

Simplified High-Rise Drift Analysis and Optimized Adjustment

WALTER H. FLEISCHER

THIS PAPER introduces a method by which drift and material-optimized drift adjustments for high-rise steel frames can be closely estimated in only a few minutes, using just simple desk calculator equipment.

This simplified method had originally been developed to provide quick and reasonably close estimates of drift amplitudes for preliminary design only; yet, comparisons with the results of "exact" methods show a remarkable degree of agreement.

The method evolved from the recognition of certain predictable patterns found in the study of numerous high-rise steel building frames. Evaluation of a wide variety of frame data showed that, in high-rise steel bents, under the type of wind loading required by major building codes, the distribution of material properties over the height of the bent (such as column stiffnesses, girder stiffnesses, cross-sectional areas of members, etc.) could be expressed by "homogenized" mathematical expressions with excellent accuracy. Slight idealizations in the wind loading pattern and the frame grid system did not create any meaningful deviations from this accuracy.

With the working equations presented in this paper, stress design is required at only three representative levels of the designer's choice (i.e., roof, intermediate, first floor) in order to calculate realistic values for total bent drift, material weights of the bent at any level, and material-optimized drift adjustments. Equations are developed for both planar rigid frames and planar braced frames.

To illustrate the simplicity of the method, an example of a typical drift analysis in a planar rigid high-rise steel bent, including material optimized drift adjustments and material take-off, is demonstrated, including a description of the required step-by-step procedures for both rigid and braced frames.

Following the illustrative example, the development of the method and working formulas is explained.

Walter H. Fleischer is Structural Consultant, Sales Engineering Division, Bethlehem Steel Corp., Bethlehem, Pa.

NOMENCLATURE

a	=	Wind load distribution exponent
b	=	Material property distribution factor
c	=	Material property distribution exponent
d	=	Story drift contribution
e	=	Distance from center of individual column to common center of gravity of all columns in a bent
i	=	Level from top of frame (at roof, $i = 1$; at first level above grade, $i = n$; see Fig. 2)
k	=	Column line or bay identification (see Fig. 2)
l	=	Length of frame component
m	=	Column line or bay identification (see Fig. 2)
n	=	Number of levels in frame
q	=	Material weight
\bar{q}	=	Material weight after drift adjustment
r	=	Radius of gyration
\bar{r}	=	Radius of gyration after drift adjustment
w	=	Wind loading (linear)
A	=	Cross-sectional area
B	=	Bent (subscript)
C	=	Column (subscript)
C'	=	Column-moment (subscript)
C''	=	Column-axial load (subscript)
D	=	Total drift; also Bracing (subscript)
\bar{D}	=	Total drift after drift adjustment
E	=	Modulus of elasticity
G	=	Girder (subscript)
H	=	Total frame height
I	=	Moment of inertia
K	=	Stiffness
L	=	Total depth of frame
M	=	Moment
P	=	Any cross-sectional material property
R	=	Axial force
T	=	Total of stiffnesses of a given material property at a given level; also, Total bent (subscript)
V	=	Shear force
αL	=	Girder span
η	=	Drift adjustment factor
ϕ	=	Drift correction factor

ILLUSTRATIVE EXAMPLE

Given:

The frame under consideration is 360 ft high, with 30 supported levels. The typical floor plan is shown in Fig. 1. The structure is to be analyzed as a rigid perimeter system, i.e., all lateral forces are to be resisted by spandrel girders connected rigidly to the perimeter columns.

The applicable building code specifies unit wind pressures of 30 psf at 30 ft above grade and 50 psf at 360 ft above grade. The total drift of the frame under these wind conditions is to be held to 1/300 of the height of the structure.

The frame is to be checked for total drift and, if necessary, adjusted in the most economical way to meet the required drift limitation.

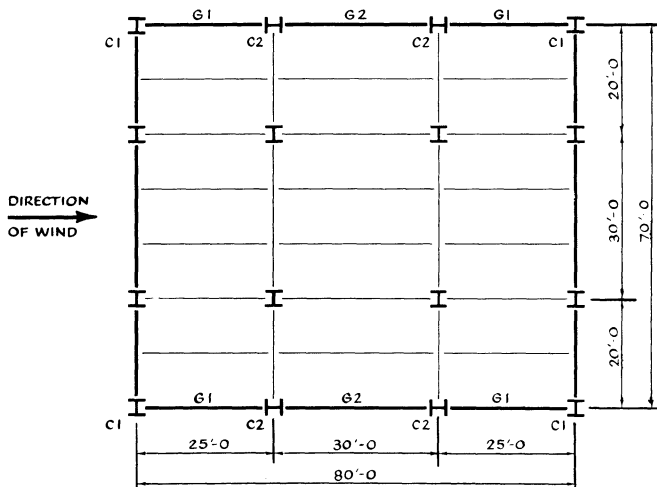


Figure 1

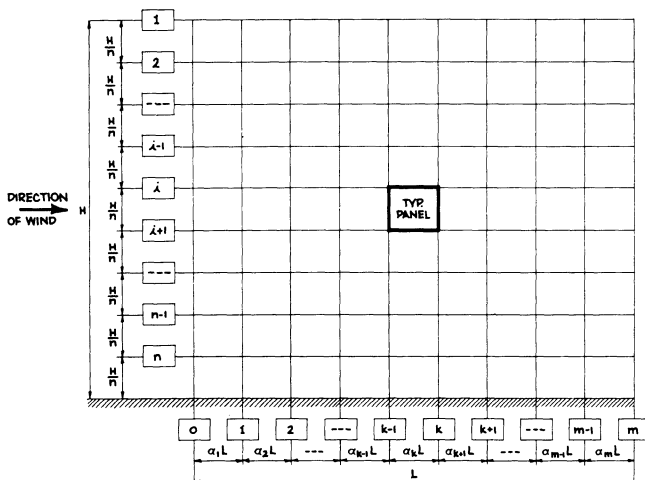


Figure 2

Procedure and Solution:

All working formulas required in the solution are shown in Tables 1 and 2. See Fig. 2 for frame grid identification nomenclature.

The rigid frame parallel to the direction of the wind will be analyzed, i.e., columns **C1** and **C2**, girders **G1** and **G2**. The following design parameters are known from the given information:

$$\text{Total frame height } (H) = 360 \text{ ft}$$

$$\text{Number of levels in frame } (n) = 30$$

$$\begin{aligned} \text{Total allowable frame drift } (\bar{D}_T) &= \frac{360 \times 12}{300} \\ &= 14.4 \text{ in.} \end{aligned}$$

$$\begin{aligned} \text{Wind force at top of frame } (w_1) &= \frac{70}{2} \times 0.05 \\ &= 1.75 \text{ kips/lin ft} \end{aligned}$$

$$\begin{aligned} \text{Wind force at El. 30 ft. } (w_{30\text{-ft}}) &= \frac{70}{2} \times 0.03 \\ &= 1.05 \text{ kips/lin ft} \end{aligned}$$

A. Check for Total Drift (Steps 1–9):

Step 1—Stress design levels 1, 15, and 30:

For this example, assume that a preliminary stress design has resulted in the steel sections shown in Fig. 3.

Step 2—Establish the wind load distribution exponent a from Eq. (3) in Table 1:

$$\begin{aligned} a &= \frac{\log (w_1/w_{30\text{-ft}})}{\log (H/30)} \\ &= \frac{\log (1.75/1.05)}{\log (360/30)} = 0.206 \end{aligned}$$

Use: $a = 0.200$

Step 3—Establish all material property values (P) at levels 1, 15 and 30 (see Table 2, lines 1–3; line 4 is not required except for braced frames):

a. Sum of girder stiffnesses:

$$T_{G(i)} = \sum_{k=1}^{k=m} \frac{I_{G(k,i)}}{\alpha_k L}$$

$$T_{G(1)} = \frac{300}{25} + \frac{513}{30} + \frac{300}{25} = 41.1 \text{ in.}^4/\text{ft}$$

$$T_{G(15)} = \frac{3270}{25} + \frac{4000}{30} + \frac{3270}{25} = 395 \text{ in.}^4/\text{ft}$$

$$T_{G(30)} = \frac{6710}{25} + \frac{9030}{30} + \frac{6710}{25} = 838 \text{ in.}^4/\text{ft}$$

b. Sum of column stiffnesses:

$$T_{C(i)} = \sum_{k=0}^{k=m} \frac{n I_{C(k,i)}}{H}$$

$$T_{C(1)} = \frac{30}{360} (133 + 542 + 542 + 133) = 113 \text{ in.}^4/\text{ft}$$

$$T_{C(15)} = \frac{30}{360} (1990 + 4400 + 4400 + 1990)$$

$$= 1070 \text{ in.}^4/\text{ft}$$

$$T_{C(30)} = \frac{30}{360} (4720 + 9450 + 9450 + 4720)$$

$$= 2360 \text{ in.}^4/\text{ft}$$

c. Moment of inertia of bent below each level (see Fig. 15 in the text section that follows):

$$I_{B(i)} = \sum_{k=0}^{k=m} A_{C(k,i)} \epsilon_k^2$$

$$I_{B(1)} = \frac{2}{144} [(21.8 \times 40^2) + (15.6 \times 15^2)]$$

$$= 533 \text{ ft}^4$$

$$I_{B(15)} = \frac{2}{144} [(109 \times 40^2) + (92.3 \times 15^2)]$$

$$= 2710 \text{ ft}^4$$

$$I_{B(30)} = \frac{2}{144} [(215 \times 40^2) + (162 \times 15^2)]$$

$$= 5280 \text{ ft}^4$$

Step 4—Establish the values for material distribution factors (b) for all applicable material properties in Table 2 from Eq. (6) in Table 1 ($b = nP_1/P_n$)

a. For girder moments:

$$b_g = \frac{nT_{G(1)}}{T_{G(30)}} = \frac{30 \times 41.1}{838} = 1.47$$

b. For column moments:

$$b_{c'} = \frac{nT_{C(1)}}{T_{C(30)}} = \frac{30 \times 113}{2360} = 1.44$$

c. For column axial forces:

$$b_{c''} = \frac{nI_{B(1)}}{I_{B(30)}} = \frac{30 \times 533}{5280} = 3.03$$

Step 5—Establish the values for the material property distribution exponent c for all applicable material properties in Table 2 from Eq. (8) in Table 1:

a. For girder moments:

$$c_g = \frac{\log\left(\frac{T_{G(15)} - T_{G(1)}}{T_{G(30)} - T_{G(1)}}\right)}{\log\left(\frac{15 - 1}{30 - 1}\right)} = \frac{\log\left(\frac{395 - 41.1}{838 - 41.1}\right)}{\log\left(\frac{15 - 1}{30 - 1}\right)} = 1.11$$

b. For column moments:

$$c_{c'} = \frac{\log\left(\frac{T_{C(15)} - T_{C(1)}}{T_{C(30)} - T_{C(1)}}\right)}{\log\left(\frac{15 - 1}{30 - 1}\right)} = \frac{\log\left(\frac{1070 - 113}{2360 - 113}\right)}{\log\left(\frac{15 - 1}{30 - 1}\right)} = 1.17$$

