Stiffened Thin Shell Domes

KENNETH P. BUCHERT

This paper was presented at the AISC National Engineering Conference, Omaha, Nebr. in May, 1964.

THE DESIGN OF ever larger clear span roofs is a challenge to structural engineers, architects, fabricators and steel producers. The type of roof proposed in this paper is a thin steel shell that is either reinforced with stiffeners or is made of two thin shells that form a double or "sandwich" shell. It is proposed that this type of roof could be used as an economical solution for covering stadiums, tanks, supermarkets, shopping centers and eventually to cover significant areas of cities.

The use of shell type structures has increased significantly in recent years. The trend in shell construction has been toward larger and thinner shells. As shells are made larger and thinner, elastic buckling becomes one of the most important design considerations and construction cost becomes of prime importance. Actually elastic buckling has been a major factor in limiting the size of certain types of shells.

One of the promising methods of raising the buckling load and reducing cost is to reduce the shell thickness and add a relatively small amount of material to the shell in the form of stiffeners. The stiffeners raise the buckling load by increasing the bending resistance of the shell while still maintaining membrane shell action. In addition the stiffeners make it possible to erect large prefabricated units without falsework or with a minimum of falsework. Each segment of a stiffened shell can be handled much more easily than an unstiffened shell segment, and preliminary studies indicate that the stiffened segments could be erected by a cantilever method. The type of shell that is proposed by the author is described herein. The following paragraphs give a brief description of the important design considerations. The details of the shell selected will depend to a large extent on the type of erection procedure that is to be used. The Bibliography at the end of this paper gives a very brief list of references that may be used for reviewing the development of the concept of stiffened shells.

Kenneth P. Buchert is Associate Professor, Department of Civil Engineering, University of Missouri at Columbia.

DESIGN CONSIDERATIONS

The professional engineer is interested in the lowest cost roof and in practical structures. Although the structure proposed will be relatively light in weight, this is not a major consideration in the suggested application.

Any engineered structural research project can be divided into the following three areas:

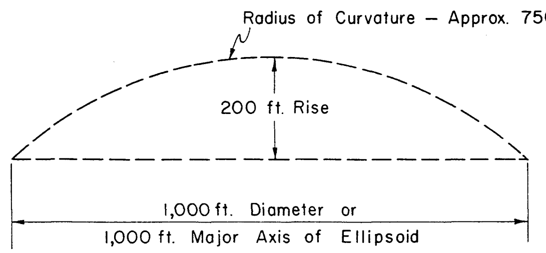
1. Strength
2. Effective Modulus
3. Geometry

There has been a multitude of good work done in recent years in developing high strength steels. The stresses in the structure proposed for large roofs are relatively small for roofs up to about 1,000 ft in base diameter; therefore mild steel is usually adequate to meet the strength requirements. For shells greater than 1,000 ft in diameter high strength steels could be used to advantage.

Many hybrid or composite structures other than those now in use or under development show considerable promise for future development. There is reason to believe that such structures, when combined with new geometric concepts, could be used for future roofs, building and bridge flooring systems, and girders. These concepts are subjects in themselves and are not discussed herein. As a result only the elastic modulus of steel is used in the work that follows. It is proposed that the higher strength steels be used for large roofs, rather than using low strength steels and the reduced effective modulus.

Although geometry frequently offers the most economical solution to a given problem, it is often the most neglected because new shapes require new structural philosophies as well as new mathematical simplifications. The shell roof proposed is old in many respects but is also new in geometrical detail and therefore requires new thinking in its use, design, fabrication and erection.

One of the most structurally efficient geometries is the doubly curved shell (i.e., spherical, elliptical, paraboloid of revolution, etc.). The doubly curved shell



Roof Dimensions for a Large Stadium or Shopping Center

Figure 1

when properly engineered is basically a membrane structure, subject to direct stresses, and is very efficient as far as stress level is concerned. In many shells the limiting factor is the critical buckling load. It is in this regard that geometry offers an area of seemingly unlimited possibilities. Fig. 1 shows a roof that might be considered for a large stadium or small shopping center. It has a base diameter of 1,000 ft and a rise of 200 ft. The loads on the roof are assumed to be 30 psf live load and 20 psf insulation load in addition to the weight of the roof itself. If this roof were designed as an unstiffened shell, buckling would be one of the major considerations and would determine the shell thickness.

