Load Factor Design of Steel Buildings

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SINCE THE EARLY 1950's there have been an increasing number of articles and books in the structural engineering literature on the application of probability and statistics in the design of structures. Since 1965 the rate of development on the subject has increased greatly. A sizeable group of experts and researchers are now working on various facets of the problem. One may be somewhat confused by all these activities, and not quite see where all this is leading. This is not at all surprising, since some of the research is only now reaching fruition and there is much activity which has not yet shaken down with respect to method or terminology. Enough work has been done, however, to justify a look at what is happening and to relate this to the present state of steel design as exemplified by the 1969 AISC Specification. This paper will attempt to give a brief review of the activities, explain some basic ideas, and examine the possibilities of these principles being applied to steel building design. The paper is not written for the experts in probability and statistics; rather, it is addressed to the structural engineer who wants to become familiar with developments on the subject.

WHAT IS STRUCTURAL DESIGN?

Most of us would agree that an acceptable definition of the task of the structural design engineer is to proportion structures which are safe from either failure or malfunction when used as intended during their design lifetime, and which are economical to build and to maintain. No one, of course, would argue that all structures are absolutely free from any chance of failure or that costs of all structures are absolute optimums. What we really mean is that the chance of failure is very small, and that the designer does his best to make his structure economical. Every designer is aware of the fact that structural design involves many uncertainties which cannot be known at the design stage, and thus he implicitly invokes probabilistic reasoning during the design process. Margins or factors of safety are introduced to account for the uncertainties. These factors of safety provide a high level of assurance that the structure should perform satisfactorily. They have evolved from past successful and unsuccessful performance, and from the results of research on structural behavior. To use a term of the literature of probabilistic design, the factors of safety appear to be "deterministic", and they seem to imply absolute safety. In reality, of course, their values are but the structural engineering profession's judgment of a factor which assures safety most of the time. The engineer knows, however, that no matter how large the safety factor is, there is always a chance, albeit almost negligible, that something unforeseen may happen and failure may occur.

Probabilistic design is directed toward formalizing and utilizing uncertainty in an explicit and consistent manner and thus quantifying the process which has gone on implicitly and intuitively for many years. Research has now shown that design based on probabilistic principles is both possible and practicable; structural design standards have been adopted which are based on the direct application of probability theory (for example, in Russia, Czechoslovakia, and Mexico). These developments have been slow in coming, but the situation is rapidly changing as the research on probabilistic concepts for design is expanded, as more meaningful statistical data are collected and evaluated, and as experience is gained in applying probabilistic concepts to special structures (for example, offshore oil drilling platforms,[16] aircraft and automobile parts). Seismic zoning is another area of recent probability-based development.[12,17]

HISTORICAL DEVELOPMENTS

In the U. S. A. and Great Britain the impetus for a new look at the safety and reliability of civil engineering structures stems from two professional committees, appointed by the American Society of Civil Engineers and the Institution of Civil Engineers, respectively, to make recommendations on how design for safety could

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be treated more rationally than in the past. These committees were appointed shortly after World War II. The membership was made up of practicing engineers and researchers, and each committee had at least one person with a background of aircraft design. Developments during World War II in the aircraft industry led to the increased realization that many of the design parameters, such as material properties and loads,were random and should not be considered as being deterministic. Structural designers had known this intuitively; the new element was the realization that some of these random parameters could be treated by the mathematics of statistics and probability.

The British committee issued its report in May 1955[19] and the American committee released its findings in 1956.[13,14] The paper by Freudenthal[13] is considered by many as the most basic and definitive paper on probabilistic design.

The British report was simple and practical, and it can be considered to be the precursor, in the Western Countries at least, of the various schemes of design which are now called Load Factor Design or Limit States Design.

The two committee reports set the stage for further developments. These seem to have pursued two directions: (1) to define the theoretical bases for the design process, based on probability theory, and (2) to arrive at practical design rules which would not greatly change the everyday design process, but would nevertheless be based on explicit probabilistic derivations.

