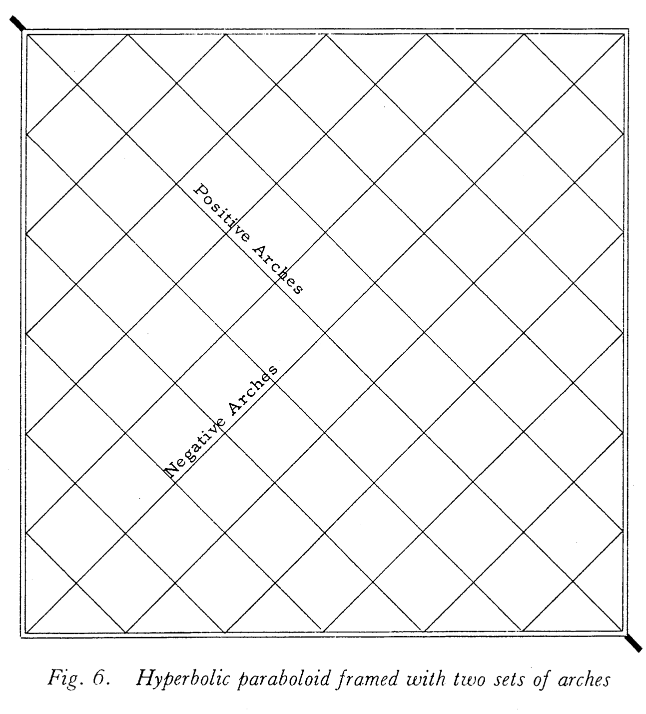

Fig. 6. Hyperbolic paraboloid framed with two sets of arches

This is equally true if the surface is a continuous hyperbolic paraboloid, but the determination of these non-membrane stresses are not so easily solved as in the case of the frame.

MOMENTS AND DEFLECTIONS

In order to deal more thoroughly with the mechanics of the hyperbolic paraboloid as a diagonal grid, it will be necessary to discuss briefly the moments and deflections of arches. When defining the shape of an arch for the support of a particular load distribution, the ideal curvature is a function of the loads imposed. In particular the curvature will be the funicular of the load. To deviate from this shape, or to change the load distribution, will induce bending moments in the arch. To determine the magnitude of these moments, recourse may be taken in the use of a sine series solution. It can be shown that for a given concentrated load P, if the moment in a beam is represented by the series

$$M = \frac{2PL}{\pi^2}\left[\sin\frac{\pi a}{L}\cdot\sin\frac{\pi x}{L} + \sum_{n+2}^{\infty}\frac{1}{2^n}\sin\frac{n\pi a}{L}\cdot\sin\frac{n\pi x}{L}\right]$$

where a is the position of the load and x is the distance to the point at which the moment is taken. The first term of the series gives that moment contributory to thrust and the sum of the remaining terms gives the magnitude of the bending moments in the arch. This gives a rather direct approach to the finding of thrusts and moments. However, it should be noted that the sum beyond the first term converges slowly, so that many terms must be used for real accuracy.

