Vibration and Deflection of Steel Bridges

RICHARD N. WRIGHT AND WILLIAM H. WALKER

This paper was presented at the AISC National Engineering Conference, Cleveland, Ohio, in May 1971.

DESIGNERS WISH to provide highway bridges with adequate strength, service lives uninterrupted by fatigue damage, durable riding surfaces, and comfortable crossings for pedestrians and occupants of moving vehicles. Among the criteria employed in the design of steel multistringer highway bridges are limitations on the slenderness and flexibility of the stringers. These limitations commonly lead to bridge designs which need extra embankment and right-of-way where vertical clearances are tight and prevent economical applications of high strength steel. It is less clear how they contribute to the desired qualities noted above. The objectives of this paper are: (1) to explore the actual effects of stringer flexibility and slenderness on the quality of highway bridges, and (2) to suggest improved criteria which may help the designer achieve a high quality, economical structure.

The limit on slenderness in the AASHO specifications,[1] Sect. 1.7.11, is $L/d \leq 25$, where L is the span between points of contraflexure for dead load and d is the depth of the steel for noncomposite design or the total depth for composite design; for composite design the specifications also require $L/d \leq 30$ where d is the depth of steel. The limit on flexibility in Sect. 1.7.13 is $L/\delta \geq 800$ for rural and non-pedestrian bridges, and preferably ≥ 1000 for urban bridges used in part by pedestrians. The deflection δ is computed for design live loading plus impact and L is the span length between supports. There are no comparable limitations applied to timber, reinforced concrete, or prestressed concrete multistringer bridges.

The ratio of span to depth and span to deflection are not independent, but are related by the expression

$$\delta/L = K\sigma(L/d) \qquad (1)$$

where σ is the flexural stress due to the loading producing δ, and K is a factor depending on the distribution of loading. Examples of K are given by Bresler, Lin, and Scalzi.[2] For highway bridge loadings, K varies only slightly with L, and σ represents the flexural stress due to live load plus impact; σ increases for stronger steels and shorter spans. A limit on L/δ tends to control for shorter spans and a limit on L/d tends to control longer spans. Flexibility and slenderness are often synonymous; the former term usually is used alone here for brevity.

What is accomplished by such deflection criteria? The deflection is limited, but such a limit by itself has no more meaning than would a limit on flexural stress imposed without relation to the stress critical for a mode of failure such as yielding, lateral buckling, or fatigue. What mode or modes of failure or unserviceability are considered in the deflection criteria? A committee of ASCE has investigated this question.[3] They reported in 1958 that no clear basis for the slenderness limit could be found, although they traced its evolution back to the AREA specification of 1905. They traced the limitation on deflection back to a railway bridge specification of 1871 which gave a limit very similar to those currently employed. However, the current limitations are nearly those recommended by the Bureau of Public Roads in the 1930's after a study of steel girder bridges which were reported to vibrate objectionably. Thus, user discomfort seems to be the only mode of behavior considered in the development of current deflection criteria. Recent, more severe limitations on bridge deflection appear to have resulted from concern about deterioration of reinforced concrete decks. Therefore, particular attention is focused on the deterioration of bridge decks and concurrent objectionable vibration.

The effects of flexibility on distribution of live load stresses and deflections and on dynamic response are described in following sections. These responses relate flexibility to the strength and fatigue resistance of the bridge deck and stringers. The influences of flexibility on deck deterioration and on human reaction to bridge motions are also discussed. Design examples illustrate the effects of relaxed deflection criteria on economy and response. Finally, a more rational deflection criterion is suggested for bridges with pedestrian traffic.

This paper is based on the report of an investigation[4] sponsored by American Iron and Steel Institute, Com-

Richard N. Wright is Professor of Civil Engineering, University of Illinois, Urbana, Ill.

William H. Walker is Associate Professor of Civil Engineering, University of Illinois, Urbana, Ill.

mittee of Structural Steel Producers and Committee of Steel Plate Producers, and conducted in the Department of Civil Engineering of the University of Illinois, Urbana. The project Task Force, Messrs. R. R. Gavin, Chairman, R. S. Fountain, W. Henneberger, K. H. Lenzen, C. E. Thunman, Jr., and I. M. Viest, was most helpful in guidance of the investigation. We are particularly grateful to Messrs. Henneberger and Thunman for advice on design practices in their states.