Table 1. Summary of Working Formulas

Equation	Description	Formulation ^{a,b}
(3)	Wind load distribution exponent	$a = \frac{\log(w_1/w_{30-ft})}{\log(H/30)}$
(6)	Material property distribution factor	$b = nP_1/P_n$
(8)	Material property distribution exponent	$c = \frac{\log\left(\frac{P_i - P_1}{P_n - P_1}\right)}{\log\left(\frac{i - 1}{n - 1}\right)}$
(14a)	Girder moment drift	$D_g = \frac{w_1 H^3}{201nT_{G(n)}} (\phi_g)$
(17a)	Column moment drift	$D_{c'} = \frac{w_1 H^3}{201nT_{C(n)}} (\phi_{c'})$
(20a)	Column chord drift	$D_{c''} = \frac{w_1 (H/38)^4}{I_{B(n)}} (\phi_{c''})$
(36a)	Bracing drift (X-bracing)	$D_D = \frac{w_1 n H}{2417T_{D(n)}} (\phi_D)$
(39a)	Bracing drift (K-bracing)	$D_D = \frac{2w_1 n H}{2417T_{D(n)}} (\phi_D)$
(32)	Column drift adjustment factor (rigid)	$\eta_c = \frac{D_c + \frac{r_g}{\bar{r}_g} \sqrt{\frac{q_{G(n)}}{q_{C(n)}}} D_g D_c}{\bar{D}_T}$
(33)	Girder drift adjustment factor (rigid)	$\eta_g = \frac{D_g}{\bar{D}_T - \left(\frac{D_c}{\eta_c}\right)}$
(46)	Column drift adjustment factor (braced)	$\eta_c = \frac{D_{c''} + \sqrt{\frac{q_{D(n)}}{q_{C(n)}}} D_D D_{c''}}{\bar{D}_T}$
(47)	Bracing drift adjustment factor (braced)	$\eta_D = \frac{D_D}{\bar{D}_T - \left(\frac{D_c}{\eta_c}\right)}$
(7)	Magnitude of material property at any level	$P_i = \frac{P_n}{n} \left[b + (n - b) \left(\frac{i - 1}{n - 1} \right)^c \right]$
(49)	Average steel weight per level	$q_{avg.} = \frac{q_n}{2} (\phi_q)$

^a Equations based on customary design units: $D = \text{in.}$, $w = \text{kips/lin ft}$, $H = \text{ft}$, $L = \text{ft}$, $l = \text{ft}$, $I_G = \text{in.}^4$, $I_C = \text{in.}^4$, $I_B = \text{ft}^4$, $A = \text{in.}^2$, $E = 29,000 \text{ ksi}$.

^b Subscripts in parentheses refer to frame grid identification nomenclature (see Fig. 2).

Table 2. Summary of Material Property References

Line	Material Property	Formulations for Values of P in Eqs. (6), (7), (8)	Frame Constants ^a		Used to Establish:
			b	c *	
(1)	Total of all girder stiffnesses at any given level	$T_{G(i)} = \sum_{k=1}^{k=m} \frac{I_{G(k,i)}}{\alpha_k L}$	b_G	c_G	Girder moment drift
(2)	Total of all column stiffnesses immediately below any given level	$T_{C(i)} = \sum_{k=0}^{k=m} \frac{nI_{C(k,i)}}{H}$	$b_{C'}$	$c_{C'}$	Column moment drift
(3)	Moment of Inertia of bent immediately below any given level	$I_{B(i)} = \sum_{k=0}^{k=m} A_{C(k,i)} e_k^2$	$b_{C''}$	$c_{C''}$	Column chord drift
(4)	Total of all bracing "stiffnesses" immediately below any given level ^b	$T_{D(i)} = \sum_{k=1}^{k=m} \frac{A_{D(k,i)} (\alpha_k L)^2}{(L_{D(k)})^3}$	b_D	c_D	Bracing drift

^a Subscripts in parentheses refer to frame grid identification nomenclature (see Fig. 2).

^b The summation shown includes bracing members stressed in tension *only*; this represents only one half of the total number of all bracing members.

c. For column axial forces:

$$c_{C''} = \frac{\log \left(\frac{I_{B(15)} - I_{B(1)}}{I_{B(30)} - I_{B(1)}} \right)}{\log \left(\frac{15 - 1}{30 - 1} \right)} = \frac{\log \left(\frac{2710 - 533}{5280 - 533} \right)}{\log \left(\frac{15 - 1}{30 - 1} \right)} = 1.07$$

Step 6—Read the corresponding drift correction factors from Tables 3, 4, and 5:

- a. For girder moments (Table 3): $\phi_G = 0.846$
- b. For column moments (Table 4): $\phi_{C'} = 0.940$
- c. For column axial forces (Table 5): $\phi_{C''} = 0.904$

Step 7—Establish material weights q by material take-off at level 30 (see Fig. 3):

a. Column weight:

$$q_{C(30)} = \frac{360}{30} (730 + 550 + 550 + 730) = 30700 \text{ lbs}$$

b. Girder weight:

$$q_{G(30)} = (25 \times 130) + (30 \times 150) + (25 \times 130) = 11000 \text{ lbs}$$

Step 8—Calculate the magnitude of the drift components from Eqs. (14a), (17a), and (20a): in Table 1 [for rigid bents; for braced bents use Eqs. (36a) or (39a), and (20a)]:

a. Girder moment drift:

$$D_G = \frac{w_1 H^3}{201 n T_{G(n)}} (\phi_G) \tag{14a}$$

$$= \frac{1.75 \times 360^3 \times 0.846}{201 \times 30 \times 838} = 13.7 \text{ in.}$$

b. Column moment drift:

$$D_{C'} = \frac{w_1 H^3}{201 n T_{C(n)}} (\phi_{C'}) \tag{17a}$$

$$= \frac{1.75 \times 360^3 \times 0.940}{201 \times 30 \times 2360} = 5.39 \text{ in.}$$

c. Column chord drift:

$$D_{C''} = \frac{w_1 (H/38)^4}{I_{B(n)}} (\phi_{C''}) \tag{20a}$$

$$= \frac{1.75 (360/38)^4 \times 0.904}{5280} = 2.41 \text{ in.}$$

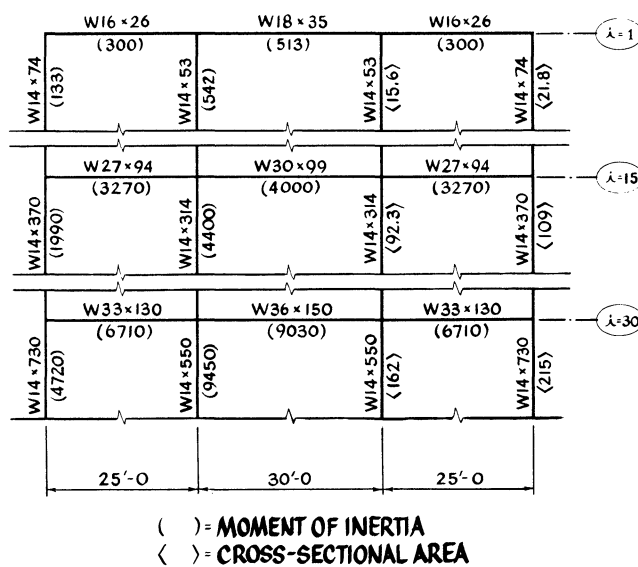


Figure 3

Table 5

a	0												0.1												0.2														
	1.0				2.0				3.0				1.0				2.0				3.0				1.0				2.0				3.0						
	1.0	1.1	1.2	1.0	1.0	1.1	1.2	1.0	1.0	1.1	1.2	1.0	1.0	1.1	1.2	1.0	1.0	1.1	1.2	1.0	1.0	1.1	1.2	1.0	1.0	1.1	1.2	1.0	1.0	1.1	1.2	1.0	1.0	1.1	1.2	1.0	1.0	1.1	1.2
b	.929	.951	.975	.896	.913	.948	.970	.869	.881	.894	.899	.921	.944	.867	.883	.900	.852	.865	.871	.892	.914	.839	.855	.871	.882	.892	.917	.943	.868	.888	.909	.948	.864	.881	.907	.948			
c	.952	.978	1.005	.927	.944	.967	.992	.926	.946	.967	.992	.932	.960	.913	.936	.960	.916	.936	.953	.910	.938	.967	.984	.918	.944	.918	.944	.967	.958	.984	.990	.958	.987	.990	.991	.923			
n	.964	.991	1.021	.944	.967	.992	1.006	1.006	.939	.962	.985	.939	.968	.923	.948	.974	.908	.930	.953	.910	.938	.967	.984	.918	.944	.918	.944	.967	.958	.984	.990	.958	.987	.990	.991	.923			
10	.976	1.005	1.037	.961	.988	1.016	.948	.972	.997	.944	.973	1.004	.930	.956	.983	.917	.941	.965	.914	.943	.973	.901	.926	.953	.960	.932	.960	.932	.960	.932	.960	.932	.960	.932	.960	.932			
15	.979	1.009	1.042	.966	.994	1.023	.955	.980	1.007	.947	.977	1.009	.935	.962	.990	.924	.949	.974	.918	.947	.978	.906	.932	.960	.960	.932	.960	.932	.960	.932	.960	.932	.960	.932	.960	.932			
20	.982	1.012	1.045	.970	.999	1.029	.960	.986	1.014	.950	.980	1.012	.939	.967	.996	.929	.955	.981	.920	.950	.981	.909	.937	.965	.965	.937	.965	.937	.965	.937	.965	.937	.965	.937	.965	.937			
25	.984	1.015	1.048	.974	1.003	1.033	.964	.991	1.019	.952	.982	1.015	.942	.970	1.000	.933	.959	.987	.922	.952	.984	.912	.940	.969	.969	.940	.969	.940	.969	.940	.969	.940	.969	.940	.969	.940			
30	.985	1.017	1.050	.976	1.006	1.037	.968	.995	1.024	.953	.984	1.017	.944	.973	1.004	.936	.963	.991	.924	.954	.986	.915	.943	.973	.973	.943	.973	.943	.973	.943	.973	.943	.973	.943	.973	.943			
35	.982	1.012	1.045	.970	.999	1.029	.960	.986	1.014	.950	.980	1.012	.939	.967	.996	.929	.955	.981	.920	.950	.981	.909	.937	.965	.965	.937	.965	.937	.965	.937	.965	.937	.965	.937	.965	.937			
40	.984	1.015	1.048	.974	1.003	1.033	.964	.991	1.019	.952	.982	1.015	.942	.970	1.000	.933	.959	.987	.922	.952	.984	.912	.940	.969	.969	.940	.969	.940	.969	.940	.969	.940	.969	.940	.969	.940			
45	.985	1.017	1.050	.976	1.006	1.037	.968	.995	1.024	.953	.984	1.017	.944	.973	1.004	.936	.963	.991	.924	.954	.986	.915	.943	.973	.973	.943	.973	.943	.973	.943	.973	.943	.973	.943	.973	.943			
50	.985	1.017	1.050	.976	1.006	1.037	.968	.995	1.024	.953	.984	1.017	.944	.973	1.004	.936	.963	.991	.924	.954	.986	.915	.943	.973	.973	.943	.973	.943	.973	.943	.973	.943	.973	.943	.973	.943			