How can one change the local geometry of the shell, still maintain the high shell membrane efficiency, and produce an economical roof? Professional structural engineers have faced similar problems with flat plates and cylindrical shells. In the cases of plate girders, aircraft and missile structures, and external pressure vessels, very economical structures have resulted from using a relatively thin plate or a cylindrical shell and adding stiffeners. Will the same economy result if stiffeners are added to the membrane shell roof? In order to approach this problem in an engineering way, the following steps must be carried out:

1. A theoretical solution for the stiffened shell must be completed.
2. The theory must be verified by testing laboratory models.
3. Full scale structures must be designed.
4. The economy of the new full scale structures must be investigated, experience factors evaluated, and the design justified.

Step 1 is essential in the case of shells where stability considerations govern the design. Model tests alone are often adequate where stress and deflection levels govern, but lead to questionable conclusions where stability considerations are important. The author has investigated the first three steps.

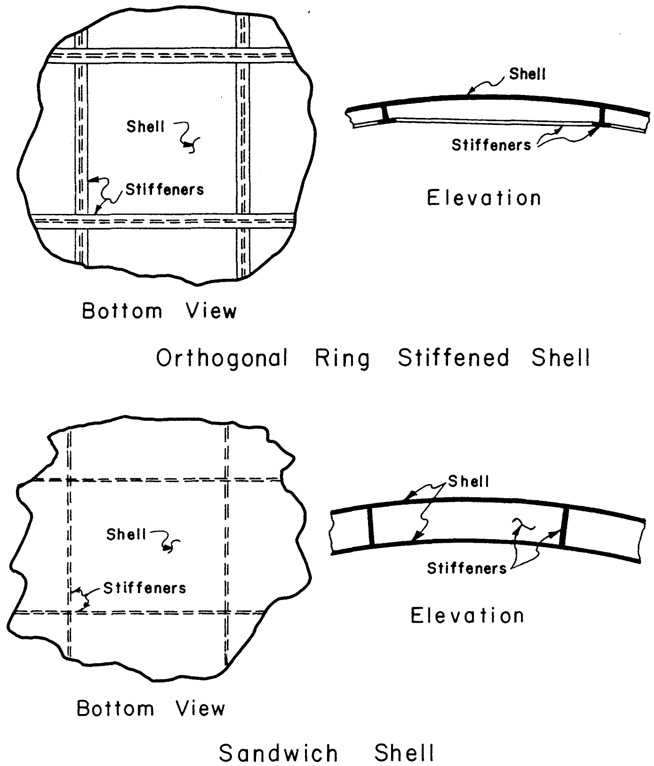


Fig. 2. Typical stiffened shells

THEORETICAL SOLUTIONS

There are many ways in which a shell may be stiffened. A shell may be stiffened by using orthogonal rings or by using a sandwich or double shell (see Fig. 2). Stiffened shells may be investigated using the split rigidity concept. That is, one can compute an effective bending thickness and an effective membrane thickness for a shell and then proceed to analyze the virtual unstiffened shell. The theoretical critical buckling pressure or load per unit area of a stiffened shell is given by *

$$P_{cr} = 0.366E\left(\frac{t_m}{R}\right)^2\left(\frac{t_b}{t_m}\right)^{3/2} \tag{1}$$

where P_{cr} = critical pressure or load
E = modulus of elasticity
t_m = effective membrane thickness
t_b = effective bending thickness
R = radius of curvature

The equation shows that very great efficiencies are possible because, with proper prudent engineering design, t_b can be made relatively large and t_m can be made relatively small.