EFFECTS ON RESPONSE TO LIVE LOAD

The flexibility of the stringers of a multibeam highway bridge has significant effects on the transverse distribution of moments and deflections as well as the amplitude of the deflection. These static responses are also related to the dynamic components of response. Sanders and Elleby[5] recently reported an extensive study of the effects of beam flexibility on transverse distribution of moment. Therefore, only a limited-range, illustrative study which includes continuous spans is reported here. It uses the grid idealization of Lightfoot and Sawko.[6]

The parameters which define the bridge characteristics are the support conditions, the number of stringers, the ratio S/L of stringer spacing S to span length L, and the stiffness parameter H. The parameter H is the ratio of the stiffness, E_bI_b, of the stringer (including any composite deck action) to the stiffness EI of the slab for the span length L, i.e.,

$$H = E_bI_b/[ELh^3/12(1 - \nu^2)] \qquad (2)$$

in which E, h, and ν are respectively the Young's modulus, thickness, and Poisson's ratio for the deck slab. In results reported here, Poisson's ratio of the deck is taken as zero. The torsional stiffness of the steel stringer is assumed to be negligible compared to that of the deck. As suggested by Rowe,[7] the torsion constant of the deck is taken as $h^3/6$ per unit length. For cross beams at the quarter points of the span, the properties of the grid members are:

Member	Torsional I	Flexural I
Longitudinal	$\frac{2}{H}\frac{S}{L}I_b$	I_b
Transverse	$I_b/2H$	$I_b/4H$

Notation is illustrated in Fig. 1. The STRESS[8] program was used to carry out the grid analyses.

The analytical studies included solutions for influence coefficients and for distributions of one, two, and three lane HS truck loadings on five-stringer bridges with stringer spacing of 8 ft and spans of 40, 80, and 160 ft for both simple-span and two-span continuous bridges. The relative stringer stiffnesses were defined by $H = 2$,

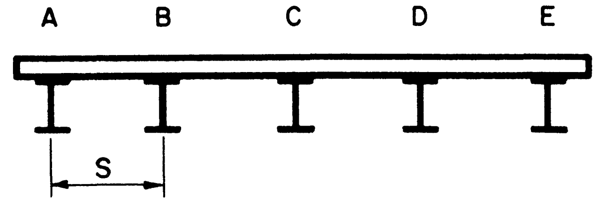

Fig. 1. Notation for bridge cross section

5, 10, and 20, which approach practical extremes of both flexibility and stiffness. These results are representative of flexibility effects in bridges with four or more stringers for the ranges of S/L and H indicated above.

The influence of flexibility of the stringers upon stringer moments follows the trend shown in Fig. 2. The distribution factor for moment is defined like that in the AASHO specifications; when multiplied by the moment for one line of wheels or one-half a lane on a single stringer, it gives the moment in the stringer for the design loading acting on the bridge. The curve shown in Fig. 2 applies to the moment near midspan in the first interior stringer of a bridge continuous over two 80-ft spans and subjected to two lanes of HS truck loading. The trend of reduction of distribution factor for moment with increased flexibility—better distribution of stringer moments—is reported in Refs. 4, 5, 9, and 10 for positive and negative moments, deflections, and shears in interior stringers. The distribution factor for moment in an outer stringer tends to increase slowly with increased flexibility, but the outer stringer moment rarely controls the design.

The transverse moments from bending of the deck slab are of concern because they can cause possibly detrimental cracking from tension at the top of the deck. Longitudinal deck moments are smaller and cause