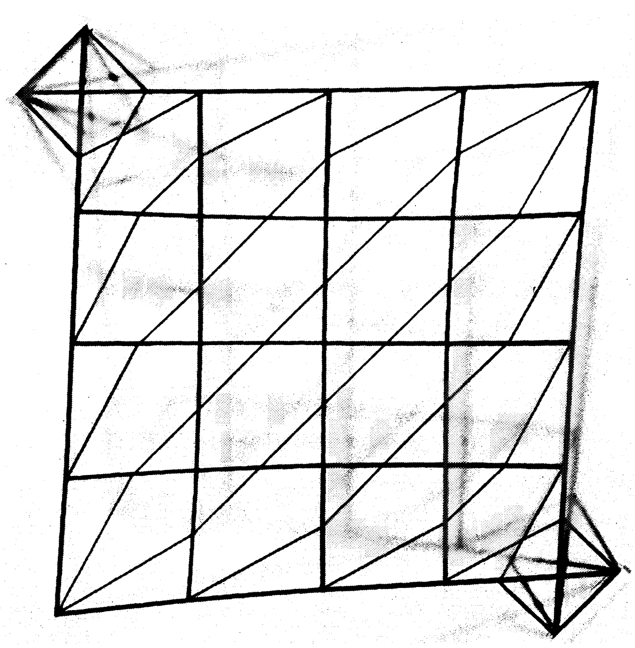

Fig. 7. Plan view of a hyperbolic paraboloid

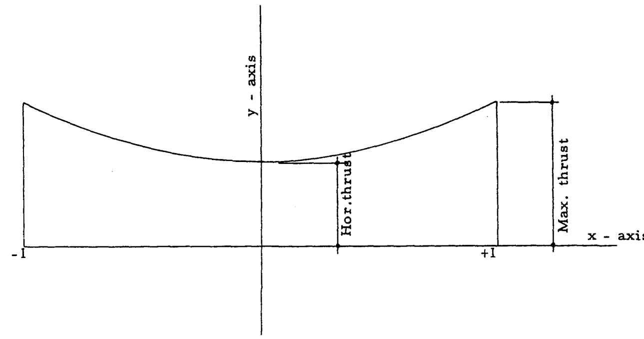

Figure 8

Coupled with the determination of moments in an arch is the determination of deflections. Deflections may stem from two causes. The first is the arch shortening and the second is deflection due to moments which may be present in the arch.

One method of finding the vertical deflections of the arch is to first compute the length of the arch, then the shortening due to the thrust and subsequently the reduced rise of the arch. The following formula will give the arch length:

$$S_0 = l\left[1 + \sum_{n=1}^{\infty}(-1)^{n+1}\cdot\frac{2^{2n+1}}{2n+1}\left(\frac{h}{l}\right)^{2n}\right]$$

where h = the rise of the arch and l = the span. This series converges rapidly so that a fair degree of accuracy may be obtained with only two terms. Since the compression in the arch at any point is a function of the horizontal thrust and the shape of the arch at the point in question, in order to find the total negative elongation the sum of the infinitesimal deflections must be taken for the length of the arch or some reasonable average of the maximum and minimum unit compressive strain multiplied by the length. One such average can be found by taking the area bounded by $x = \pm 1$, the x-axis and a parabola with its crown a distance from the x-axis equal to the horizontal thrust and intersecting $x = \pm 1$ a distance equal to the maximum thrust (see Fig. 8). Once the negative elongation is found, the new rise may be determined. Taking the first two terms of the series

$$S_0 = l\left[1 + \frac{8}{3}\left(\frac{h}{l}\right)^2 - \frac{32}{5}\left(\frac{h}{l}\right)^4\right]$$

and solving for h,

$$h = \sqrt{0.208333 - 0.625\sqrt{0.111111 - 0.4\left(\frac{S_0}{l} - 1\right)}}$$

If greater accuracy is required for h, it will be necessary to solve a cubic equation for $(h/l)^2$ or to make up a table for a desired range of h. Such accuracy, however, should seldom be required.

Deflections due to moments in the arch can be found by the trigonometric series for deflection associated with the moment series. Therefore, the formula will be the same as for a beam minus the first term:

$$\delta_P = \frac{2PL^3}{\pi^4 EI} \sum_{n=2}^{\infty} \frac{1}{2^{2n}} \sin \frac{n\pi a}{L} \cdot \sin \frac{n\pi x}{L}$$

Letting Δ_P be the deflection due to the rib shortening, the total deflection at a joint will be

$$\delta_T = \delta_P + \Delta_P$$

These formulas give the tools necessary for the solution of both the grid dome and the hyperbolic paraboloid diagonal grid when concentrated or partial loadings are applied. To find the load distribution it becomes necessary to find the total deflection due to all the loads on an arch at a joint. A series of simultaneous equations will result from setting the deflections of the ribs at each joint equal. It is not to be suggested that this is a job for long hand computation, but with the aid of high speed computors, the solutions to these problems are readily available.

DESIGN

The design of the orthogonal frame avoids the problems inherent in the frames which have grid characteristics. Surface loads are supported by ribs spanning between the ties which run parallel to the peaks. The rib members take on the dual function of beam and of strut, since they also serve as anchors for the ties. As a consequence of this arrangement, the ties take the entire roof load. The design of these ties becomes a cable problem with a series of equal concentrated loads.

An arrangement of ribs in the form of an orthogonal layout can be placed in static equilibrium by means of a series of ties (Fig. 9). The ties will cause the ribs, including the exterior rib or fascia to be in compression. It will be assumed that the ties are fastened only at the ends and are not rigidly fixed at the intersections of the surface. The thrust may be determined by

$$H = M/h$$

where H is the horizontal thrust, M is the maximum moment and h is the rise. The slope α at any point along the cable (see Fig. 10) is given as

$$\alpha = \arctan\, (8hx^2)/l$$

The hyperbolic paraboloid has the property that the horizontal thrust H of each tie will be the same when the vertical component of the superimposed load is constant over the entire surface. Since the anchorage for the ties are the ribs and the fascia, the horizontal component of the thrusts produces in these members a constant horizontal component of compression. The value of this compression is

$$C = H \cos \beta$$

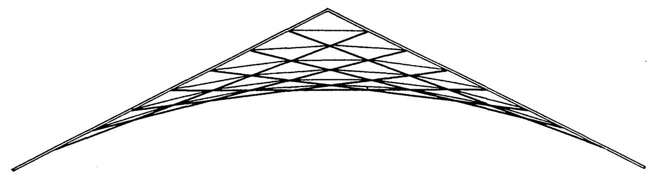

Fig. 9. The double lines represent the fascia; the heavy single lines represent the ribs; the light single lines represent the ties.