Table 6

a	0												0.1												0.2														
	1.0				2.0				3.0				1.0				2.0				3.0				1.0				2.0				3.0						
	1.0	1.1	1.2	1.0	1.0	1.1	1.2	1.0	1.0	1.1	1.2	1.0	1.0	1.1	1.2	1.0	1.0	1.1	1.2	1.0	1.0	1.1	1.2	1.0	1.0	1.1	1.2	1.0	1.0	1.1	1.2	1.0	1.0	1.1	1.2	1.0	1.0	1.1	1.2
b	1.062	1.028	1.150	1.114	1.082	1.200	1.166	1.136	1.190	1.218	1.250	1.218	1.190	1.300	1.271	1.244	1.350	1.323	1.298	1.400	1.375	1.352	1.352	1.400	1.400	1.375	1.375	1.400	1.400	1.375	1.375	1.400	1.400	1.375	1.375	1.400			
c	1.067	1.026	1.100	1.060	1.024	1.133	1.095	1.060	1.096	1.130	1.167	1.130	1.096	1.200	1.165	1.133	1.233	1.200	1.169	1.267	1.234	1.205	1.205	1.267	1.267	1.234	1.234	1.267	1.267	1.234	1.234	1.267	1.267	1.234	1.234	1.267			
n	1.050	1.007	1.075	1.033	1.000	1.100	1.059	1.023	1.050	1.100	1.125	1.086	1.050	1.150	1.112	1.077	1.175	1.138	1.104	1.200	1.164	1.131	1.131	1.200	1.200	1.164	1.164	1.200	1.200	1.164	1.164	1.200	1.200	1.164	1.164	1.200			
10	1.040	.996	1.060	1.017	.978	1.080	1.038	1.000	1.022	1.059	1.100	1.059	1.022	1.120	1.080	1.043	1.140	1.101	1.065	1.160	1.122	1.087	1.087	1.160	1.160	1.122	1.122	1.160	1.160	1.122	1.122	1.160	1.160	1.122	1.122	1.160			
15	1.033	.989	1.050	1.006	.967	1.067	1.024	.985	1.003	1.041	1.083	1.041	1.003	1.100	1.059	1.021	1.117	1.076	1.039	1.133	1.094	1.057	1.057	1.133	1.133	1.094	1.094	1.133	1.133	1.094	1.094	1.133	1.133	1.094	1.094	1.133			
20	1.029	.984	1.043	.999	.958	1.057	1.014	.974	.990	1.029	1.071	1.029	.990	1.086	1.043	1.005	1.100	1.058	1.021	1.114	1.073	1.036	1.036	1.114	1.114	1.073	1.073	1.114	1.114	1.073	1.073	1.114	1.114	1.073	1.073	1.114			
25	1.025	.980	1.037	.993	.952	1.050	1.006	.966	1.002	1.019	1.062	1.019	.979	1.075	1.032	.993	1.087	1.045	1.007	1.100	1.058	1.020	1.020	1.100	1.100	1.058	1.058	1.100	1.100	1.058	1.058	1.100	1.100	1.058	1.058	1.100			
30	1.022	.977	1.033	.988	.947	1.044	1.000	.960	1.056	1.012	1.056	1.012	.972	1.067	1.023	.984	1.078	1.035	.996	1.089	1.047	1.008	1.008	1.089	1.089	1.047	1.047	1.089	1.089	1.047	1.047	1.089	1.089	1.047	1.047	1.089			
35	1.020	.974	1.030	.985	.944	1.040	.995	.955	1.050	1.006	1.050	1.006	.965	1.060	1.016	.976	1.070	1.027	.987	1.080	1.037	.998	.998	1.080	1.080	1.037	1.037	1.080	1.080	1.037	1.037	1.080	1.080	1.037	1.037	1.080			
40	1.020	.974	1.030	.985	.944	1.040	.995	.955	1.050	1.006	1.050	1.006	.965	1.060	1.016	.976	1.070	1.027	.987	1.080	1.037	.998	.998	1.080	1.080	1.037	1.037	1.080	1.080	1.037	1.037	1.080	1.080	1.037	1.037	1.080			
45	1.020	.974	1.030	.985	.944	1.040	.995	.955	1.050	1.006	1.050	1.006	.965	1.060	1.016	.976	1.070	1.027	.987	1.080	1.037	.998	.998	1.080	1.080	1.037	1.037	1.080	1.080	1.037	1.037	1.080	1.080	1.037	1.037	1.080			
50	1.020	.974	1.030	.985	.944	1.040	.995	.955	1.050	1.006	1.050	1.006	.965	1.060	1.016	.976	1.070	1.027	.987	1.080	1.037	.998	.998	1.080	1.080	1.037	1.037	1.080	1.080	1.037	1.037	1.080	1.080	1.037	1.037	1.080			

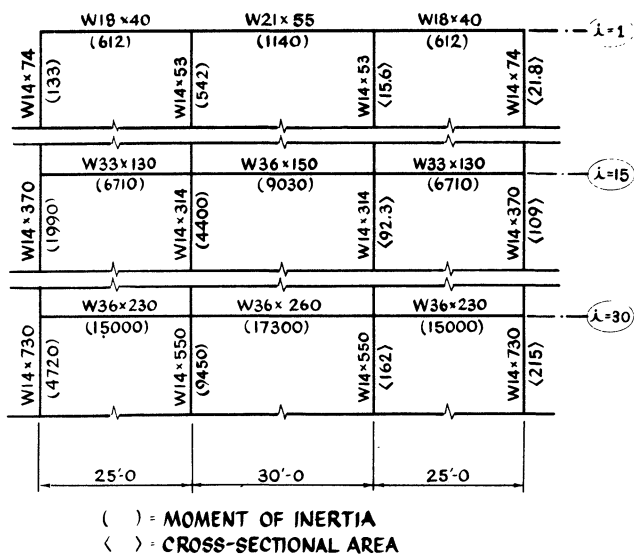


Figure 4

Step 9—Determine the total drift (sum of components from step 8):

$$D_T = 13.7 + 5.39 + 2.41 = 21.5 \text{ in.}$$

B. Determine Most Economical Drift Adjustment (Steps 10–14):

Step 10—Calculate the optimized column drift adjustment factor η_C from Eq. (32) in Table 1 [for rigid bents; for braced bents use Eq. (46)]:

$$\eta_C = \frac{D_C + \frac{r_G}{r_G} \sqrt{\frac{q_{G(n)}}{q_{C(n)}}} D_G D_C}{\bar{D}_T} \quad (32)$$

$$= \frac{7.80 + \sqrt{\frac{11000}{30700}} \times 13.7 \times 7.80}{14.4}$$

$$= 0.971$$

Step 11—Choose a convenient column drift adjustment factor η_C (≥ 1.0): The optimized factor (step 10) is smaller than unity. Therefore, use $\eta_C = 1.00$.

Step 12—Calculate the girder drift adjustment factor η_G as per Eq. (33) in Table 1 [for rigid bents; for braced bents find η_D from Eq. (47)]:

$$\eta_G = \frac{D_G}{\bar{D}_T - \left(\frac{D_C}{\eta_C}\right)} \quad (33)$$

$$= \frac{13.7}{14.4 - \left(\frac{7.80}{1.00}\right)}$$

$$= 2.08$$

Step 13—Adjust all stress-designed sections by multiplying their relevant material properties (rigid: moment of inertia—braced: cross-sectional area) by their pertinent drift adjustment factors η ; choose new drift-adjusted sections to meet these new requirements:

The column drift adjustment factor $\eta_C = 1.00$ indicates that no changes in the column sections will be necessary.

The moments of inertia of all girders throughout the bent will be multiplied by the girder drift adjustment factor $\eta_G = 2.08$. It will not be necessary to select steel sections with moments of inertia always greater than obtained from this multiplication; “eyeball” scatter selection closely above and below this value is satisfactory as long as the total sum of the girder stiffnesses (T_G) will be reasonably close to the value $\bar{T}_{G(i)} = \eta_G T_{G(i)}$.

Level 1	G1	$\bar{I} = 2.08 \times 300 = 624$	W18×40 (612)
	G2	$\bar{I} = 2.08 \times 513 = 1070$	W21×55 (1140)
Level 15	G1	$\bar{I} = 2.08 \times 3270 = 6800$	W33×130 (6710)
	G2	$\bar{I} = 2.08 \times 4000 = 8320$	W36×150 (9030)
Level 30	G1	$\bar{I} = 2.08 \times 6710 = 14000$	W36×230 (15000)
	G2	$\bar{I} = 2.08 \times 9030 = 18800$	W36×260 (17300)

The drift-adjusted frame is shown in Fig. 4.

Step 14—Establish sectional requirements at all other levels by interpolation as per Eq. (7) (see Table 1):

The technique of interpolation is the same for all sections; it will, therefore, only be demonstrated for one section: girder G1. The material property in Eq. (7) will be the moment of inertia I :

$$\bar{I}_i = \frac{\bar{I}_{30}}{30} \left[b + (30 - b) \left(\frac{i - 1}{30 - 1} \right)^c \right]$$

Entering the value for b_G from Eq. (6):

$$b_G = 30 \left(\frac{\bar{I}_1}{\bar{I}_{30}} \right)$$

and the value for c_G from Eq. (8):

$$c_G = \frac{\log \left(\frac{\bar{I}_{15} - \bar{I}_1}{\bar{I}_{30} - \bar{I}_1} \right)}{\log \left(\frac{15 - 1}{30 - 1} \right)}$$

in which, from step 13, $\bar{I}_1 = 624$; $\bar{I}_{15} = 6800$ and $\bar{I}_{30} = 14000$. The general material property distribution equation above will take the form:

$$\bar{I}_i = 624 + 375(i - 1)^{1.06} \text{ in.}^4$$

The moduli of inertia of any level are obtained by entering the pertinent value of i into the above equation. Tables 7 and 8 will assist in the selection of steel sections according to their moment of inertia.