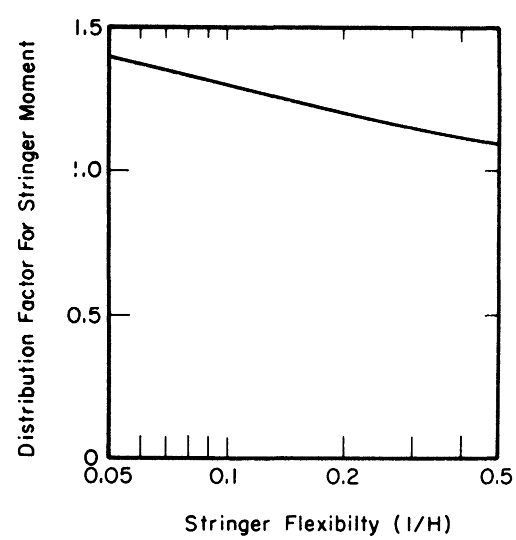

Fig. 2. Effect of flexibility on moment in stringer

compression at the top of the deck. Because damage appears most likely to arise from repeated stressing of the deck, effects of stringer flexibility on transverse moment in the deck are shown in Fig. 3 for a single axle loading. The figure is derived from influence coefficients given by Newmark and Siess[9] for simple span bridges for a span of 80 ft and stringer spacing of 8 ft. The trend of reduction in peak deck moment with increased stringer flexibility also is seen for other spans, spacings, and loadings. The range in deck moments reduces with increased stringer flexibility for a single axle loading, moving longitudinally, but staying in the transverse position for peak negative moment. When multiple truck loadings are considered in variable transverse positions, the range of deck moment increases slowly with increased stringer flexibility.

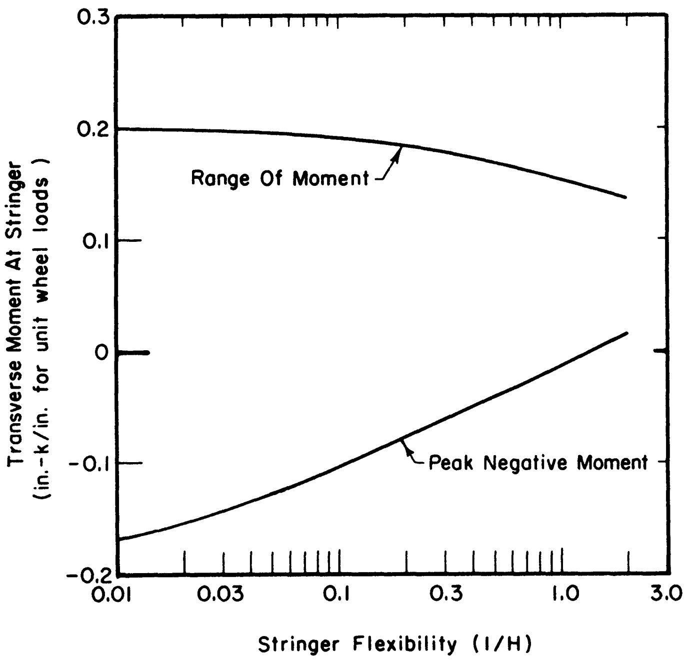

Fig. 3. Effect of flexibility on moment in deck

Human reaction to bridge deflection may be evaluated for a loading of one heavy vehicle, since synchronous multiple-vehicle loadings are too rare to cause frequent complaints of human discomfort. However, the critical transverse position for the one heavy vehicle is difficult to define. The sidewalk may be assumed to be over an outer stringer such as **A** of Fig. 1. A representative view of the influence of stringer flexibility on the distribution of deflection is shown in Fig. 4 for a simple span with $S/L = 0.1$. The deflection coefficient is the multiplier of the deflection computed for one line of wheels acting on a single stringer.

The curves in Fig. 4 show graphically the variation of the deflection coefficient for transverse load position with the relative stringer stiffness parameter H. The ambiguity of the effect of H is evident. High speed vehicles tend to be in the high speed lanes—roughly the deck span **B-C**. For such a position, the distribution to the outer stringer is greater for smaller H. Low speed vehicles in the outer deck span **A-B** have greater distribution for greater H. If a random vehicle location between stringers **A** and **C** is considered, the picture is clarified. Average deflection coefficients between **A** and **C** are nearly independent of H and only slightly dependent on S/L.

A distribution factor of $2 \times 0.35 = 0.70$ lines of wheel loads is suggested for the deflection caused by one vehicle as a conservative representation for a mean transverse position of vehicle for all values of S/L and H. This distribution factor is to be used with high vehicle speed which, as discussed in the next section, leads to maximum dynamic response. It does not seem appropriate to combine high vehicle speed with a vehicle position close to the pedestrian. In such event the pedestrian would be more concerned about being hit than about bridge motions.