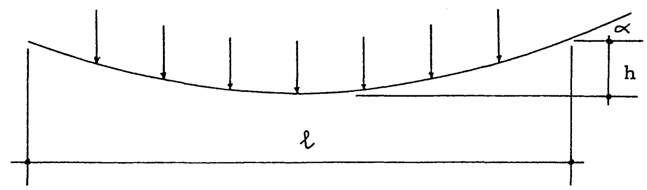

Figure 10

where β is the angle made between the tie and the fascia. The compression in the ribs will be nearly equal; however, the rib with the greatest slope will have the maximum compression.

The only remaining member to be defined is the horizontal tie which takes up the thrust of the supports. Simply, this force is the vector sum of the compressive forces in the last panel of the fascia.

This value will be

$$H' = nC/\cos \beta$$

where n is the number of panels.

The design of the ribs and the fascia will be according to the usual methods of designing steel frames. The lower set of ribs will be designed as members under combined stress. However, there is one advantage in that the upper ribs will receive a certain amount of lateral stability when attached to the decking; therefore, maximum stress may be determined on the basis of the greatest r. In general the fascia member will have to be stiffened laterally near the downpoints of the frame.

The deflection of the hyperbolic paraboloid is dependent upon the elongation of the ties and the compression of the ribs. Once S_0 has been determined, the increase in length is given by

$$\Delta S_0 = PS_0/AE$$

In order to evaluate P, which is dependent upon the thrust H, as was mentioned previously an average value will be taken. Therefore the elongation of the tie will be

$$\Delta S_0 = H_{avg} S_0/AE$$

The compression of the rib and fascia will also contribute to vertical deflection of the tie by shifting the fascia joint.

It should be remembered that as part of the total system, vertical elements are required as tiedowns either in the form of mullions or cables at the fascia edge. These tiedowns most logically should be attached when the frame is erected and before the decking or any other superimposed load is applied. These superimposed loads will place these tiedowns in tension and consequently stabilize the entire roof frame in form.

Whereas the ribs may be designed with the highest strength steel, if deflections must be kept to a minimum, lower strength steel would be used for the ties. The amount of thrust in the tie cables will be dependent upon the shape of the parabola, and not on the length of the tie. Theoretically, there is no limit to the size of the arch which will produce the same thrust. What will govern, however, will be the deflection of the tie arch. Since the total elongation increases, the vertical deflection at the center will increase.

Concentrated loads caused by mechanical equipment, or partial loads caused by snow, may be evaluated as readily as those evenly distributed loads discussed. For example, a concentrated load at **A** (Fig. 11) would be reflected in the members $\mathbf{T}_1$, $\mathbf{R}_1$, $\mathbf{T}_2$ and $\mathbf{R}_2$. A load at point **B** would be distributed through $\mathbf{T}_1$, $\mathbf{R}_1$, $\mathbf{T}_2$, $\mathbf{R}_2$, $\mathbf{T}_4$, $\mathbf{R}_3$, $\mathbf{T}_5$ and $\mathbf{R}_4$. Partial loadings may be similarly traced.

The design of the hyperbolic paraboloid is a compromise between the structural efficiency and the peak height that the architecture demands of the building. The greater the slope of the fascia the greater will be the curvature of the surface and consequently the less will be the stress developed within the surface. But at the same time, more square area will be required for the enclosing walls. For this reason heights will normally be kept as low as possible.