* *The equations presented are derived from those given in the author's Ph.D. dissertation.*

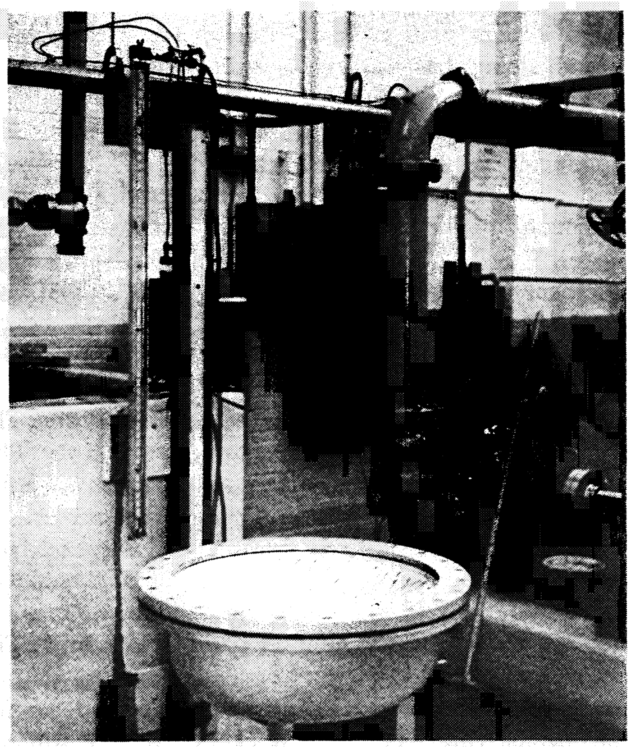


Fig. 3. Laboratory assembly

Designs of full scale roofs have shown, for example, that the buckling load of an unstiffened shell may be increased by a factor of 3 to 5 by adding 5 to 25 percent of the weight of the shell in stiffeners. The solution given in equation (1) was made in general form so that as different designs are developed in the future, effective bending and membrane thicknesses may be calculated and the critical load found with relative ease, accurately enough for engineering design purposes.

Referring again to Fig. 1, one could consider using a shell with orthogonal ring stiffeners. At this point several questions arise. How does one apply the theoretical solution to this geometry? How close should the stiffeners be? Won't the shell buckle between stiffeners? Is the lightest structure the most economical one?

The composite critical buckling load (buckling of shell and stiffeners) for a shell stiffened with orthogonal rings is

$$P_{cr} = 0.366E\left(\frac{t}{R}\right)^2\left(1 + \frac{12I}{dt^3}\right)^{1/2}\left(1 + \frac{A}{dt}\right)^{1/2} \quad (2)$$

where d = distance between rings
A = area of the stiffener that is added to the shell
I = effective moment of inertia of the stiffener
t = actual thickness of the shell

If orthogonal stiffeners are used, one must place the stiffeners close enough together so that the shell does not buckle between stiffeners. The theoretical buckling load for this type of buckling is

Fig. 4 Special edge ring

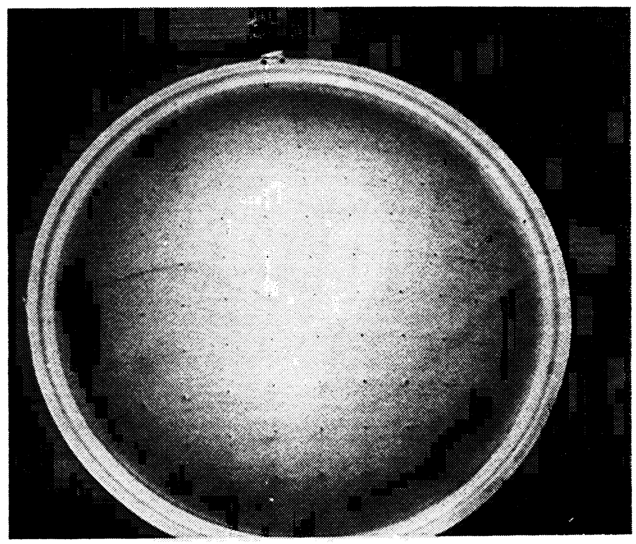


Fig. 5. Unstiffened shell with center buckle

$$P_{crL} \approx 7.42\frac{Et^3}{Rd^2} \quad (3)$$

where P_{crL} = the critical load.

LABORATORY TESTS

The second step in the development was to conduct laboratory model tests. These tests proved to be most successful in that both equations (2) and (3) were verified.