Much of this work has culminated in several conferences and special courses, a session of the 1968 IABSE Congress in New York, a symposium on structural safety in London in September 1969 (sponsored by the ICE and the IABSE), a series of lectures at the University of Waterloo in Canada, an ASCE-EMD conference and lecture series in November 1969 at Purdue, etc. The number of papers at ASCE conferences and in the ACI and ASCE Journals, and elsewhere, has increased in recent years.*

CONCEPTS OF STRUCTURAL SAFETY

Following is a very simplified but useful view of the basic ideas of structural safety concepts as provided in Pugsley's book.[18] It is assumed that structural safety can be characterized by two independent properties: (1) the "load effect", that is, the forces (moments, shear forces, axial forces) acting in the structure or its components as a result of the applied loads, and (2) the "strength" or "resistance" of the structure. In the literature, the load effect is usually denoted by S (from the French *sollicitation*) and the strength by R (for *resistance*). Because S is too easily confused with "strength", the symbol Q is used to represent the "load effect" and R is retained for "resistance" or "strength".

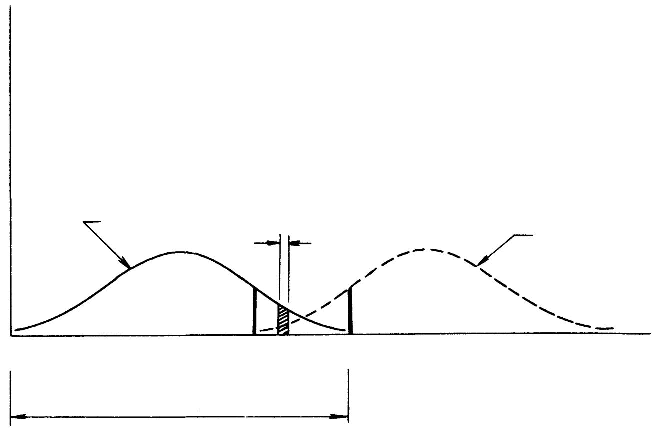

Fig. 1. Probabilistic model of resistance and load effect

Both R and Q are "random" variables involving, in the case of the load effect Q, the randomness of the forces of nature and the forces imposed by man, and in the case of the strength R, the randomness of the mechanical properties of the material, for example. Other effects such as errors due to idealization of the structural theory, construction imperfections, etc., also contribute to the uncertain natures of R and Q. While all of these sources of uncertainty are not random in the sense of, say, repeated coin flippings, modern probability theory has been developed to treat all kinds of uncertainty in a uniform manner.

Since Q and R are random variables, they will have a probability distribution as illustrated in Fig. 1. In principle, these distributions may be obtained from results of tests and measurements of sufficient numbers of samples. The structure is safe when $Q < R$, that is, the loading is smaller than the resistance. Because the two curves may overlap, there is a possibility that $Q > R$, and the structure or structural element fails. This chance of failure must be acceptably small.

The traditional concept of the factors of safety concedes from the outset that probability distributions of Q and R exist, but that they are not known. Thus a large value of the loading $Q = Q_1$ and a low value of the resistance $R = R_1$ is specified, and a factor F.S. = R_1/Q_1, is obtained to implicitly account for the uncertainties. This is the basis, for example, for the allowable stress values of the AISC Specification being considerably less than the specified yield stress of the material.

The probabilistic approach assumes that the probability distributions of $R = f(R)$ and $Q = f(Q)$ are known, and thus by the following considerations the

* *See the list of references at the end of this paper. A study guide for engineers endeavoring to become more familiar with the subject is included in this paper.*

probability of $Q > R$ (that is, the probability of failure) can be determined:

(a) The probability of Q being with the limits of Q and $(Q + dQ)$ is the shaded area, $f(Q)dQ$, in Fig. 1.

(b) The probability of R being less than Q is the area under the $f(R)$ curve to the left of Q, or $\int_0^Q f(R)dR$.