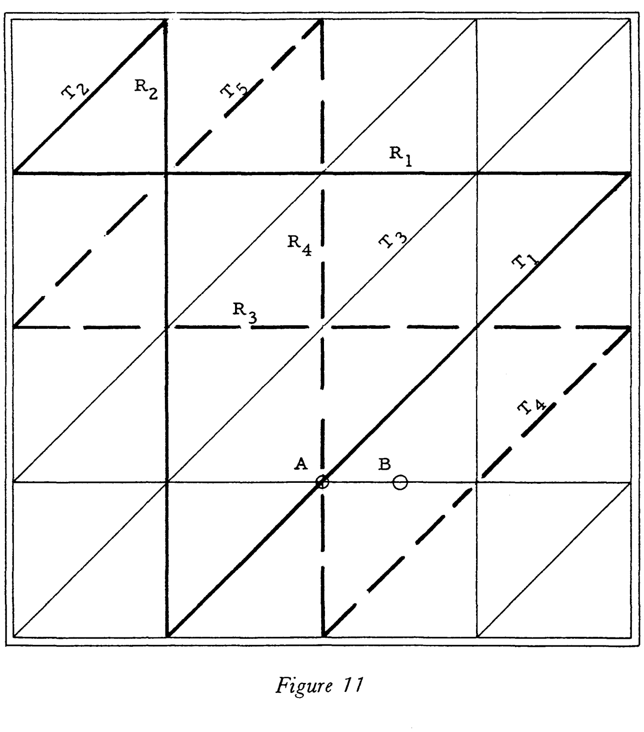

Figure 11

Although the double arched hyperbolic paraboloid is perhaps the most efficient way in which a frame may be put together, efficiency of material is not necessarily the only criteria for the use of a structural system. It may be readily seen that this method will require that the rib members be cambered, and that the field assembly would require erection in the air on temporary supports. With the orthogonal frame not only is the design simplified, for all types of loadings, but the erection is also made easier.

CONSTRUCTION

The monkey saddle, the hyperbolic paraboloid with an orthogonal frame, and some domes, either with regular boundaries or as skewed surfaces, may be assembled on a plane and subsequently raised to form the particular curved surface. When these frames are assembled on a plane there will be a curvature induced into all members except those on the centerline. This temporary curvature in the ribs and fascias will not overstress the members because of their length and their lack of lateral stiffness. In the case of the hyperbolic paraboloid this temporary curvature in the ribs and fascia may be removed when the frame is erected. However, in the case of the monkey saddle and the dome this curvature will not be completely eliminated. In general, then, these lateral curvatures will have to be taken into account in the design of these two types of surfaces, especially if the members are necessarily stiff because of the size of the shell or because of some unusual loading.

The lengths of the ribs may be determined mathematically or they can be determined by projection. The accuracy of the location of the joints is not a highly critical matter and therefore the normal tolerances which are permitted in steel construction can be observed. In the hyperbolic paraboloid the joints will be evenly spaced for the length of a given rib. In the domes the location of the joints can be made on the basis of an assumption that the portion of the rib within the grid section for each rib is equally divided. It should be born in mind that all joints except those connected to the center-most ribs will rotate in the process of erection and therefore must either be pinned or left free until erection is complete.

The system of construction as applied to the hyperbolic paraboloid was demonstrated on the Calumet Campus of Purdue University, in cooperation with the Inland Steel Co., during the winter of 1963 (see Fig. 12). The reason for the construction of this shell unit was to prove the method as a simple and direct way of using steel for shells without elaborate facilities. The shell is 80.5 ft square in projected plan with a rise of 23 ft from the top of the support to the peak. The area covered by the roof is 6,480 sq ft. The size of the ribs is 6B8.5 and size of the fascia is 14B22, with a 3 × 2 × $\frac{3}{16}$ angle welded to the top side of the fascia to work structurally and to serve as a closure for the metal deck. A 3-in. metal deck spans between the ribs. The framing steel is INX70 (70 ksi yield). The ties alternate between 2–3 × 2 × $\frac{3}{16}$ angles and 1$\frac{1}{8}$ in. round bars. The structural frame weighs 3.2 psf and the roof deck weighs 2.8 psf, so that as shown the total weight of the structure is 6 psf. Procedure for construction of this frame was first to lay out all of the members as shown in Fig. 13. All of the ribs were welded to the centermost ribs, and the center ribs were welded to the fascia. Also the mitred corners were welded. The remaining ribs were attached to the fascia with studs. The studs were placed through holes in the fascia and held in place with a nut so as to permit rotation of the ribs with respect to the fascia. The tie which runs from peak to peak was secured. The frame was then ready for erection. The frame was lifted in place with a winch truck and temporarily supported on pipe columns (Fig. 14). The ground tie was secured and jacked up tight until the frame was lifted off the columns. Guy cables were then fastened to deadmen.

STRUCTURAL MEMBERS

The type of structural shape used for the ribs may have to be taken into account in the design of these surfaces in regard to the amount of twist. Since the ribs will remain tangent to the surface, there will be twist induced into these members. When rolled shapes are used, the amount of resistance to twist is small, and the stresses resulting are of secondary importance. Tubular sections are somewhat more ideal than rolled shapes for these surfaces because of their lateral stiffness. However, they are highly resistant to twist, and therefore the stresses developed must be taken into account in the final design as part of the total combined stress.