The difference between theory and experiment was less than 10 percent for composite buckling and less than 20 percent for local buckling. The experiments also confirmed the importance of preventing buckling between stiffeners. Buckling pressures of less than one-fourth of that given by equation (2) have been demonstrated when local buckling occurs prior to the theoretical composite value. Fig. 3 shows the laboratory assembly used for the tests. The shells were tested using hydrostatic

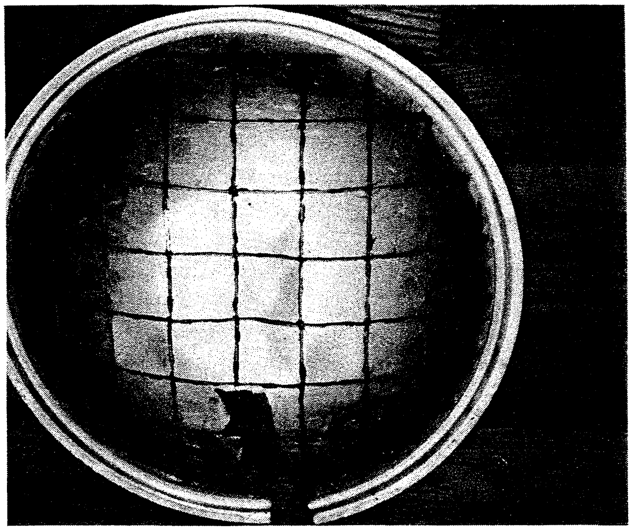


Fig. 6. Stiffened shell designed to buckle locally prior to composite buckling

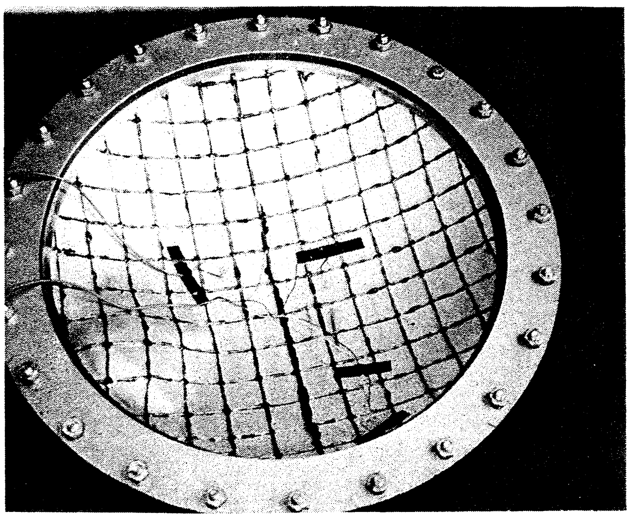


Fig. 7. Buckled stiffened shell

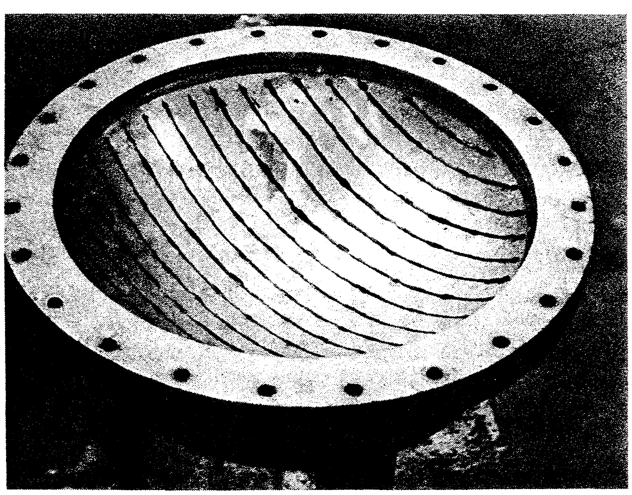


Fig. 8. Buckled stiffened shell with orthogonal rings on both sides of shell

pressure and the pressure differentials were measured using a differential mercury manometer. Fig. 4 shows an edge ring that was used to stiffen the shell edge to prevent the undesirable edge effects that have been present in many model and full scale tests that have been conducted previously on unstiffened shells. A similar stiffening effect may be obtained in a practical full scale structure by the prudent design of the shell near the edges. Fig. 5 shows an unstiffened shell with the symmetrical buckle in the center. Figs. 6, 7 and 8 show buckled stiffened specimens.