EFFECTS ON DYNAMIC RESPONSE

The response of highway bridges to moving vehicles can be separated into static and dynamic components. This separation is illustrated in Fig. 5 by comparing field deflection measurements from the AASHO Road

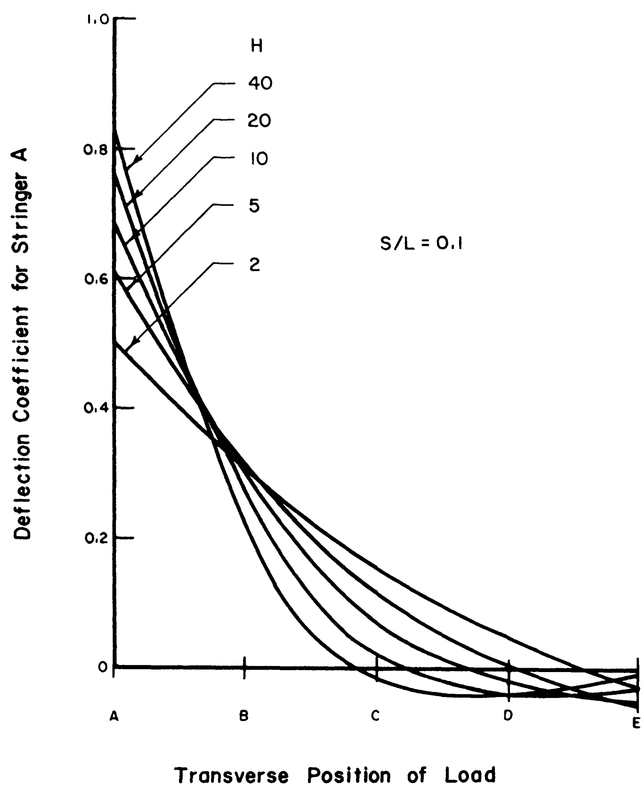

Fig. 4. Effect of flexibility on sidewalk deflection

Test[11] for a vehicle crossing a bridge very slowly (Curve **a**) and at about 50 mph (Curve **b**). The response to static loads is given by the crawl curve (curve **a**); the difference between this and the total response at normal speed is the dynamic component of motion (Curve **c**). When the amplitude of the dynamic component is expressed as a proportion of the peak static response, it is called a *dynamic increment.* Bridge deflections, moments, and shears can be expressed as a static component plus a dynamic component defined by a dynamic increment times the static component. There are two aspects of the effect of stringer flexibility on the dynamic component of response—the effect on the static component, described in the previous section and the effect on the dynamic increment. This section describes the influence of stringer flexibility on the dynamic increments for bridge responses and the total effect on the dynamic component of response.

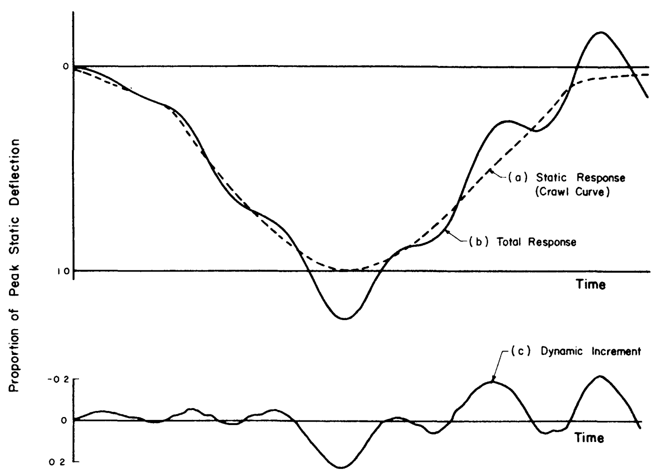

Fig. 5. Deflection-time history for a heavy vehicle

Factors of major concern in the dynamic response of bridges are: (a) dynamic contribution to peak stresses for considerations of strength, (2) dynamic contribution to stress ranges for considerations of fatigue life, and (3) dynamic contribution to the deflection-time history for consideration of human reaction to bridge movement. The effects of stringer flexibility on these responses are considered here. Attention to dynamic stresses is limited to the longitudinal stresses in bridges (those related to the beam moments). The dynamic components of moments in the deck are essentially independent of stringer flexibility.