MULTI-BAY SYSTEMS

Any structural system has a kind of optimum range, within which the materials used seem to best achieve their purpose and function. Since this optimum does exist, and since an architectural problem in most cases demands a space development which is beyond this natural unit, much of the energy of an architect is spent in considering how units of one sort or another may be put together. One of the most important aspects of the steel rib frame is the ease with which it may be combined to form a multi-bayed system (Fig. 16). Four units of the hyperbolic paraboloid may be combined in such a manner that the downpoints of the individual surfaces are in the middle of the span. Designwise this will offer no disturbance to the stress distribution and they would be designed as individual units. It is apparent that these

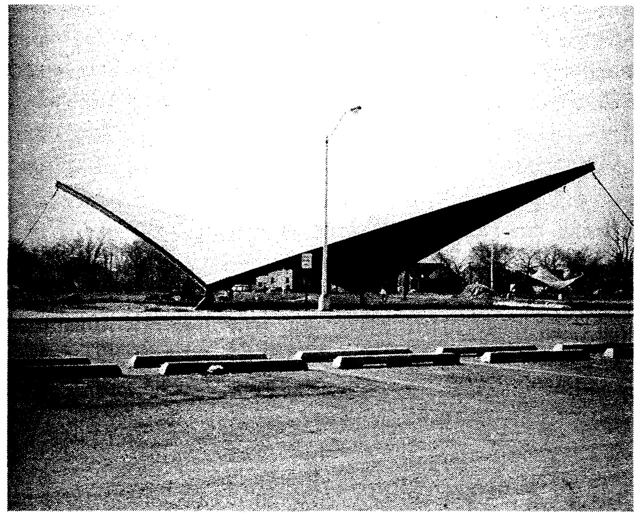

Figure 12

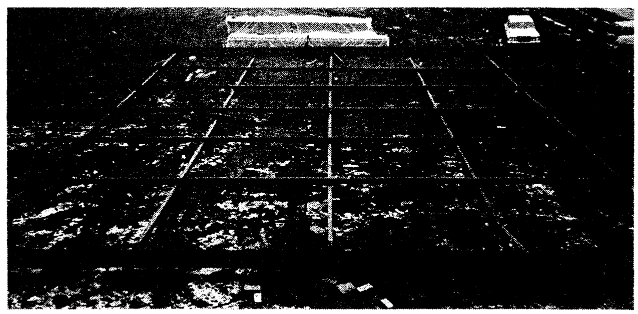

Fig. 13. The frame laid out on the ground

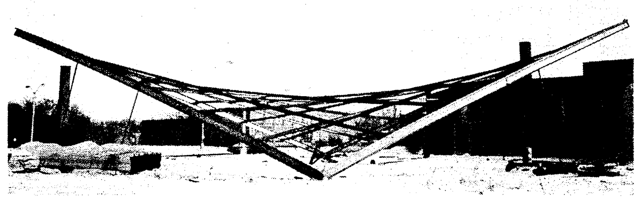

Fig. 14. The frame after being lifted in place and temporarily supported on pipe columns

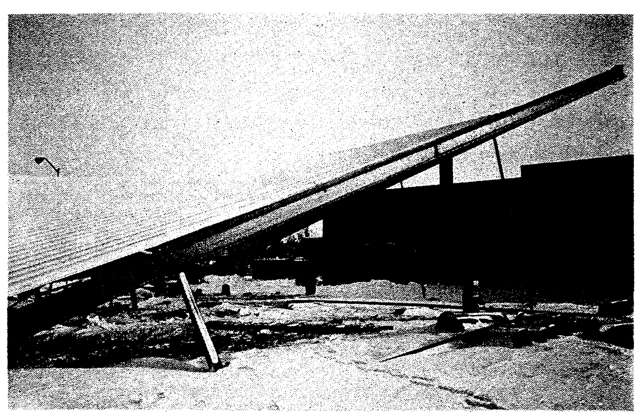

Fig. 15. Frame partially sheathed. Although all of the decking was placed while the frame was in position, it was realized that about half of it could have been secured while the frame was on the ground.