(c) The probability of both events (a) and (b) occurring simultaneously is the product

$$\left[\int_0^Q f(R)dR\right]f(Q)dQ$$

(d) All such probabilities of R being less than Q represent the probability of failure P_F, i.e.,

$$P_F = \int_0^\infty f(Q)\left[\int_0^Q f(R)dR\right]dQ \qquad (1)$$

Conversely, the probability of survival is

$$P_S = 1 - P_F \qquad (2)$$

The concept of Eq. (1) can be also illustrated by Fig. 2, where the difference $R - Q$ is plotted. When $R - Q < 0$, the structure "fails" and the area below the $R - Q$ curve to the left of the zero axis (shown shaded) represents the probability of failure.

The traditional factor of safety,

$$\text{F.S.} = \frac{R_1}{Q_1} \qquad (3)$$

and the concept of the probability of survival represent idealized extremes. The first implies that *nothing* is statistically quantifiable, and the second implies that *everything* is statistically quantifiable. The truth lies somewhere in between.. Some data can be quantified, for example, the variation of material properties, the live loads, the wind loads; increasingly, more data are being accumulated. Such data draws on past experience and must be tempered with judgment as to provisions for the future. Some things, such as fabrication and construction imperfections cannot be adequately quantified, and so judgment factors and quantifiable statistical data must be considered.

The factor of safety, by not utilizing the available statistical data and the probabilistic methodology, does not account for the better knowledge which has been accumulated through recent research. Clearly, a better job can be done by applying what knowledge we have in providing more consistent and perhaps even more economical designs. On the other hand, fully probabilistic design is as yet an unattainable ideal. The probability distributions of even the more common inputs, such as the variation of the actual yield stress of the material, the live loads, and the cross-sectional properties of structural sections, are not yet fully available. Such

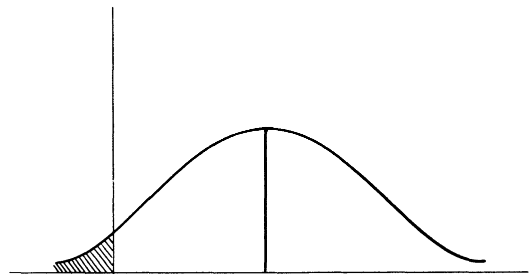

Fig. 2. Illustration of survival and failure

information can be collected, however, and attention is being directed toward collection and classification of such data. Other types of information, such as the uncertainties introduced by analytical idealizations, random construction imperfections, and stresses introduced by fabrication, are much more difficult to obtain and may never be completely known. Thus the implementation of fully probabilistic design would be impossible now, and one can only hope to achieve better degrees of conformance with the ideal, but never the ideal itself.

In the realization that present design methodology can be improved but that the ideal probabilistic method is not yet practical, much recent work has involved devising compromise schemes which retain the traditional format of design using load factors, instead of factors of safety. These factors themselves are now not determined from judgment and experience alone, but with statistical considerations, using the theory of probability as a formal vehicle for manipulating the information.[2,9,15] These schemes for adapting probability to present forms of design criteria are being referred to as *Load Factor Design*.

LOAD FACTOR DESIGN

Load Factor Design (LFD) is a method of proportioning members in a structure to resist at their limit of structural usefulness ("limit state") multiples of the forces and moments resulting from the applied loads.[4] Symbolically, the following relationship must be satisfied:

$$\sum_1^k \gamma_k Q_{nk} \leq \phi R_n \qquad (4)$$

The right hand side of Eq. (4) represents the strength which must be equal to or larger than the left hand side, which represents the load effect, for a satisfactory design. The strength R_n is a value given in a standard, such as the AISC Specification, and is often called the "nominal resistance". For example, the nominal plastic moment resistance of a beam, M_P, is F_yZ, where Z is the theoretical value of the plastic section modulus of the beam and F_y is the specified minimum yield stress of the steel. The load factor design equation for a fixed end beam which fails by the formation of a plastic mechanism is equal to