Dynamic response of the stringers is studied by considering the whole bridge to respond as a unit at any cross section and using distribution factors to account for transversely non-uniform moments, shears and deflections. Extensive research, such as reported by Walker and Veletsos,[12] has shown that this approach gives reliable results. The structural parameters include the dead weight, the flexural stiffness, the span length or lengths, and the support conditions (simple, continuous, etc.) which define the natural modes and natural frequencies of vibration. Ordinarily, vibration in the fundamental mode (lowest natural frequency) predominates in the dynamic response. Unless it is specifically defined otherwise in the following development, the term natural frequency denotes the lowest natural frequency. The roughness of the approach and the roadway surface also are important parameters in the dynamic response of the structure. These qualities are difficult to predict in advance and are more subject to variation during the life of the bridge than the structural parameters which define the natural frequency. Damping is an important parameter in human reaction to motions, but practically attainable damping has little effect on the peak dynamic response of bridges.

The parameters describing the vehicle producing the dynamic response include the number of axles, axle spacing, effective axle loads, tire or tire-spring stiffness, and vehicle speed as parameters that can be evaluated with reasonable confidence. Additional parameters, known to be significant but random in magnitude, are the spring friction and the initial conditions defining the vertical motion of the vehicle on its suspension as it enters the span. Walker and Veletsos[12] and Veletsos and Huang[13] give a thorough analytic evaluation for most of the parameters cited. The analytic procedures were confirmed in evaluation of bridge dynamic response during the AASHO Road Test.[11] The range of bridge properties considered is sufficiently broad to include the properties likely to be encountered with relaxed bridge deflection criteria. The dynamic response is expressed in terms of dynamic increment, which is defined as the difference between the instantaneous value of a dynamic effect and the corresponding value of the static effect, normalized by the maximum static value of that effect. The dynamic increment varies with time; it is conservative but reasonable to design for a peak total response given by (1 + Peak Dynamic Increment) × Peak Static Response. Used in this way, the peak value of dynamic increment corresponds exactly to the impact factor I conventionally used in bridge design. In the development which follows, the notation DI is used for the peak dynamic increment.

The development in Ref. 12 shows that the effects on dynamic increment of parameters definable at the design stage (i.e., excluding the effects of vehicle oscillation and roadway roughness) can be represented adequately by solutions for moving forces of constant magnitude. The dominant parameter, the speed parameter α, is related to the vehicle speed and the natural frequency of the bridge by

$$\alpha = v/2f_bL \qquad (3)$$

in which v is vehicle speed (fps), f_b is the bridge natural frequency (cps), and L is the span (ft). For example, the speed parameter α is equal to about 0.15 for a vehicle speed of 60 mph on a steel-beam, 60-ft span, simply-supported bridge with composite reinforced concrete deck; α becomes gradually smaller as span increases. For a single smoothly moving force the peak value of dynamic increment for deflection and moment is approximately

$$DI \cong \alpha \tag{4}$$

The effects on dynamic increment of approach and surface roughness and initial vehicle conditions tend to be relatively independent of other bridge properties. Reference 12 suggests that these effects for good deck surface conditions be accounted for by addition of a constant term to the expression for peak dynamic increment.

$$DI \cong 0.15 + \alpha \tag{5}$$

These approximate relationships for dynamic increment are derived for simple span bridges. Studies of dynamic response of multi-span bridges reported by Nieto-Ramirez and Veletsos[14] have yielded expressions for dynamic increments in terms of the same parameters. Values of DI are smaller with continuity; therefore, it is conservative to use the above expressions for continuous bridges. This conservative approach does not unduly penalize design of continuous bridges.

The speed parameter α and the dynamic increment are affected by the stringer flexibility. For simple span bridges or continuous bridges of approximately equal spans, the natural frequency f_b is given by

$$f_b = \frac{\pi}{2L^2}\sqrt{\frac{E_b I_b g}{w}} \tag{6}$$

where $E_b I_b$ is the flexural rigidity of the stringer acting with any composite deck, L is the span, g is the acceleration due to gravity, and w is the weight per unit length of the stringer and its share of the deck. Although most bridges actually have non-uniform $E_b I_b$, use of the value appropriate at midspan gives good results.