Table 7

<div style="display: flex; align-items: center;"> <div style="font-size: 2em; margin-right: 10px;">I</div> <div style="font-size: 2em; margin-right: 10px;">x</div> <div>SECTIONS SHOWN IN BOLD FACE ARE "WEIGHT ECONOMY SECTIONS"</div> </div>							
Moment of Inertia, I _x In. ⁴	Beam Designation	Moment of Inertia, I _x In. ⁴	Beam Designation	Moment of Inertia, I _x In. ⁴	Beam Designation	Moment of Inertia, I _x In. ⁴	Beam Designation
20300	W36 x 300	3270	W27 x 94	843	W21 x 44	130	W12 x 19
		3230	W14 x 246	836	W16 x 64	126	W8 x 35
18900	W36 x 280	3080	W14 x 237	802	W18 x 50	110	W8 x 31
17300	W36 x 260	3020	W21 x 127	797	W14 x 74	107	W10 x 21
		3000	W24 x 100	789	W12 x 92		
16100	W36 x 245	2940	W14 x 228	748	W16 x 58	105	W12 x 16.5
				724	W14 x 68	97.8	W8 x 28
15000	W36 x 230	2830	W27 x 84	723	W12 x 85	96.3	W10 x 19
14400	W14 x 730	2800	W14 x 219	719	W10 x 112		
13600	W33 x 240	2690	W24 x 94	706	W18 x 45	88.0	W12 x 14
12500	W14 x 665	2670	W14 x 211	663	W12 x 79	82.5	W8 x 24
		2620	W21 x 112	657	W16 x 50	81.9	W10 x 17
12300	W33 x 220	2540	W14 x 202	641	W14 x 61	69.4	W8 x 20
		2400	W14 x 193	625	W10 x 100	68.9	W10 x 15
12100	W36 x 194	2370	W24 x 84	612	W18 x 40	56.6	W8 x 17
		2270	W14 x 184	597	W12 x 72	53.3	W6 x 25
11300	W36 x 182	2150	W14 x 176	584	W16 x 45	52.0	W10 x 11.5
11100	W33 x 200	2100	W24 x 76	542	W10 x 89	48.1	W8 x 15
10900	W14 x 605	2100	W21 x 96	542	W14 x 53	41.5	W6 x 20
		2040	W18 x 114	533	W12 x 65	39.6	W8 x 13
10500	W36 x 170	2020	W14 x 167	517	W16 x 40	31.7	W6 x 16
9890	W30 x 210	1900	W14 x 158	513	W18 x 35	30.8	W8 x 10
		1890	W12 x 190	485	W14 x 48	30.1	W6 x 15.5
9760	W36 x 160	1850	W18 x 105	476	W12 x 58	25.4	W5 x 18.5
9450	W14 x 550	1820	W24 x 68	457	W10 x 77	21.7	W6 x 12
		1790	W14 x 150	447	W16 x 36	21.3	W5 x 16
9030	W36 x 150	1760	W21 x 82	429	W14 x 43	14.8	W6 x 8.5
8850	W30 x 190	1680	W18 x 96	426	W12 x 53	11.3	W4 x 13
8250	W14 x 500	1670	W14 x 142	421	W10 x 72		
8160	W33 x 152	1600	W21 x 73	395	W12 x 50		
7910	W30 x 172	1590	W14 x 136	386	W14 x 38		
		1540	W24 x 61	382	W10 x 66		
7820	W36 x 135	1540	W12 x 161	374	W16 x 31		
7460	W33 x 141	1480	W21 x 68	351	W12 x 45		
7220	W14 x 455	1480	W14 x 127	344	W10 x 60		
6740	W27 x 177	1440	W18 x 85	340	W14 x 34		
		1370	W14 x 119	310	W12 x 40		
6710	W33 x 130	1360	W16 x 96	306	W10 x 54		
6610	W14 x 426	1340	W24 x 55	300	W16 x 26		
6030	W27 x 160	1330	W21 x 62	290	W14 x 30		
6010	W14 x 398	1290	W18 x 77	281	W12 x 36		
		1270	W14 x 111	273	W10 x 49		
5900	W33 x 118	1220	W16 x 88	272	W8 x 67		
5760	W30 x 132	1220	W12 x 133	249	W10 x 45		
5450	W14 x 370	1170	W14 x 103	244	W14 x 26		
5430	W27 x 145	1160	W18 x 70	239	W12 x 31		
5360	W30 x 124	1140	W21 x 55	227	W8 x 58		
5120	W24 x 160	1070	W12 x 120	210	W10 x 39		
		1060	W14 x 95	204	W12 x 27		
4930	W30 x 116	1050	W18 x 64	198	W14 x 22		
4910	W14 x 342	986	W16 x 78	184	W8 x 48		
4570	W24 x 145	971	W18 x 60	171	W10 x 33		
		967	W21 x 49	158	W10 x 29		
4470	W30 x 108	941	W14 x 87	156	W12 x 22		
4400	W14 x 314	931	W16 x 71	146	W8 x 40		
4140	W14 x 320	928	W12 x 106	133	W10 x 25		
4090	W27 x 114	891	W14 x 84				
4020	W24 x 130	891	W18 x 55				
		859	W16 x 78				
4000	W30 x 99	851	W12 x 99				
3910	W14 x 287		W14 x 78				
3650	W24 x 120						
3610	W27 x 102						
3530	W14 x 264						
3410	W21 x 142						
3330	W24 x 110						

Table 8

<div style="display: flex; align-items: center;"> <div style="font-size: 2em; margin-right: 10px;">I_y</div> <div>SECTIONS SHOWN IN BOLD FACE ARE "WEIGHT ECONOMY SECTIONS"</div> </div>							
Moment of Inertia, I _y In. ⁴	Beam Designation	Moment of Inertia, I _y In. ⁴	Beam Designation	Moment of Inertia, I _y In. ⁴	Beam Designation	Moment of Inertia, I _y In. ⁴	Beam Designation
4720	W14 x 730	492	W14 x 119	116	W10 x 60	21.6	W8 x 28
4170	W14 x 665	486	W12 x 161	115	W21 x 96	21.6	W12 x 31
3680	W14 x 605	471	W24 x 145	108	W24 x 94	20.7	W21 x 44
3260	W14 x 550	455	W14 x 111	107	W12 x 58	19.5	W14 x 30
2880	W14 x 500	443	W27 x 145	107	W14 x 61	19.1	W18 x 40
2560	W14 x 455	420	W14 x 103	105	W27 x 84	18.3	W12 x 27
2360	W14 x 426	414	W21 x 142	105	W18 x 85	18.2	W8 x 24
2170	W14 x 398	412	W24 x 130	104	W10 x 54	17.1	W6 x 25
1990	W14 x 370	390	W12 x 133	96.1	W12 x 53	16.3	W10 x 29
1810	W14 x 342	384	W14 x 95	95.6	W21 x 82	15.5	W18 x 35
1640	W14 x 320	375	W36 x 194	94.5	W24 x 84	13.7	W10 x 25
1630	W14 x 314	366	W21 x 127	94.1	W18 x 77	13.3	W6 x 20
1470	W14 x 287	350	W14 x 87	93.0	W10 x 49	12.5	W16 x 31
1330	W14 x 264	347	W36 x 182	92.5	W16 x 78	10.8	W10 x 21
1300	W36 x 300	345	W12 x 120	88.6	W8 x 67	9.67	W6 x 15.5
1230	W14 x 246	320	W36 x 170	84.0	W18 x 70	9.59	W16 x 26
1200	W36 x 280	317	W21 x 112	82.8	W16 x 71	9.22	W8 x 20
1170	W14 x 237	301	W12 x 106	82.6	W24 x 76	8.89	W5 x 18.5
1120	W14 x 228	295	W36 x 160	75.8	W18 x 64	8.86	W14 x 26
1090	W36 x 260	278	W12 x 99	74.9	W8 x 58	7.51	W5 x 16
1070	W14 x 219	274	W18 x 114	73.3	W16 x 64	7.44	W8 x 17
1030	W14 x 211	274	W24 x 120	70.6	W21 x 73	7.00	W14 x 22
1010	W36 x 245	273	W33 x 152	70.0	W24 x 68	4.64	W12 x 22
980	W14 x 202	270	W36 x 150	65.3	W16 x 58	4.42	W6 x 16
940	W36 x 230	256	W12 x 92	64.7	W21 x 68	4.28	W10 x 19
933	W33 x 240	249	W18 x 105	60.9	W8 x 48	3.76	W4 x 13
930	W14 x 193	249	W24 x 110	57.5	W21 x 62	3.76	W12 x 19
883	W14 x 184	246	W33 x 141	57.5	W14 x 53	3.55	W10 x 17
841	W33 x 220	235	W12 x 85	56.4	W12 x 50	3.40	W8 x 15
838	W14 x 176	235	W10 x 112	53.2	W10 x 45	2.98	W6 x 12
790	W14 x 167	226	W36 x 135	51.3	W14 x 48	2.88	W10 x 15
757	W30 x 210	225	W14 x 84	50.1	W18 x 60	2.88	W12 x 16.5
750	W33 x 200	225	W18 x 96	50.0	W12 x 45	2.72	W8 x 13
745	W14 x 158	224	W16 x 96	49.0	W8 x 40	2.34	W12 x 14
703	W14 x 150	223	W24 x 100	48.3	W21 x 55	2.10	W10 x 11.5
673	W30 x 190	218	W33 x 130	45.1	W14 x 43	2.08	W8 x 10
660	W14 x 142	216	W12 x 79	45.0	W18 x 55	1.98	W6 x 8.5
598	W30 x 172	207	W14 x 78	44.9	W10 x 39		
590	W12 x 190	207	W10 x 100	44.1	W12 x 40		
568	W14 x 136	202	W16 x 88	42.5	W8 x 35		
556	W27 x 177	196	W30 x 132	40.2	W18 x 50		
530	W24 x 160	195	W12 x 72	37.1	W16 x 50		
528	W14 x 127	187	W33 x 118	37.0	W8 x 31		
495	W27 x 160	181	W10 x 89	36.5	W10 x 33		
		181	W30 x 124	34.8	W18 x 45		
		175	W12 x 65	34.3	W24 x 61		
		164	W30 x 116	32.8	W16 x 45		
		159	W27 x 114	28.9	W24 x 55		
		153	W10 x 77	28.8	W16 x 40		
		146	W30 x 108	26.6	W14 x 38		
		142	W10 x 72	25.5	W12 x 36		
		139	W27 x 102	24.7	W21 x 49		
		133	W14 x 74	24.4	W16 x 36		
		129	W10 x 66	23.3	W14 x 34		
		128	W30 x 99				
		124	W27 x 94				
		121	W14 x 68				

Frame Weight Estimate—Material take-off of the steel sections as shown in Fig. 4 will amount to the following quantities:

$$\bar{q}_1 = (74 \times 24) + (53 \times 24) + (40 \times 50) + (55 \times 30) = 6700 \text{ lbs}$$

$$\bar{q}_{15} = (370 \times 24) + (314 \times 24) + (130 \times 50) + (150 \times 30) = 27400 \text{ lbs}$$

$$\bar{q}_{30} = (730 \times 24) + (550 \times 24) + (230 \times 50) + (260 \times 30) = 50000 \text{ lbs}$$

From Eq. (6): $b_q = \frac{30 \times 6700}{50000} = 4.02$

From Eq. (8): $c_q = \frac{\log \left(\frac{27400 - 6700}{50000 - 6700} \right)}{\log \left(\frac{15 - 1}{30 - 1} \right)} = 1.01$

From Table 6: $\phi_q = 1.13$

The average steel weight of the bent per level is obtained from Eq. (49):

$$q_{avg} = \frac{q_n}{2} (\phi_q) = \frac{50000}{2} \times 1.13 = 283000 \text{ lbs}$$

DEVELOPMENT OF METHOD AND WORKING FORMULAS

Wind Loading—There are many different wind specifications in force. Almost all of them require a steplike increase of the wind force at designated elevations. Without any meaningful sacrifice of accuracy, these requirements can be expressed by the equation

$$w_x = w_1(x/H)^a \quad (1)$$

in which w_x designates the wind pressure at distance x above grade and w_1 is the wind pressure at the top of the bent (level 1). Most, if not all, building codes express wind requirements in relation to the wind load at a specified elevation x , usually 30 ft. Rearranging Eq. (1) and substituting $x = 30$,

$$w_1 = w_{30-ft} (H/30)^a \quad (2)$$

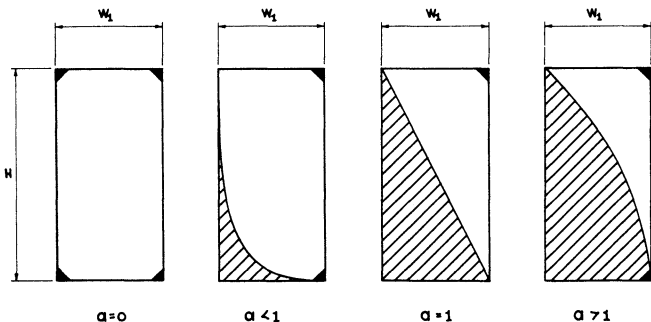


Figure 5

The value for the wind exponent a is obtained as follows:

$$a = \frac{\log (w_1/w_{30-ft})}{\log (H/30)} \quad (3)$$

Equation (5) below can be used to demonstrate that exponent a is the ratio of the shaded area to the unshaded area in Fig. 5, where wind load configurations based on different values of a are shown. Most common values for exponent a are in the range between (0.15) and (0.20), usually closer to the latter value.