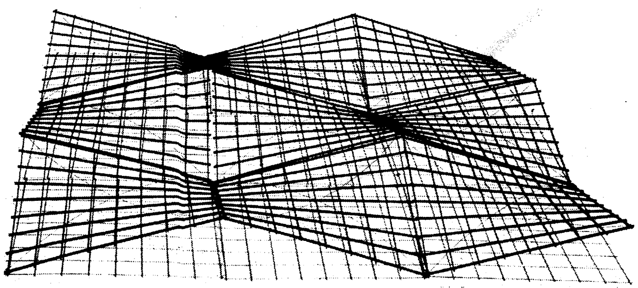

Figure 16

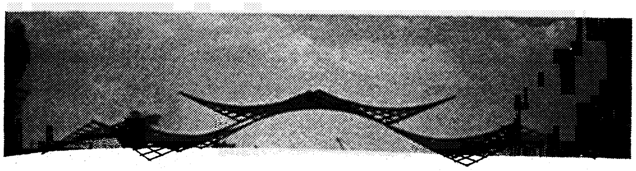

Figure 17

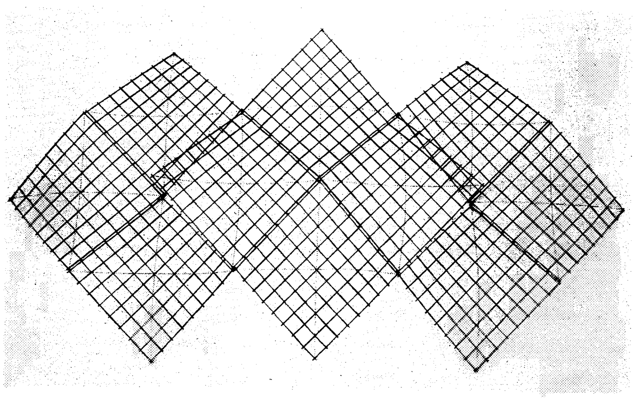

Figure 18

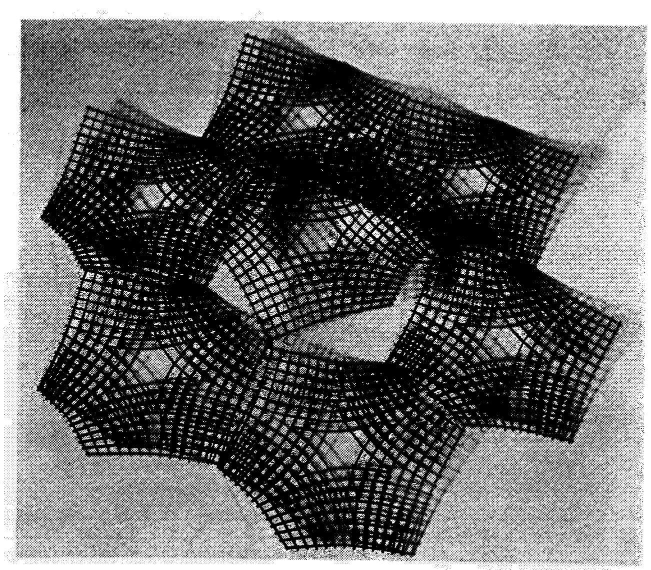

Figure 19

units may be placed together indefinitely in any direction with no change of principle. It should be noted, however, that the bay thus formed is four times greater in area than the individual unit.

Because of the fact that the steel frames are light in weight they will admit of raised units supported by a series of units or a ring of units below. In the case of the hyperbolic paraboloid combination shown in Figs. 17 and 18, the thrust of the upper units is translated directly to the ground through the fascias of the lower units. If there is no column at the juncture of the upper and lower units, there will be peak loads induced into the lower surfaces. These peak loads will have the effect of possibly producing an upward deflection in the lower frames. In the combination of monkey saddles (Fig. 19), no columns will be required for the center of the elevated unit, since the vertical reactions of this upper unit as well as the thrust will be translated through the tripodal action of the fascias below.

As a final example of the versatility of the ribbed frame, Figs. 20 and 21 show the application of a combination of units to a concert hall. Again, because of the relative lightness of the roof structure, these units may be suspended. The complex is prestressed by vertical cables attached to the down points of the lower rings of units. This project was designed to span a column-free space of about 500 ft. The thrust of the individual frames is taken up by cables from down point to down point. Many such direct combinations of units are possible, using either the monkey saddle or the hyperbolic paraboloid as the unit type.

METAL DECK COVERING

The use of a metal deck is probably the most direct way of providing cover over the frame. This will permit a rather wide rib spacing. If the deck runs parallel to one of the edges, thus spanning from rib to rib, the set of ribs receiving the decking will be under combined stresses, while the other set will be only under direct compressive load.

If the ribs are closely spaced, other materials may be used to span the frame, and ideally, any poured material should be made to span in both directions. Although this would place both sets of ribs under combined loading, it would have the advantage of reducing the bending stresses in each set of ribs. There could be some question, however, as to whether the surface should not now be designed on the basis of membrane theory rather than frame analysis.

To make the most efficient use of the metal decking, it should be combined structurally with the frame, which could be done in one of three ways. First, the deck could replace one of the sets of ribs, with the result that the deck would be placed in combined stress. Second, the deck could be laid perpendicular to the ties, that is, to arch from down point to down point. This would result in a division of load between the ties and the deck which would substantially reduce the stresses in the tie members and in the ribs. The third method is to use a double lamination of decking, one layer laid perpendicular to the other. This may be either parallel to or at an angle of forty-five degrees to the fascias.