$$\gamma_D\left(\frac{w_D l^2}{16}\right) + \gamma_L\left(\frac{w_L l^2}{16}\right) \leq \phi F_y Z \qquad (5)$$

where γ_D and γ_L are the dead and live load factors, w_D and w_L are the distributed dead and live loads, respectively, and l is the span. This equation can be written as

$$\frac{l^2}{16}\left[\frac{\gamma_D w_D}{\phi} + \frac{\gamma_L w_L}{\phi}\right] \leq F_y Z \qquad (6)$$

and if $\gamma_D/\phi = \gamma_L/\phi = 1.7$, the load factor design procedure of Part 2 of the 1969 AISC Specification is obtained.

The actual values of Z and F_y are random variables. There are also additional uncertainties due to the assumptions relating to the theory used in the analysis and uncertainties related to the actual fabrication and erection of the beam. The factor ϕ, also called the "strength factor", takes account of the randomness of the actual strength. It has been demonstrated[2,9] by a process called "first order reliability approach" that ϕ can be derived from the knowledge of the mean strength and its coefficient of variation. The derivation is not given here because it is beyond the scope of this paper; however, it is apparent that a method and sufficient data are available to determine ϕ for structural steel components.

The left hand side of Eq. (4) represents the load, or more precisely, the "load effect", and it is equal to the sum of the products of the "load factor" γ and the specified, or "nominal load effect" Q_n, for different types of loads. If, for example, R_n is a nominal bending moment (resisting), then Q_n is also a bending moment (applied). If we have only dead and live load, then

$$\sum_{1}^{k} \gamma_k Q_{nk} = \gamma_D D_r + \gamma_L L_n \qquad (7)$$

and D_n is the moment due to the nominal dead load, L_n is the moment due to the nominal live load, and γ_D and γ_L are the corresponding load factors. The values of D_n and L_n are specified for a given structure and occupancy, and the γ factors account for the various uncertainties which underlie the loads and the idealizations which were made to translate the loads into bending moments, shears, and axial forces. The load factors γ and the strength factors ϕ can be derived from a knowledge of the mean load effects and their coefficients of variation.[2,9] Other load combinations, such as dead load, live load, wind load, etc., can be handled by incorporating additional terms to the summation of Eq. (4).

The LFD formulation represented by Eq. (4) is not novel to the designer. Load factors are used as the basis for plastic design in steel (Part 2 of the AISC Specification). The format of Eq. (4) has been used by the ACI *Building Code Requirements for Reinforced Concrete* (*ACI 318–71*) and it was recently adopted in the LFD procedure for steel highway bridges.[1] What is new, however, is the fact that the determination of the strength factors ϕ and the load factors γ is guided by probabilistic procedures. As a result the design criterion can be adjusted to give a more nearly uniform degree of reliability to all structural elements in a given class of structures.

The argument that ϕ and γ can be chosen so that structures have similar degrees of reliability is not the only basis for recommending the LFD procedure. The advantage of loading combinations having different γ values permits a greater flexibility than present practice. Furthermore, ϕ and γ values can be adjusted to account for the different consequences of failure or malfunction. In present practice we treat identically structures which would completely fail when one member failed and structures for which the loss of one member would involve only local damage. By an adjustment of the ϕ and γ factors the various "limit" states of the design can be handled in a consistent manner.

In fact, the load factor design format is ideally suited to implement the concept of "limit states design" in which load effects and resistances are determined for the specific limit of structural usefulness under consideration. There are essentially two types of limits of structural usefulness: (1) "collapse" or "ultimate" limit states, which result in the loss of the structure or some part of it, and (2) "unserviceability" limit states, which concern failure phenomena that impair the usefulness and function of the structure, but do not result in its loss. The formation of a plastic mechanism, frame instability, member instability, fracture, incremental collapse, etc., are some of the ultimate limit states of steel structures. Unserviceability limit states comprise: (1) excessive elastic deflections and drift under service live and wind loads, respectively, (2) annoying vibrations, (3) permanent deflections due to yielding under service loads, etc.