FULL SCALE STRUCTURES

The third step was to design a full scale structure. The lightest structure will not be the most economical one. This opinion is based upon previous experience with large full scale stiffened cylindrical shells and plate girders. A ½-in. thick shell for the roof shown in Fig. 2 was selected for reasons of economy. Theoretically the shell could be much thinner. With the help of the experienced fabricator a more economical solution could be obtained. The structural-T stiffeners (¼ x 12-in. webs and ½ x 10-in. flanges on 4-ft centers) acting with the shell form the orthogonal rings. The shell thickness and the stiffener size were selected in order to provide a minimum theoretical factor of safety of three against buckling between stiffeners and also against composite shell and stiffener buckling.

At this stage one begins to look at the economy of the structure and many questions and ideas present themselves. Won't the shell be expensive to erect if it is necessary to shore the structure during erection? Why not erect the shell by the cantilever method with no shoring? A brief study of this problem has indicated that the cantilever method appears feasible because of the relatively high bending stiffness of the stiffened shell segments.

Are full penetration butt welds in the shell required? Are lap welds satisfactory? What is the minimum amount of weld required between the stiffeners and shell? Do the intersections of the orthogonal rings need to be welded? Engineered structural research in a few of these areas is underway at present and the results to date have been promising. The theory needs to be checked by experiments. The rate of progress is very slow because of lack of funds.

Although the weight of a structure often bears little resemblance to cost, a weight curve is shown in Fig. 9. It should be pointed out here that there are many types of stiffened shells possible. Therefore the weight can vary considerably. Curves for steel framed domes, presented by Mr. G. Odom at the 1963 AISC National Engineering Conference, and for the stiffened shell are shown. The circles represent roofs that have been built or are under construction. The relative weights are not too significant because the unit cost of the two

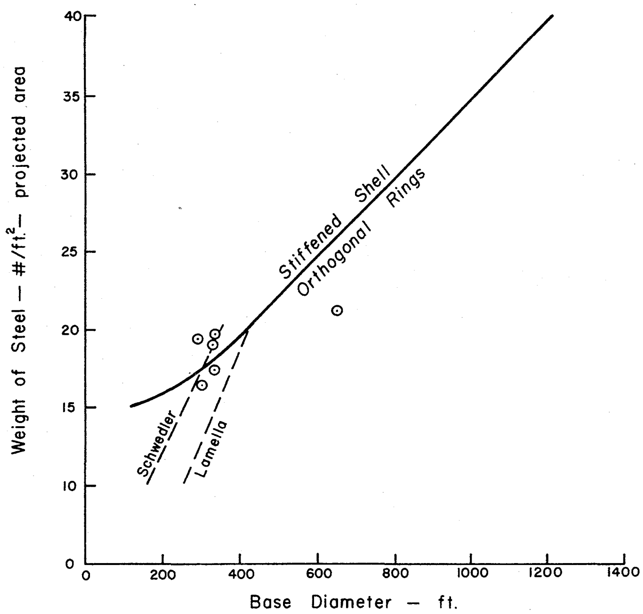


Figure 9

types of designs could be quite different. In addition the stiffened shell automatically provides a waterproof covered roof, whereas additional material must be added to the steel framed roof to provide a covered roof. The curves do indicate that the stiffened shell roof can be designed so that the weight will be the same as, or less than, that of the framed dome.

What kind of a roof might be designed and built ten years from now? How could a large shell be used? Fig. 10 is a copy of a painting made by artist Ed Collings. This painting illustrates "University 1975". The shell in the foreground would be about one mile in diameter and about 400 ft high at the center. Please note that the entire area could be air-conditioned and interior walls and ceilings would not be required to be insulated, resist wind or be waterproofed. In many cases walls would be eliminated. How thin would this "thin shell" be? Some preliminary calculations indicate that a sandwich shell 16 ft thick might be appropriate. The sandwich would consist of two 2-in. thick alloy steel shells (yield point about 100,000 psi) separated by trusses on about 150 ft centers. It is interesting to note that in this case the point where stability no longer governs has been passed. Here the shell is operating at its maximum efficiency and is the lightest structure obtainable.