Immediately the question arises as to whether the skin or decking material may not be used structurally in order to improve the efficiency of the system even more. In order to give an affirmative answer, it becomes necessary to solve the very important problem of making suitable structural connections of the deck to the frame and to itself to form a continuous sheet. There are presently two standard methods of fastening sheet metal decking to a frame: one is by means of the sheet metal screws and the other is with welding washers. In order to check the merits of these connections, I have conducted a series of tests. Welding washers were used in various combinations which fastened 18 and 16 ga. sheets to bars and the assembly subjected to tension. The results demonstrated that the welding washer for a structural connection was unreliable. Probably the reason is that there is a high percentage of galvanizing and sulfur mixed with the base metal and the rod. Good penetration into the bar occasionally became a problem. These results are corroborated by a series of tests which were sponsored by the Inland Steel Company, where the Milcor deck in combination with concrete was used to study horizontal diaphragm action, in which welding washers were used to fasten the deck to the structural frame. Although the tests were otherwise successful, occasionally such a connector would release itself. The use of sheet metal screws is reliable both for single and double bearing. However, since the bearing values of sheet metal are low, some auxiliary edge strip would have to be placed on the end of a sheet to reduce the quantity of screws.

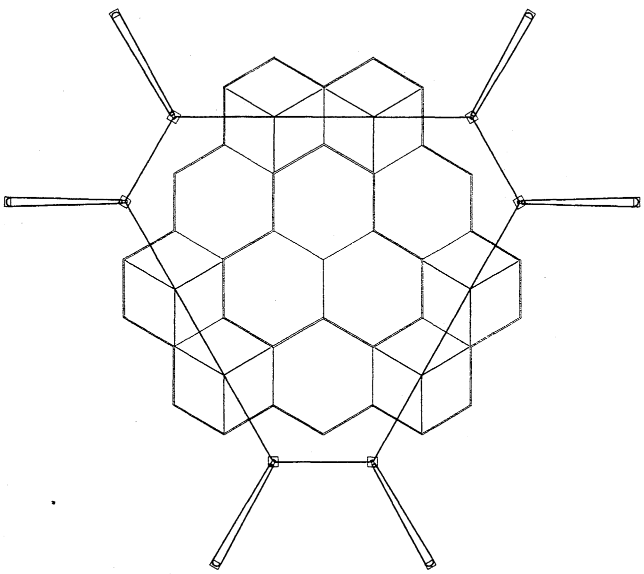

Figure 20

Another type of welded connection has been tested using a steel flat 1-in. wide and ⅛-in. thick, where the welder's pass is made along the edge of the flat, puddling the flat, the sheet metal and the structural frame (see Fig. 22). Ten tests were run using flats 2-in. in length, five with the flat laid on the sheet and five where the flat was secured to the frame with a single sheet metal screw. In this latter case the purpose of the screw was not structural but rather as a convenience for holding the flat in place and making sure that the flat is held close to the bar frame. In all cases failure occurred in the sheet, with the sheet tearing out near the weld.