Both types of limiting structural usefulness represent "failure", and in good design the chance of reaching any limit state should be acceptably small. This requirement can be achieved by the proper combinations of γ, ϕ, the relevant loads and load levels, and the resistance corresponding to the type of limit state under investigation.

THE 1969 AISC SPECIFICATION AND LFD

Since its first edition in 1921, the AISC Specification has provided satisfactory structures. It is used in the U. S. for many types of steel structures other than buildings and has served as the model for design standards in other countries. Judging, then, by the lack of any serious and consistent complaints, we must assume that the probability of failure and malfunctioning inherent in the AISC Specification is at least at an acceptable level now.

While the present AISC Specification is apparently satisfactory, resulting in acceptable structures, it cannot readily utilize the facts regarding the probabilistic nature of loads and resistances in its current form. As the probability-based LFD approach offers a promise of design flexibility and economy through more consistent safety, research work was started at Washington University in St. Louis under the sponsorship of the American Iron and Steel Institute to develop load factor criteria for steel building design. This paper is the initial outcome of that effort.

NOMENCLATURE

D_n = Nominal dead load
l = Span
L_n = Nominal live load
P_F = Probability of failure
P_S = Probability of survival, or reliability
Q = Load effect, a random variable
Q_n = Nominal load effect
Q_1 = Load effect used in present design practice
R = Strength or resistance, a random variable
R_n = Nominal load effect
R_1 = Strength used in present design practice
w_D = Dead load
w_L = Live load
γ = Load factor
γ_D = Dead load factor
γ_L = Live load factor
ϕ = Strength factor

GLOSSARY

First order probabilistic approach—a probabilistic design procedure using only the mean and the coefficient of variation of a random variable.

Limit states design—a design method aimed at providing adequate safety against the structure being rendered unfit for use when it reaches a limiting state of structural usefulness.

Load effect—bending moment, axial force or shear force induced at a cross section by an external load. The load effect is a random variable owing to uncertainty in the load and in the estimation of the induced cross-sectional forces.

Load factor design—a design procedure where the sum of the products of the nominal load effects and their respective load factors γ is equal to or less than the nominal strength times a strength factor ϕ. The procedure accounts for different load combinations and limits of structural usefulness.

Load factor—a factor γ by which the nominal load effect is multiplied to account for overloads and for the uncertainties of load effect determination.

Nominal load effect—the load effect used in LFD and calculated by the designer on the basis of specified loads.

Nominal strength—the strength used in LFD and calculated using theoretical section properties, specified minimum material properties, and specified design formulas.

Probabilistic analysis—a method of structural analysis where statistical information and the mathematics of probability theory are used to determine the probability of survival (reliability) or failure of a structure or structural component.

Probabilistic design—a method of structural design where the mathematics of probability theory is used to process information about the uncertainty in the various components of the problem yielding structural designs which are consistent with this information.

Random variable—a quantity having values characteristic of a probability distribution; a variable where value cannot be predicted with certainty.

Strength (or resistance)—limiting capacity of a structure or structural element. It is a random variable.

Strength factor—a factor ϕ by which the nominal strength or resistance is multiplied to account for the uncertainties of strength determination.

STUDY GUIDE

The following guide is suggested for the engineer wanting to become more familiar with probabilistic methods applied to design.

The reader could start his study with Asplund's[3] and Benjamin's[5] papers and with the small and delightful book by Pugsley[18] which would give him a considerable background and philosophy. He could then read the committee reports on structural safety of the committees of the ASCE[13,14] and the Institution of Civil Engineers.[19]

More recent developments can then be tackled by reading the papers by Cornell,[9,10] Turkstra,[20] Ang[2] and Lind.[15] These are only representative papers by these authors; much more published literature by these and other authors exists. It is expected that the ASCE Committee on Structural Safety will soon issue a comprehensive bibliography on this subject.