The term I_b in Eq. (6) is inversely proportional to the flexibility of the stringer. Increasing flexibility amounts to reducing I_b, which reduces f_b by about one-half the percent change in I_b, and increases the speed parameter of Eq. (2) proportionally. Thus, a 10 percent increase in stringer flexibility produces approximately a 5 percent increase in α. The peak dynamic increment of Eq. (5) receives approximately equal contributions from each term, so DI would increase by about 2.5 percent. The static component of deflection is inversely proportional to I_b, so the dynamic component of deflection, $\delta_d = DI\delta_s$, would increase about 12.5 percent with a 10 percent increase in flexibility. The amplitude of the dynamic component of acceleration in the fundamental mode, which will be related to human response to vibrations, is given by

$$a = \delta_D(2\pi f_b)^2 = DI\delta_s(2\pi f_b)^2 \tag{7}$$

Noting above that the change in $f_b{}^2$ is inversely proportional to the change in flexibility and cancels the effect on δ_s, the dynamic component of acceleration increases only in proportion to DI, or about 2.5 percent for a 10 percent increase in flexibility.

The static component of deflection is linearly related to stringer flexibility, the dynamic component of deflection is slightly more than linearly proportional to stringer flexibility, and the dynamic component of acceleration is only slightly increased by increased stringer flexibility.

The dynamic component of moment in the stringers is a product of the dynamic increment, Eq. (5), and the statical moment of the live loading. The dynamic increment of Eq. (5) corresponds to the traditional impact factor and is generally more conservative. Increased flexibility does increase dynamic increment slightly as noted above. However, increased flexibility also reduces the static component of moment as shown in Figs. 2 and 3. Therefore, changes in stringer flexibility have little effect on the dynamic component of moment; the total moment may be reduced by increased flexibility as a result of the reduced static component.

Field measurements reported by Wright and Green[15] include peak dynamic increments for deflection in a number of bridges of various types—including truss and beam bridges with simple, cantilever, and continuous spans. They sought empirical relationships between dynamic increment and span to depth ratio as well as dynamic increment and ratio of span to deflection for design live load. No systematic variation in dynamic increment was apparent for span to depth ratios $5 \leq L/d \leq 20$ and span to live load deflection ratios $200 \leq L/\delta \leq 4000$. Thus, both analytical and empirical evidence currently available suggests that bridge flexibility and slenderness criteria have a limited effect on dynamic components of deflections and stresses.

DECK DETERIORATION

It seems that no other problem has plagued highway bridge departments in most states as much as deterioration of bridge decks. The direct costs in terms of maintenance expenses for repairs have been great; indirect costs related to public dissatisfaction, inconvenience, and safety hazards perhaps have been even greater. Moreover, additional direct costs in new construction are involved, as bridge design and construction procedures are made more conservative in an effort to mitigate the deterioration problem before the fundamental causes are clearly defined, and effective, economical means for avoiding deterioration are developed, proven, and accepted. It has been suggested, e.g. Refs. 16, 17, and 18, that the flexibility of steel bridges may contribute to deck deterioration; restraints on bridge flexibility have been made more severe as a part of the general effort to avoid deck deterioration. Following is a review of recently available reports from both field and laboratory investigations, to ascertain whether there is any correlation between deck deterioration and the flexibility and slenderness of steel bridges. The notation for the principal types of deck deterioration is taken from the definitions in Refs. 19–22:

Scaling: Loss of surface mortar and exposure of surface of coarse aggregate; as scaling becomes more severe mortar is lost between coarse aggregate particles.

Transverse cracking: Reasonably straight cracks perpendicular to the center line of the roadway or parallel to the transverse reinforcement in skew bridges.

Longitudinal cracking: Fairly straight cracks roughly parallel to the center line of the roadway.

Surface spalling: A roughly circular or oval depression caused by separation and removal of a portion of the surface concrete revealing a roughly horizontal or inclined fracture.

The flexibility of bridge beams could contribute to deck deterioration through an increase in the peak longitudinal stresses due to live load plus impact or through increased stress ranges. However, the view of flexibility effects on static and dynamic stresses discussed previously shows that the slight increase in dynamic increment with increased flexibility tends to be counteracted by the associated reduction of peak live load stresses. The flexibility of bridge beams has very little effect on transverse flexural stresses in the deck. With increased flexibility the transverse moments distributing loads between beams are increased, but these moments remain similar to those from local effects of wheel loads. Additional factors contributing to deck deterioration are of interest here to the extent to which it appears that such factors, rather than bridge flexibility, are consistent with field and laboratory observations of deck deterioration. Among these factors reported in Ref. 23 are procedures of concrete placement and finishing which may leave voids around reinforcing bars, poor distribution of air voids in the surface layer, shrinkage cracking, use of deicing chemicals which may promote deterioration of the concrete and corrosion of reinforcement, and local wheel load effects on fatigue-induced cracking and spalling.