Obviously considerable work must be done prior to designing and building shells with bases of 1,000 ft or miles. In addition to the items listed previously, the author proposes testing a "large model" shell that would have a base diameter of 60 ft and a thickness of about ¼-in. After the test a detailed set of design rules should be formulated and then applied to the larger shells.

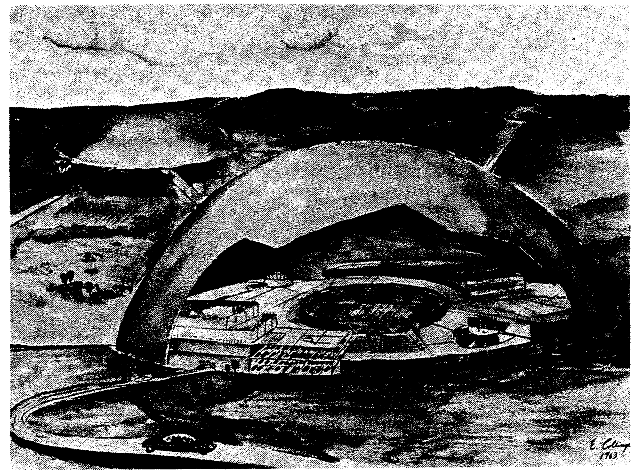


Fig. 10. University 1975

The clear span roof of the future is a challenge for the structural engineer, the architect, the steel fabricator and the steel producer. It seems that roofs are getting larger and larger and structures of this general type will be built. Will they be made of steel or of some other material?

BIBLIOGRAPHY

1. *Buchert, K. P.* Stability of Doubly Curved Stiffened Shells, *Dissertation, University of Missouri at Columbia, Jan. 1964.*
2. *Klöppel K. and Jungbluth, O.* Beitrag zum Durchschlagproblem dunnwandiger Kugelschalen, *Der Stahlbau, Berlin, June 1953, pp. 121–130.*
3. *Von Karman, T., and Tsien, H. S.* The Buckling of Spherical Shells by External Pressure, *Journal of the Aeronautical Sciences, Vol. 7, No. 2, 1939, pp. 43–50.*
4. *Friedrichs, K. O.* On the Minimum Buckling Load for Spherical Shells, *Theodore von Karman Anniversary Volume, Pasadena, Calif., 1941, pp. 258–272.*
5. *Csonka, P.* The Buckling of a Spheroidal Shell Curved in Two Directions, *Acta Technika, Academic Scientiarum Hungaricae (Budapest), Vol. 15, No. 3–4, 1956, pp. 425–437.*
6. *Herber, Karl-Heinz* Bemessung von Rippenkuppeln und Pippenschalen fur Tankdacher, *Der Stahlbau, Berlin, Sept. 1956, pp. 216–225.*
7. *Klöppel, K. and Roos, E.* Beitrag zum Durchschlagproblem dunnwandiger versteifter und unversteifter Kugelschalen fur Voll- und halbseitige Belastung, *Der Stahlbau, Berlin, March 1956, pp. 49–60.*
8. *Mitchell, L. H.* A Shell Analogy for Framed Domes, *Structures and Materials Note 208, Aeronautical Research Laboratories, Department of Supply (Melbourne, Australia), Dec. 1953.*
9. *Zick, L. P. and Carlson, C. E.* Vacuum Test of Sphere to Failure, *The Water Tower, Nov. 1947, pp. 6–7.*
10. *Thompson, J. M. T.* The Elastic Instability of a Complete Spherical Shell, *Aeronautical Quarterly, Vol. XIII, Part 2, May 1962.*
11. *Feodosyev, V. I.* On the Stability of a Spherical Shell Under the Action of an External Uniformly Distributed Pressure, *Prikladniya Matematika Mckhanike, Vol. XVIII, No. 1, 1954.*