Practical applications for the treatment of wind loads[11] and for the design of offshore structures[16] can then be considered. The mathematical background for a thorough study of the use of probability and statistics in Civil Engineering is given in the book by Benjamin and Cornell.[6] European and Russian developments are available through the books by Borges and Castanheta[8] and by Bolotin.[7] The former deals with many practical problems including probabilistic design and loading data. The latter, while discussing briefly code developments in Russia, emphasizes more theoretical topics.

ACKNOWLEDGMENTS

The author expresses thanks for the contributions from and the discussions by members of the Advisory Task Force of AISI Project 163 "Load Factor Design of Steel Buildings". The members of this group are Messrs. I. M. Viest (Chairman), L. S. Beedle, C. A. Cornell, E. H. Gaylord, J. A. Gilligan, W. C. Hansell, I. M. Hooper, W. A. Milek, Jr., C. W. Pinkham, and G. Winter. The many helpful suggestions made by Dr. M. K. Ravindra are also acknowledged with thanks.

REFERENCES

1. AASHO Interim Specifications, 1971 "Interim 7, Load Factor Design" *American Association of State Highway Officials, Washington, D. C., 1971.*
2. *Ang, A. H.-S. and Ellingwood, B. R.* Critical Analysis of Reliability Principles Relative to Design *Proceedings of the Conference on Applications of Statistics and Probability to Soil and Structural Engineering, Hong Kong, Sept. 1971.*
3. *Asplund, S. O.* The Risk of Failure *Structural Engineer, London, Vol. 36, No. 8, Aug. 1958.*
4. *Beedle, L. S., L. W. Lu, and L. C. Lim* Recent Developments in Plastic Design Practice *Journal of the Structural Division, ASCE, Vol. 95, No. ST9, Sept. 1969.*
5. *Benjamin, J. R.* Probabilistic Structural Analysis and Design *Journal of the Structural Division, ASCE, Vol. 94, ST7, July 1968.*
6. *Benjamin, J. R. and Cornell, C. A.* Probability, Statistics and Decision for Civil Engineers *McGraw-Hill, New York, 1970.*
7. *Bolotin, V. V.* Statistical Methods in Structural Mechanics *Holden-Day, Inc., San Francisco, 1969.*
8. *Borges, F. J. and Castanheta, M.* Structural Safety *National Laboratory of Civil Engineering, Lisbon, Portugal, 1971.*
9. *Cornell, C. A.* A Reliability Based Code Format *ACI Journal, Dec. 1969.*
10. *Cornell, C. A.* Bounds on the Reliability of Structural Systems *Journal of the Structural Division, ASCE, Vol. 93, ST1, Feb. 1967.*
11. *Davenport, G. A.* Gust Loading Factors *Journal of the Structural Division, ASCE, Vol. 93, No. ST3, June 1967.*
12. *Ferahian, R. H.* Commentary No. 3, Earthquake Loads *Canadian Structural Design Manual, 1970.*
13. *Freudenthal, A. M.* Safety and the Probability of Structural Failure *Transactions, ASCE, Vol. 121, 1956.*
14. *Julian, O. G.* Synopsis of First Progress Report of the Committee on Factors of Safety *Journal of the Structural Division, ASCE, Vol. 83, ST4, July 1957.*
15. *Lind, N. C.* Consistent Partial Safety Factors *Journal of the Structural Division, ASCE, Vol. 97, No. ST6, June 1971.*
16. *Marshall, P. W.* Risk Evaluation of Offshore Structures *Journal of the Structural Division, ASCE, Vol. 95, ST12, Dec. 1969.*
17. *Milne, W. G. and Davenport, A. G.* Distribution of Earthquake Risk in Canada *Bulletin of the Seismological Society of America, Vol. S9, No. 2, April 1969.*
18. *Pugsley, A.* The Safety of Structures *Edw. Arnold Ltd., London, 1966.*
19. *Special Committee of the ICE* Report on Structural Safety Structural Engineer, London, *Vol. 33, No. 5, May 1955.*
20. *Turkstra, C. J.* Choice of Failure Probabilities *Journal of the Structural Division, ASCE, Vol. 93, ST6, Dec. 1967.*