Other general statements can now be obtained (see Fig. 6):

Overtuning Moment at Elevation ρH above Grade:

$$\begin{aligned} M_\rho &= H^2 \int_\rho^1 [w_\xi(\xi - \rho)d\xi] \\ &= \frac{w_1 H^2}{(a+1)(a+2)} [\rho^{(a+2)} - \rho(a+2) + (a+1)] \end{aligned} \quad (4)$$

Wind Shear at Elevation ρH above Grade:

$$\begin{aligned} V_\rho &= H \int_\rho^1 (w_\xi d\xi) \\ &= \frac{w_1 H}{(a+1)} [1 - \rho^{(a+1)}] \end{aligned} \quad (5)$$

Material Properties—In a normally proportioned steel bent, cross-sectional material properties (hereinafter in general called P), such as cross-sectional area, section modulus, moment of inertia, unit weight, usually increase from a minimum value at the roof level, P_1 , in a steplike fashion to a maximum value at the level above grade, P_n . These steps are caused by the finite choice of material selection. In steel bents where drift investigations become desirable (starting at about 10 to 15 levels), the importance of these little steps diminishes rather drastically. It is justifiable to express the increase and dis-

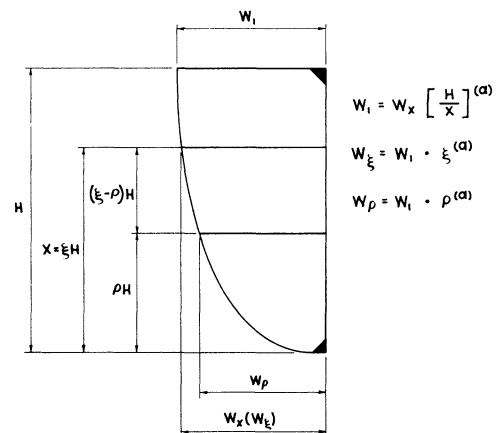


Figure 6

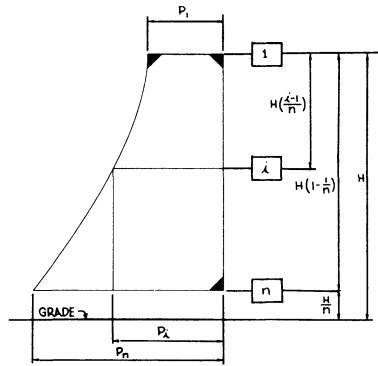


Figure 7

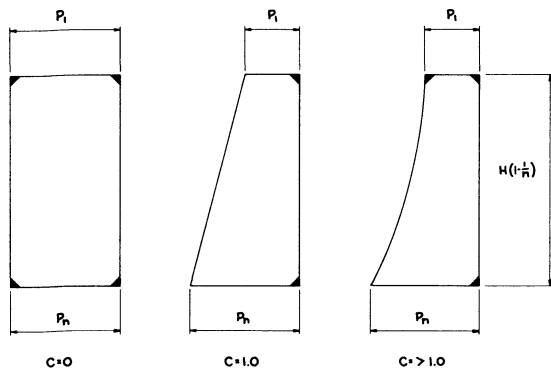


Figure 8

tribution of cross-sectional properties as a continuous mathematical relationship, as indicated in Fig. 7.

Introducing a Material Property Distribution Factor,

$$b = nP_1/P_n \quad (6)$$

where P_1 signifies the magnitude of a given material property at level 1 and P_n stands for the size of this material property at level n, it is possible to state the magnitude of any material property at any level throughout the height of the bent:

$$P_i = \frac{P_n}{n} \left[b + (n - b) \left(\frac{i - 1}{n - 1} \right)^c \right] \quad (7)$$

The exponent c usually has a magnitude of from 1.0 to about 1.2. It can be found by establishing the property in question on any other convenient level, say i :

$$c = \frac{\log \left(\frac{P_i - P_1}{P_n - P_1} \right)}{\log \left(\frac{i - 1}{n - 1} \right)} \quad (8)$$

Material property distribution patterns for different exponents c are shown in Fig. 8.

For drift computation, specific material property values for girder stiffnesses, column stiffnesses, bent moments of inertia, and bracing "stiffnesses," at levels 1, i , and n , will be substituted for the general expression P in Eqs. (6), (7), and (8). The mathematical expressions for these values are summarized in Table 2.

If it is felt that the material properties at level 1 and/or level n are not representative of the remainder of the frame (such as greater story height at the grade level and/or penthouse conditions at the roof), the following procedure is recommended:

The inter-relationship of properties on three different levels, say i , s , and t , in a mathematical configuration as shown in Fig. 7 and stated in Eq. (7), can be expressed by the following equation:

$$P_i[(s - 1)^c - (t - 1)^c] + P_s[(t - 1)^c - (i - 1)^c] + P_t[(i - 1)^c - (s - 1)^c] = 0$$

For the purpose of establishing idealized values for P_1 and P_n , it is satisfactory to assume an initial value of $c = 1.0$, which will resolve this equation into:

$$P_1 = \frac{P_s(t - 1) - P_t(s - 1)}{(t - s)} \quad \text{for } i = 1 \quad (9)$$

and

$$P_n = \frac{P_t(n - s) - P_s(n - t)}{(t - s)} \quad \text{for } i = n \quad (10)$$

These idealized values for P_1 from Eq. (9) and for P_n from Eq. (10) can then be used in Eqs. (6), (7), and (8). To make the described approximation as accurate as possible, it is advisable to choose level s as close to level 1 as possible, level t as close to level n as possible, and level i as close to the mid-height of the frame as possible. The *exact* values for P_1 and P_n have to be established for their special conditions. They will, however, not be used in subsequent calculations within the framework of this article. Only the *idealized* values from Eqs. (9) and (10) should find future application.

DRIFT IN PLANAR RIGID BENTS

In this frame type, four different components contribute to the lateral motion of the bent:

- (a) Girder Moment Drift (caused by the flexing of the girders)
- (b) Column Moment Drift (caused by the flexing of the columns)
- (c) Column Chord Drift (caused by axial deformation of the columns)
- (d) Girder Axial Drift (caused by axial deformation of the girders)

Girder Moment Drift—The following idealizing assumptions will facilitate the establishment of this component:

- (1) Girders have points of contraflexure at mid-span.
- (2) Columns have points of contraflexure at mid-story height.
- (3) Columns have an infinitely great stiffness and cannot bend.
- (4) No change in the length of any member will occur.

The resulting condition is illustrated in Fig. 9. In Fig. 9, β_k denotes an unspecified proportion of the total wind shear $V_{B(i)}$ at level i , and d_G denotes the contribution of level i to the total amount of girder moment drift for the whole bent. Since, on each level, the leeward column of bay 1 becomes the windward column of bay 2, the leeward column of bay 2 in turn becomes the windward column of bay 3, etc., and since, within the same bay, the adjacent columns must be parallel to each other (because of assumptions 1 and 4 above), all columns on the same level must be parallel to each other after girder deformation.

For this reason, the girder moment drift contribution d_G will be identical for all bays and a constant for the level under investigation. The typical girder will have a free body diagram as shown in Fig. 10.

Geometrically, the following proportion can be read from Fig. 9:

$$d_G = \frac{2Hf_{(k,i)}}{n\alpha_k L}$$

The forces shown in Fig. 10 will result in the following elastic deformation:

$$f_{(k,i)} = \frac{\alpha_k \beta_k H L}{48nEK_{G(k,i)}} [V_{B(i)} + V_{B(i-1)}]$$

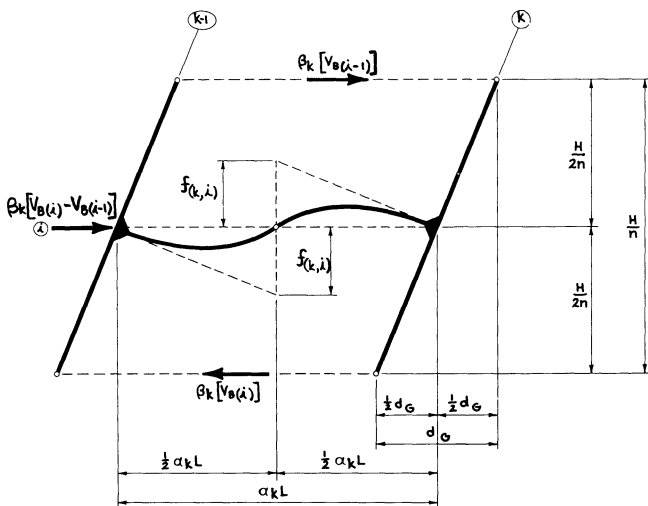


Figure 9

Substituting,

$$d_G = \frac{\beta_k H^2}{24n^2 EK_{G(k,i)}} [V_{B(i)} + V_{B(i-1)}] \quad (11)$$

It will be noted that the proportion $\beta_k/K_{G(k,i)}$ must be a constant for each level because d_G is also a constant for each level. Therefore, $\beta_k = (\text{constant}) \times K_{G(k,i)}$. Summating both sides of this equation within the limits $k = 1$ and $k = m$, $1 = (\text{constant}) \times T_{G(i)}$. From this follows: $(\text{constant}) = 1/T_{G(i)}$. Then, $\beta_k = K_{G(k,i)}/T_{G(i)}$ and Eq. (11) can be modified to the following:

$$d_G = \frac{H^2}{24n^2 ET_{G(i)}} [V_{B(i)} + V_{B(i-1)}] \quad (12)$$

Equation (12) represents the contribution of level i to the total girder moment drift of the whole frame.

It is obvious that the condition shown in Fig. 9 does not represent the condition at level 1 (top of bent). The actual condition at level 1 is illustrated in Fig. 11.

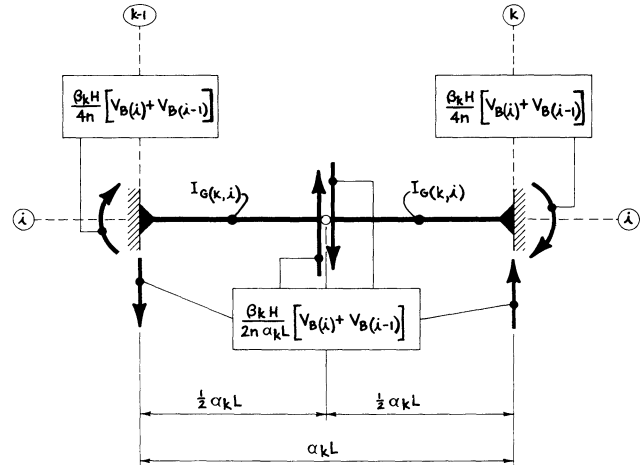


Figure 10

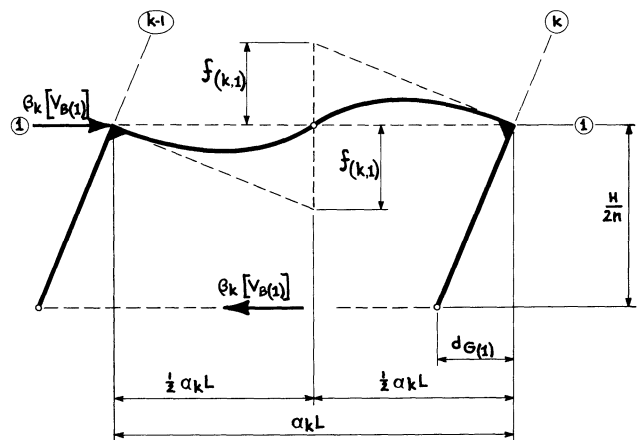


Figure 11

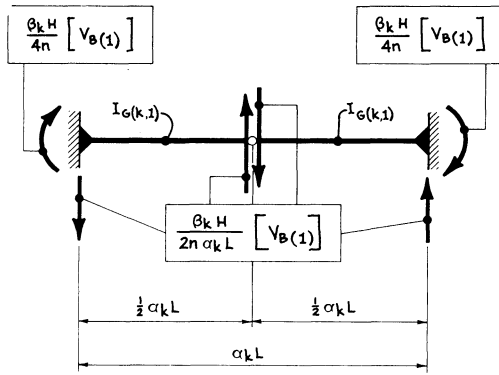


Figure 12

The free body diagram at level 1 is shown in Fig. 12. Treatment of this special condition corresponding to the treatment of the general condition results in the following equation:

$$d_{G(1)} = \frac{H^2}{48n^2 ET_{G(1)}} [V_{B(1)}] \quad (13)$$

The total girder moment drift is obtained by taking the following steps:

- (1) Express the material property $T_{G(i)}$ as prescribed in Eq. (7).
- (2) Formulate building shears as per Eq. (5) by the substitution of:

$$\left(1 - \frac{i - 0.5}{n}\right) \text{ for } \rho \text{ to obtain } V_{B(i)}$$

$$\left(1 - \frac{i - 1.5}{n}\right) \text{ for } \rho \text{ to obtain } V_{B(i-1)}$$

$$\left(1 - \frac{0.5}{n}\right) \text{ for } \rho \text{ to obtain } V_{B(1)}$$

- (3) Insert the results of steps (1) and (2) above into Eqs. (12) and (13), form the summation of Eq. (12) from $i = 2$ to $i = n$, and add the adjustment as per Eq. (13). The total girder moment drift then becomes:

$$D_G = \frac{w_1 H^3}{12n ET_{G(n)}} (\phi_G) \quad (14)$$

or, expressed in customary design units (see Table 1),

$$D_G = \frac{w_1 H^3}{201n T_{G(n)}} (\phi_G) \quad (14a)$$

in which

$$\phi_G = \left[\frac{1 - \left(1 - \frac{0.5}{n}\right)^{(a+1)}}{4b_G^{(a+1)}} \right] + \sum_{i=2}^n \left\{ \frac{2 - \left(1 - \frac{i - 0.5}{n}\right)^{(a+1)} - \left(1 - \frac{i - 1.5}{n}\right)^{(a+1)}}{2(a+1) \left[b_G + (n - b_G) \left(\frac{i-1}{n-1}\right)^{c_G} \right]} \right\} \quad (15)$$

Values of ϕ_G are tabulated in Table 3 for various values of a , b_G , and c_G . They represent frame constants.

Column Moment Drift—For the establishment of this drift component, the following idealizing assumptions will be used:

- (1) Columns have points of contraflexure at mid-story height.
- (2) Girders have an infinitely great stiffness and cannot bend.
- (3) No change in the length of any member will occur.

The resulting condition is illustrated in Fig. 13. In Fig. 13, γ_k denotes an unspecified proportion of the total wind shear $V_{B(i)}$ below level i and $d_{C'}$ denotes the contribution of level i to the total amount of the column moment drift for the whole bent. The assumptions listed above again dictate that the size of $d_{C'}$ must be a constant for all columns on each level under investigation. The forces on the column as shown in Fig. 13 will result in the following elastic deformation:

$$d_{C'} = \frac{\gamma_k H^2}{12n^2 EK_{C(k,i)}} (V_{B(i)})$$

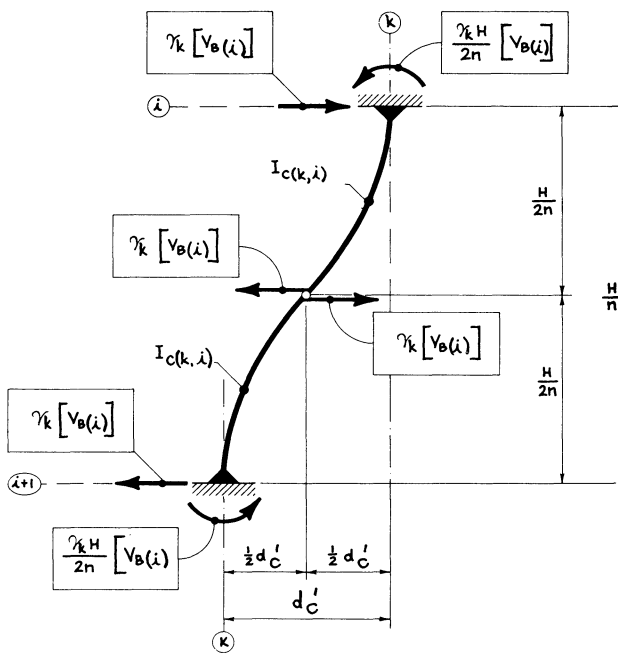


Figure 13

The same reasoning used in the formulation of β_k (in the discussion of girder moment drift) applies. Then, $\gamma_k = K_{C(k,i)}/T_{C(i)}$, and

$$d_{C'} = \frac{H^2}{12n^2ET_{C(i)}} (V_{B(i)}) \quad (16)$$

Equation (16) represents the contribution of level i to the total column moment drift of the whole frame.

The total column moment drift is obtained as follows:

- (1) Express the material property $T_{C(i)}$ as prescribed in Eq. (7).
- (2) Formulate the building shear as per Eq. (5) by the substitution of $1 - [(i - 0.5)/n]$ for ρ to obtain $V_{B(i)}$.
- (3) Insert the results of steps (1) and (2) above into Eq. (16) and form the summation of Eq. (16) from $i = 1$ to $i = n$. The total column moment drift then becomes:

$$D_{C'} = \frac{w_1 H^3}{12nET_{C(n)}} (\phi_{C'}) \quad (17)$$

or, expressed in customary design units (see Table 1),

$$D_{C'} = \frac{w_1 H^3}{201nT_{C(n)}} (\phi_{C'}) \quad (17a)$$

in which

$$\phi_{C'} = \sum_{i=1}^n \left\{ \frac{1 - \left(1 - \frac{i - 0.5}{n}\right)^{(a+1)}}{(a+1) \left[b_c + (n - b_c) \left(\frac{i-1}{n-1}\right)^{c_{C'}} \right]} \right\} \quad (18)$$

Values of $\phi_{C'}$ are tabulated in Table 4 for various values of a , $b_{C'}$, and $c_{C'}$. They represent frame constants.

Column Chord Drift—So far, the effects of the wind load in its attempt to push the frame to the side have been investigated. In this section, the effect of the wind load in attempting to overturn the frame will be analyzed. Because of the assumption that the frame is a vertical truss in which the girders are the web members and the columns are the chords of this truss, the resulting drift component will be called Column Chord Drift. This component is caused by the lengthening of the windward columns and the shortening of the leeward columns. Again, simplifying assumptions are used:

- (1) The plane of floor levels remains plane after bending; therefore, columns deform axially in proportion to their distance to the common center of gravity of the cross-sectional areas of all columns.
- (2) The radius of curvature of the bent in its deflected state is constant for each individual floor level.

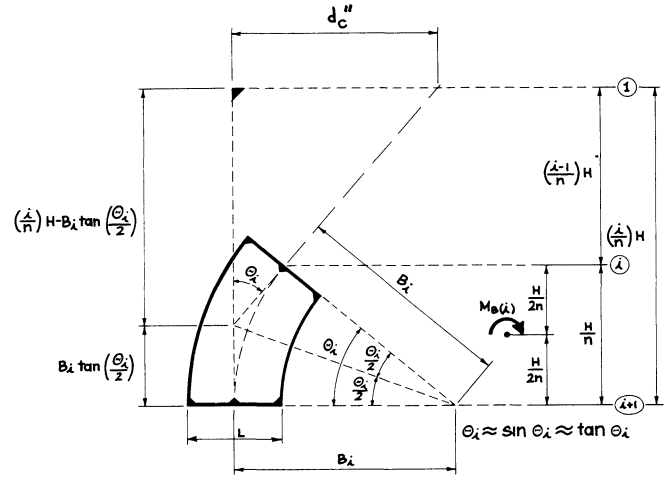


Figure 14

The resulting condition is illustrated in Fig. 14. In Fig. 14, B_i denotes the radius of curvature of the bent, θ_i denotes the angle of change of the elastic line at level i , and $d_{C''}$ denotes the column chord drift contribution of level i . The concept of "Moment of Inertia of the Bent" will be introduced here: The Moment of Inertia of the Bent, $I_{B(i)}$, is defined as the sum of the individual cross-sectional areas of the columns in the bent times the square of their distance to their common center of gravity (see Fig. 15).

From Fig. 14, by geometric proportion,

$$d_{C''} = \left[\left(\frac{i}{n}\right)H - \frac{B_i \theta_i}{2} \right] [\theta_i] \quad \text{and} \quad \theta_i = \frac{H}{nB_i}$$

The elastic deformation is

$$\theta_i = \frac{HM_{B(i)}}{nEI_{B(i)}}$$

Combining,

$$d_{C''} = \frac{H^2 M_{B(i)} (i - 0.5)}{n^2 EI_{B(i)}} \quad (19)$$

Equation (19) represents the contribution of level i to the total column chord drift of the whole bent.

The total column chord drift is obtained by taking the following steps:

- (1) Express the material property $I_{B(i)}$ as prescribed in Eq. (7).
- (2) Formulate the overturning moment as per Eq. (4) by the substitution of $1 - [(i - 0.5)/n]$ for ρ to obtain $M_{B(i)}$.
- (3) Insert the results of steps (1) and (2) above into Eq. (19) and form the summation of Eq. (19)

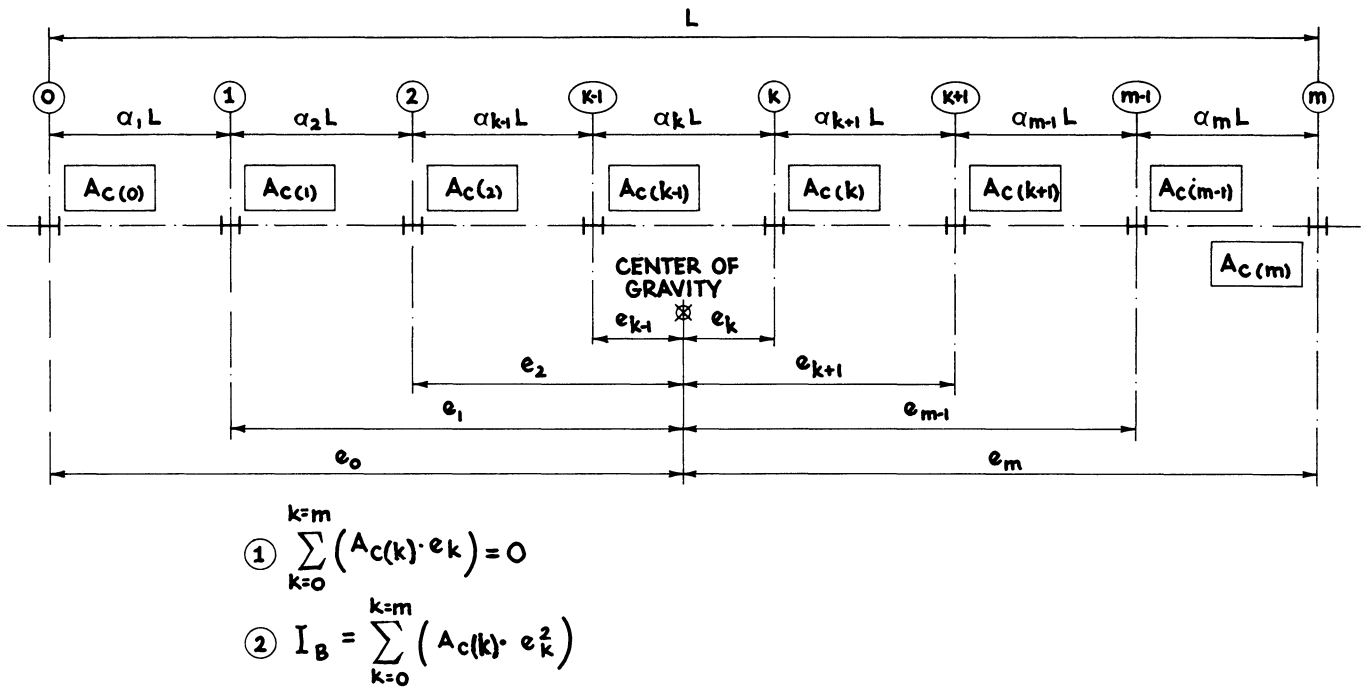


Figure 15

from $i = 1$ to $i = n$. The equation for column chord drift then becomes:

$$D_{C''} = \frac{w_1 H^4}{6EI_{B(n)}} (\phi_{C''}) \quad (20)$$

or, expressed in customary design units (see Table 1),

$$D_{C''} = \frac{w_1 (H/38)^4}{I_{B(n)}} (\phi_{C''}) \quad (20a)$$

in which $\phi_{C''}$ is expressed by Eq. (21) below. Values of $\phi_{C''}$ are tabulated in Table 5 for various values of a , $b_{C''}$, and $c_{C''}$. They represent frame constants.