A second set of tests were run using this type of con-

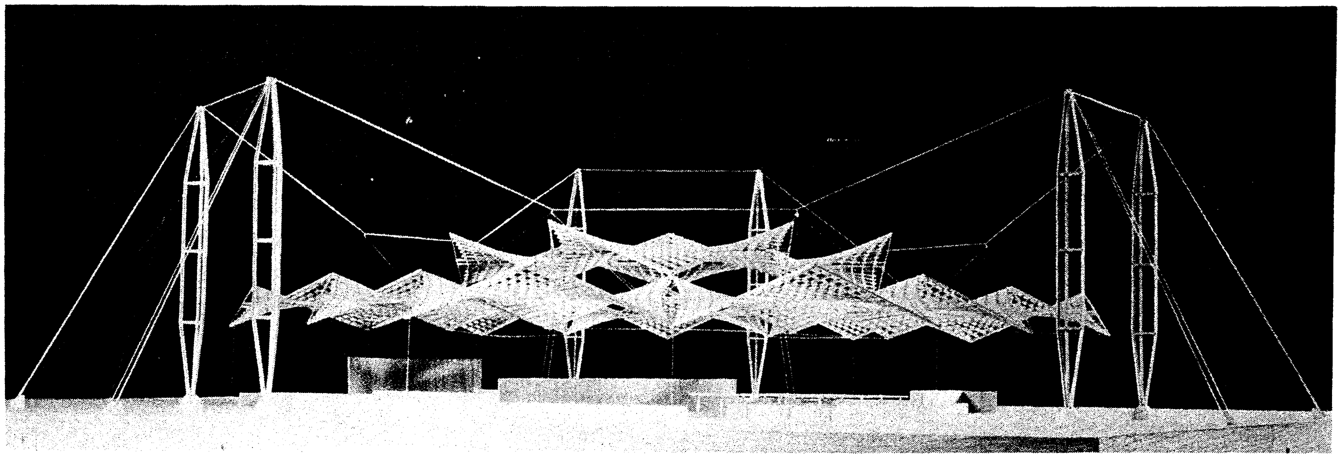

Figure 21

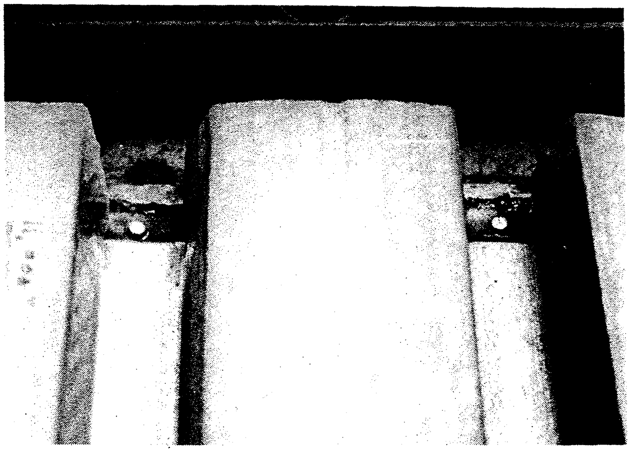

Fig. 22. View showing the buckled panel at one of the connections. The connection is a flat, $1\frac{3}{4}$ x 1 x $\frac{1}{8}$ in., welded to the frame.

nector with short Hi-Bond 3-in. metal deck made from 20 ga. galvanized sheet. Five panels were loaded in compression. The flats measured $1\frac{3}{4} \times 1 \times \frac{1}{8}$ in. in order to fit the valley of the deck. Four of the sheets measured 68 in. and one sheet 54 in. The panels were loaded in compression until there was no significant increase in the gage reading as the deck continued to cripple. In all cases this was about 11,000 lbs plus or minus a few hundred pounds. The two center valleys of the deck were fastened with the welded flat, and the two outside valleys were restrained from lateral movement by a single sheet metal screw in each valley. All buckling was local around the welded connectors.

Obviously yielding occurred some distance beyond the width of the flat. What that distance is would be difficult to ascertain at this time. Taking the yield point of the metal as 33,000 psi, and considering only the width of the connector, yielding would begin with a load of 2,170 lbs. Visual crippling did not occur until approximately 3,600 lbs per connector. If a value of 20,000 psi could be assumed for the sheet for the width of the connector, this would mean that 1,220 lbs could be sustained for each welded flat for the 20 ga. metal deck, 1,730 lbs for 18 ga. and 2,280 lbs for 16 ga. With the assumption of 20 psi it would be expected that these latter values would be conservative, since the relative stiffness of the panel will be greater for the heavier sheet since the dimensions of the valleys remain the same. Hi-Bond deck was originally developed to be used with concrete, but the fluting of the web also permits the possibility of making continuous sheets by the interlocking of the flutes, with the aid of the sheet metal screw to hold the flutes tight to each other.

What is implied in the above results is that one may develop a connection which will permit the use of the metal deck as a compression member. A gage thickness may be used sufficient to constrain the ends of a panel, and a lighter gage of sufficient strength to handle the axial load to prevent buckling may be used for most of the length.

Since the deck must necessarily be attached on the bottom flange of thc panel, any axial load applied will be an eccentric load. The effect will be such as to produce a negative deflection. The superimposed load will produce a positive deflection. The combined efforts of the two loads will be such as to (in effect) reduce the amount of eccentricity within the panel. However, this eccentricity will be entirely eliminated, and therefore must be taken into account in the design of the panel, but will not affect the design of the connector. The resulting moment condition will be local, and will decrease fairly rapidly as the distance from the connector is increased. For this reason, as well as making a connection, it would be necessary to increase the gage of the deck near the support.

Where local buckling of the deck at the connection of the fascia becomes a problem, the gage of the deck may be changed for a short distance, or the deck could have an additional plate secured to the bottom flange in the shop.

CONCLUSION

The presentation of the foregoing material was not intended to be thorough or complete. There are other ribbed frames and other concepts of design. However, if the ideas discussed are truly important, the topic of Curvilinear Grid Frames will prove to be a rich field for further investigation and refinement.