Girder Axial Drift—This component is caused by axial deformation of the girders. It must, however, be realized that horizontal wind forces under actual field conditions will have to flow through the floor membrane in order to reach the components of the bent. By code requirement, this floor membrane must be firmly and permanently connected to the girders.

Since the membrane possesses a far greater cross-sectional area in the direction of the wind force than do the floor girders and, since the floor membrane usually con-

sists of a poured concrete slab which tightly surrounds all columns, it is to be assumed that the far greater amount of the shear forces does not flow through the girders at all, but is directly introduced into the columns by the floor membrane. For these reasons, it is felt that neglecting this drift component in subsequent discussions is justified.

The total drift of the planar rigid bent will therefore be the sum of the three components discussed previously: girder moment drift, column moment drift, and column chord drift, or

$$D_T = D_G + D_{C'} + D_{C''} \quad (22)$$

Drift Adjustments—At times, the total drift of the bent will exceed the desirable amount as required by code or established engineering practice. The usual procedure in such cases has been to increase the stiffness of the girders only until a satisfactory drift performance is obtained. It was argued that experience has shown that the greatest benefits and the best economies could be achieved by adjusting only the girder stiffnesses.

This is only partly true. As the ratio H/L increases, the axial column forces due to wind increase rapidly.

$$\phi_{C''} = \sum_{i=1}^{i=n} \left\{ \frac{6 \left(\frac{i-0.5}{n} \right) \left[\left(1 - \frac{i-0.5}{n} \right)^{(a+2)} - \left(1 - \frac{i-0.5}{n} \right) (a+2) + (a+1) \right]}{(a+1)(a+2) \left[b_{C''} + (n - b_{C''}) \left(\frac{i-1}{n-1} \right)^{(c_{C''})} \right]} \right\} \quad (21)$$

It is possible to find slender steel frames where the column drift alone, based on stress-designed column sections, exceeds the total specified drift for the frame.

Obviously, no amount of increase in girder stiffnesses will remedy this situation. In cases where the total column drift does not yet exceed the total desirable drift, it would still be necessary to "pack" the girders to a wasteful extent to obtain satisfactory drift readings. The results would be an unnecessarily uneconomical steel bent.

In this section, a reasonable and economical method of optimized drift adjustment will be described.

Referring to Fig. 16, if a stiffness distribution (column and/or girder) of $K_1-K_i-K_n$ will result in a total drift D , then a stiffness distribution (column and/or girder) of $\eta K_1-\eta K_i-\eta K_n$ will result in a total drift D/η . This drift adjustment factor η is assumed to be constant for the total bent. This procedure will not represent an ultimate drift adjustment since the same adjustment factor, η , is used for the entire bent. Yet, no appreciable savings have been realized by establishing a level adjustment factor, η_i . By mathematical optimization, one adjustment factor for columns, η_c , and one adjustment factor for girders, η_g , is obtained.

One more assumption is made, that the ratio of radius of gyration of all members before adjustment to radius of gyration of all members after adjustment will remain fairly constant. Using a bar over the symbol to signify conditions after drift adjustment, this assumption can be expressed in the following form:

$$\frac{\bar{r}_G}{r_G} \approx \text{constant} \quad \text{and} \quad \frac{\bar{r}_C}{r_C} \approx \text{constant}$$

After-adjustment conditions then become:

$$\bar{I} = \eta I \quad (23)$$

$$\bar{A} = \eta A \left(\frac{\bar{r}}{r} \right)^2 \quad (24)$$

The following relationships can be stated:

Adjusting Eq. (14):

$$\bar{D}_G = \frac{D_G}{\eta_G} \quad (25)$$

Adjusting Eq. (17):

$$\bar{D}_{C'} = \frac{D_{C'}}{\eta_C} \quad (26)$$

Adjusting Eq. (20):

$$\bar{D}_{C''} = \frac{D_{C''}}{\eta_C} \left(\frac{\bar{r}_C}{r_C} \right)^2 \quad (27)$$

Let $q_{C(n)}$ denote the total material weight of the columns below level n and $q_{G(n)}$ denote the total material weight of the girders at level n , the total material weight of level n will be: $q_{T(n)} = q_{C(n)} + q_{G(n)}$. The after-adjustment weight will appear as:

$$\bar{q}_{T(n)} = q_{C(n)} \eta_C \left(\frac{r_C}{\bar{r}_C} \right)^2 + q_{G(n)} \eta_G \left(\frac{r_G}{\bar{r}_G} \right)^2 \quad (28)$$

The total frame drift before adjustment is shown in Eq. (22). If \bar{D}_T denotes the total drift of the bent after adjustment, the after-adjustment equation can be expressed as:

$$\bar{D}_T = \frac{D_G}{\eta_G} + \frac{D_{C'}}{\eta_C} + \frac{D_{C''}}{\eta_C} \left(\frac{\bar{r}_C}{r_C} \right)^2 \quad (29)$$

Resolving Eq. (29) for η_G and introducing it into Eq. (28) will show the general statement of the total material weight for level n after adjustment:

$$\bar{q}_{T(n)} = \left[q_{C(n)} \eta_C \left(\frac{r_C}{\bar{r}_C} \right)^2 \right] + \left[\frac{q_{G(n)} D_G \left(\frac{r_G}{\bar{r}_G} \right)^2}{\bar{D}_T - \frac{1}{\eta_C} \left[D_{C''} \left(\frac{\bar{r}_C}{r_C} \right)^2 + D_{C'} \right]} \right] \quad (30)$$

This expression has to be optimized to obtain the desired material adjustment economies:

$$\frac{d(q_{T(n)})}{d(\eta_C)} = 0$$

Resolving the result of this optimization for η_C will then yield the column drift adjustment factor (see Eq. (31) below.)

As mentioned earlier, the adjustments to the girders are usually of greater magnitude than the adjustments to the columns. Furthermore, of the two column drift components, the column moment drift customarily is of a pronouncedly greater importance than the column chord drift (very slender frames excepted). Also, in structural steel W-shapes, the radius of gyration is of almost constant proportion to the outside dimension of the shapes. Finally, the column adjustment requirements normally

$$\eta_C = \frac{\left[D_{C'} + \left(\frac{\bar{r}_C}{r_C} \right)^2 D_{C''} \right] + \left(\frac{r_G}{\bar{r}_G} \right) \left(\frac{\bar{r}_C}{r_C} \right) \sqrt{\frac{q_{G(n)}}{q_{C(n)}} D_G \left[D_{C'} + \left(\frac{\bar{r}_C}{r_C} \right)^2 D_{C''} \right]}{\bar{D}_T} \quad (31)$$

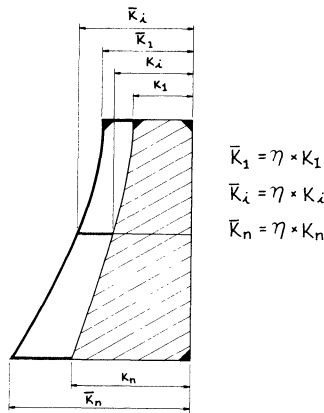


Figure 16

do not increase the outside dimensions of the column shapes. The ratio of the radii of gyration of the columns before adjustment to after adjustment will indeed be close to unity. For these reasons, it is recommended to assign unity to this ratio. With this assumption, Eq. (31) will simplify into:

$$\eta_c = \frac{D_c + \frac{r_g}{\bar{r}_g} \sqrt{\frac{q_{G(n)}}{q_{C(n)}}} D_G D_C}{\bar{D}_T} \quad (32)$$

Equation (32) is included as a working equation in Table 1.

Introducing this column drift adjustment factor into Eq. (23) results in the required drift adjusted column sizes. This means that all the designer has to do is to multiply the moments of inertia of all stress-designed columns in the entire bent by the same factor, η_c , to obtain the moments of inertia of economically drift-adjusted columns.

If Eq. (32) should yield a value for η_c smaller than unity, that would mean that stress design governs the sizes of the columns and no drift adjustment will be made to the column sections.

The establishment of the girder drift adjustment factor, η_g , will not present any difficulties. Equation (25) can be rearranged to read $\eta_g = D_G/\bar{D}_G$. In this expression, D_G can also be expressed as $\bar{D}_G = \bar{D}_T - \bar{D}_C$, in which, from Eq. (26), $\bar{D}_C = D_C/\eta_c$. Substituting in the expression above for η_c , will result in:

$$\eta_g = \frac{D_G}{\bar{D}_T - \left(\frac{D_C}{\eta_c}\right)} \quad (33)$$

Equation (33) is included as a working equation in Table 1.

In Eq. (33), the expression η_c will be either the result of Eq. (32) or unity, whichever of the two values is larger.

In most cases, it will be advisable to use a ratio of $r_g/\bar{r}_g = \text{unity}$ in Eq. (32) to obtain a preliminary column drift adjustment factor. Then introduce this factor (or unity if η_c is smaller than 1) into Eq. (33) to obtain a preliminary girder drift adjustment factor. Then multiply the moments of inertia of the unadjusted girders at level n by this factor to obtain preliminary adjusted girder sizes. Now establish the proper ratio r_g/\bar{r}_g to be used in the final determination of η_c in Eq. (32). A second adjustment will rarely be necessary.

The adjusted girder sizes are obtained by multiplying the moments of inertia of all stress-designed girders in the entire bent by the same factor η_g to obtain the moments of inertia of economically drift adjusted girders. The adjusted drift of the total bent will amount to:

$$\bar{D}_T = \left(\frac{D_G}{\eta_g}\right) + \left(\frac{D_C}{\eta_c}\right) \quad (34)$$

PLANAR BRACED BENT

The establishment of drift and drift adjustment statements for braced bents will follow about the same pattern as was used in rigid bents. Triangulated braced bents, however, resolve all forces axially. No primary bending will occur and no points of contraflexure exist.

The components contributing to the lateral motion of the braced bent are:

- (a) Bracing Drift (caused by axial deformation of the bracing).
- (b) Column Chord Drift (caused by axial deformation of the columns).
- (c) Girder Axial Drift (caused by axial deformation of the girders).

Handling last things first, it can be stated that the argument in favor of neglecting girder axial drift is still valid. Also, the assumption of Fig. 14 concerning the establishment of column chord drift is considered accurate enough to be incorporated into the braced bent. Equations (20) and (21) will, therefore, also apply in the case of the braced bent. Therefore, only the bracing drift remains to be established.

Planar Cross-Braced Bent—A typical panel of this type is illustrated in Fig. 17. The assumption has been made that all effective bracing members are to be stressed in tension only and that, for this reason, the bracing members descending from the windward side to the leeward side will buckle under compressive stresses and become totally ineffective.

Again, some idealizing assumptions will facilitate the calculations:

- (1) Any point of intersection of frame components is a pinned joint, allowing unrestrained rotation of any joining frame component.

(2) No change in column lengths will occur.

In Fig. 17, β_k denotes an unspecified proportion of the total panel load $V_{B(i)}$; $R_{D(k,i)}$ denotes the axial force in the diagonal caused by the panel load; $A_{D(k,i)}$ denotes the cross-sectional area of the diagonal; Δ_D denotes the deformation due to axial stress in the diagonal; and d_D denotes the contribution of level i to the total amount of bracing drift for the whole frame. Since all panel points at the same level under service conditions have to retain their distance from each other, the value d_D must be a constant for each individual level.

Geometrically, the following proportions can be read from Fig. 17:

$$d_D = \Delta_D \left(\frac{l_{D(k)}}{\alpha_k L} \right) \quad \text{and} \quad R_{D(k,i)} = \beta_k \left(\frac{l_{D(k)}}{\alpha_k L} \right) (V_{B(i)})$$

The elastic deformation in the diagonal will be:

$$\Delta_D = \frac{R_{D(k,i)} l_{D(k)}}{EA_{D(k,i)}}$$

Resolving these three equations for d_D :

$$d_D = \frac{\beta_k V_{B(i)}}{E \left[\frac{A_{D(k,i)} (\alpha_k L)^2}{(l_{D(k)})^3} \right]}$$

The argument used to establish β_k in rigid frames also applies here; $\beta_k = K_{D(k,i)} / T_{D(i)}$, when inserted into the previous equation, will state the contribution of level i to the total bracing drift of the whole frame:

$$d_D = \frac{V_{B(i)}}{ET_{D(i)}} \quad (35)$$

The total bracing drift is obtained by taking the following steps:

(1) Express the material property $T_{D(i)}$ as prescribed in Eq. (7).

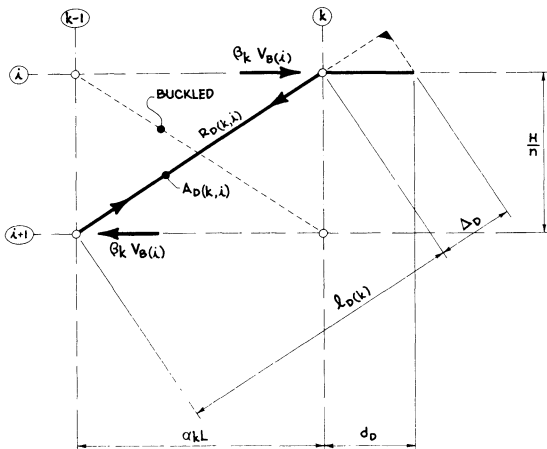


Figure 17

(2) Formulate the building shear as per Eq. (5) by the substitution of $1 - [(i - 0.5)/n]$ for ρ to obtain $V_{B(i)}$.

(3) Insert the results of steps (1) and (2) above into Eq. (35) and form the summation of Eq. (35) from $i = 1$ to $i = n$. The equation for total bracing crossbracing drift then becomes:

$$D_D = \frac{w_1 H n}{ET_{D(n)}} (\phi_D) \quad (36)$$

or, expressed in customary design units (see Table 1),

$$D_D = \frac{w_1 H n}{2417 T_{D(n)}} (\phi_D) \quad (36a)$$

in which ϕ_D has the same magnitude as $\phi_{C'}$ in Eq. (18). Values of ϕ_D are tabulated in Table 4 for various values of a , $b_{C'}$, and $c_{C'}$. They represent frame constants.

The total drift of the planar cross-braced bent will be the sum of the bracing drift component and the column chord drift component, or

$$D_T = D_D + D_{C'} \quad (37)$$

Planar K-Braced Bent—A typical panel of this type is illustrated in Fig. 18. Both the tensile and the compressive diagonals will be needed to justify the idealizing assumptions used in the cross-braced frame analysis. The notations used here are the same as for the cross-braced frame.

Geometrically, the following proportions can be read from Fig. 18:

$$d_D = \Delta_D \left(\frac{2l_{D(k)}}{\alpha_k L} \right) \quad \text{and} \quad R_{D(k,i)} = \beta_k \left(\frac{l_{D(k)}}{\alpha_k L} \right) (V_{B(i)})$$

The elastic deformation in the diagonal will be:

$$\Delta_D = \frac{R_{D(k,i)} l_{D(k)}}{EA_{D(k,i)}}$$

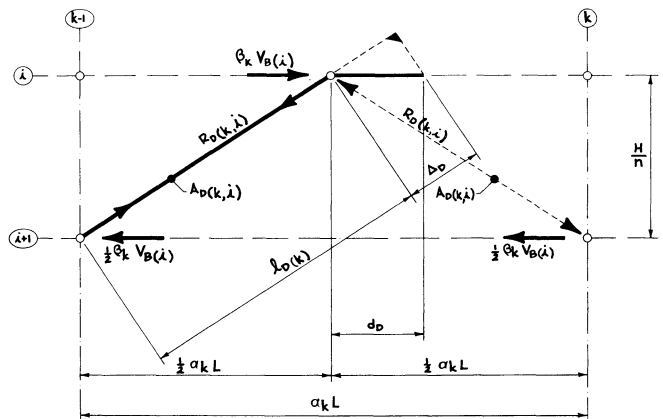


Figure 18

Resolving these three equations for d_D and using the familiar expression for β_k again will yield a statement on the contribution of level \mathbf{i} to the total bracing drift of the whole frame:

$$d_D = \frac{2V_{B(i)}}{ET_{D(i)}} \quad (38)$$

Comparing Eq. (38) for K-braced bents with Eq. (35) for cross-braced bents will show that the only notation difference between these two equations is the factor 2 in the numerator of Eq. (38). Since the further development of the K-braced type will follow exactly the treatment given to the cross-braced condition, the only difference in the final statement will be this multiplier, 2. For this reason, it will be possible to immediately state the total bracing drift of the whole bent for K-braced conditions:

$$D_D = \frac{2w_1Hn}{ET_{D(n)}} (\phi_D) \quad (39)$$

or, expressed in customary design units (see Table 1),

$$D_D = \frac{2w_1Hn}{2417T_{d(n)}} (\phi_D) \quad (39a)$$

The values for ϕ_D have the same magnitude as $\phi_{c'}$ in Eq. (18) and represent frame constants. See Table 4.

The total drift of the planar K-braced bent will be the sum of the components, as previously expressed for cross-braced bents:

$$D_T = D_D + D_{c''} \quad (37)$$

Drift Adjustments for Braced Bents—The drift adjustment procedure for braced frames is basically the same as for rigid bents. The difference consists in the fact that all exterior forces in the braced bent are resolved into axial forces in the components of the bent. No primary bending takes place. For this reason, stiffness of members becomes irrelevant. In the ensuing calculations, the drift adjustment techniques will only be based on the cross-sectional areas of the frame components. The adjustment condition will, therefore, read:

$$\bar{A} = \eta A \quad (40)$$

in which the bar over the symbol again denotes the condition after drift adjustment. The following relationships can now be stated:

Adjusting Eq. (36) or (39):

$$\bar{D}_D = D_D/\eta_D \quad (41)$$

Adjusting Eq. (20):

$$\bar{D}_{c''} = D_{c''}/\eta_C \quad (42)$$

Letting $q_{C(n)}$ denote the total material weight of the columns below level \mathbf{n} and $q_{D(n)}$ denote the *total* (not only the tension members) material weight of the bracing below level \mathbf{n} , the total material weight before adjustment

would be $q_{T(n)} = q_{C(n)} + q_{D(n)}$. The after-adjustment weight will then appear as:

$$\bar{q}_{T(n)} = q_{C(n)}\eta_C + q_{D(n)}\eta_D \quad (43)$$

The total frame drift before adjustments is shown in Eq. (37). If \bar{D}_T denotes the total drift of the bent after adjustment, Eq. (37) will convert into:

$$\bar{D}_T = \frac{D_D}{\eta_D} + \frac{D_{c''}}{\eta_C} \quad (44)$$

Resolving Eq. (44) for η_D and introducing it into Eq. (43) will result in the general statement of the total material weight for level \mathbf{n} after adjustment:

$$\bar{q}_{T(n)} = q_{C(n)}\eta_C + \frac{q_{D(n)}D_D}{\left(\bar{D}_T - \frac{D_{c''}}{\eta_C}\right)} \quad (45)$$

This expression must be optimized to obtain the desired material adjustment economies:

$$\frac{d(\bar{q}_{T(n)})}{d(\eta_C)} = 0$$

Resolving the result of this optimization for η_C will then yield the column drift adjustment factor:

$$\eta_C = \frac{D_{c''} + \sqrt{\frac{q_{D(n)}}{q_{C(n)}} D_D D_{c''}}}{\bar{D}_T} \quad (46)$$

The adjustment procedure consists of multiplying the cross-sectional areas of all stress-designed columns by the same factor, η_C , to obtain the cross-sectional areas of economically drift adjusted columns. If Eq. (46) should yield a value for η_C that is smaller than unity, it would indicate that stress design governs the size of the columns and no drift adjustment will be made to the column sections.

To establish the bracing drift adjustment factor, η_D , first rearrange Eq. (41) to read $\eta_D = D_D/\bar{D}_D$. In this expression, \bar{D}_D can also be stated as $\bar{D}_D = \bar{D}_T - \bar{D}_{c''}$ in which, from Eq. 42, $\bar{D}_{c''} = D_{c''}/\eta_C$. Then the bracing drift adjustment factor can be expressed as:

$$\eta_D = \frac{D_D}{\bar{D}_T - \left(\frac{D_{c''}}{\eta_C}\right)} \quad (47)$$

The economically adjusted bracing member sizes are obtained by multiplying the cross-sectional areas of all stress-designed bracing members in the entire frame by the same factor η_D .

The adjusted drift of the total bent will amount to:

$$\bar{D}_T = \frac{D_D}{\eta_D} + \frac{D_{c''}}{\eta_C} \quad (48)$$

MATERIAL WEIGHT ESTIMATING

The techniques used to express the ratios and relationships of material properties as expressed in Eq. (7) can, of course, also be applied to obtain accurate material unit weights for the total steel frame.

Let q_n denote a material unit weight at level n . The average material unit weight per level for the entire steel bent will then be:

$$q_{avg.} = \frac{q_n}{2}(\phi_q) \quad (49)$$

in which the weight correction factor ϕ_q , applying Eq. (7), has the value:

$$\phi_q = \frac{2}{n^2} \sum_{i=1}^{i=n} \left[b_q + (n - b_q) \left(\frac{i-1}{n-1} \right)^{c_q} \right] \quad (50)$$

Values of ϕ_q are tabulated in Table 6 for various values of b_q and c_q . They represent frame constants.

CONCLUSIONS

It is firmly believed that the accuracies obtained by the use of this simplified method fully justify its application in the very important field of preliminary design and drift estimations. Recommendations to use this method for the final design of steel frames are not necessarily

made at this time; more experience in its use might be necessary. The user is invited to exercise his own judgment in this decision.

In retrospect, it becomes quite apparent that frame components do not exist by themselves but are, in their performance, very much a part of their group. The summation expressions which are encountered throughout the entire investigation of the high-rise frame bear witness to this fact.

More future emphasis on the development and use of standards for multi-component design would appear to be very desirable and necessary.

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