The cooperative studies conducted by the Bureau of Public Roads, Portland Cement Association, and state highway departments, Refs. 19–22, provide well correlated interpretive studies of deck deterioration in both steel and concrete bridges for wide ranges of climactic conditions, maintenance practices, and traffic conditions. The following paragraphs summarize observations in these studies.

Deterioration by scaling of bridge decks subjected to cyclic freezing and thawing is strongly attributed to inadequate air entrainment in the surface layer in field[24] and laboratory studies.[25] No correlation with longitudinal stresses or beam flexibility is reported.

Transverse cracking may be expected to occur in negative moment regions for any degree of stringer flexibility, but excessive crack width might be related to flexibility. In continuous bridges, transverse cracking from live load plus impact stresses should be concentrated in the negative moment region where tension is induced in the deck. Such response is observed in decks of concrete bridges, but all reports note that transverse cracks are evenly spaced throughout the length of decks of continuous steel bridges. An example distribution of cracking from Ref. 19 is shown in Fig. 6. Cracks are prevalent in the positive moment region and in the negative moment region. It is suggested that transverse cracking in decks of steel beam bridges is caused by shrinkage of the deck concrete in the longitudinal direction. The hypothesis is supported by the observation in the Michigan report[20] that fewer visible transverse cracks appear in composite deck bridges for which shear connectors contribute to control of shrinkage cracking. The California report[21] notes that transverse cracking is more prevalent in heavily traveled lanes of steel beam bridges; because no concentration of cracking in the negative moment region of these lanes is reported, it is possible that local wheel load stresses are responsible, rather than general longitudinal stresses.

Longitudinal cracking also appears to be more associated with local effects of wheel loads than with overall bridge response as influenced by beam flexibility. The cooperative study reports a lower incidence of longitudinal cracking than transverse cracking which tends to support the hypothesis that shrinkage causes most cracks. Peak stresses in the deck are essentially the same longitudinally and transversely, but the much

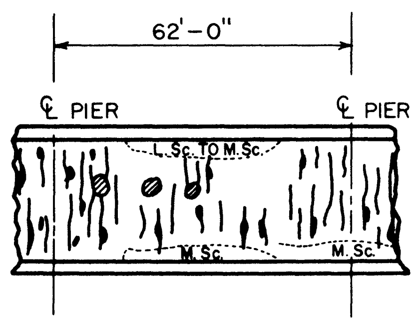

Fig. 6. View of deck deterioration

greater restraint to shrinkage strains in the longitudinal direction contributes to transverse cracking. Kansas[19] and California[21] report a definitely greater incidence of longitudinal cracking in the decks of concrete slab bridges which have longitudinally oriented reinforcement. California reports a correlation of incidence of longitudinal cracking with traffic density. There appears to be no evidence for association of longitudinal cracking with the flexibility of steel bridge beams.

Surface spalling is tentatively correlated with the flexibility of the deck spanning between stringers, but not with the flexibility of the stringers in the Kansas report,[19] and all four reports identify an association with subsidence cracks over reinforcing and with corrosion of the reinforcing. No correlation of spalling with traffic density is reported. There appears to be no evidence for association of spalling with the flexibility of steel bridge beams.

Available evidence on the causes of bridge deck deteriorations does not point to longitudinal stresses due to live load plus impact as primary factors in deck durability. These are the only stresses which may be affected by the stringer flexibility. The observations of maintenance personnel that bridges with deteriorated decks undergo excessive vibrations may be explained quite directly by the strong effect of surface roughness on the amplitude of the dynamic response of bridges. The strong effect of surface roughness on the dynamic increment has been observed in field tests reported by Oehler[26] and explored analytically in Refs. 12 and 27. The evidence supports the conclusion in Ref. 24 that materials and construction procedures are principal contributors to the problem.

HUMAN RESPONSE

Much of the available literature on bridge vibrations and on human response to vibrations has been reviewed in an effort to define the effects of flexibility and slenderness of bridges on their serviceability in terms of the comfort of their users. No effort is made to present a bibliography of the literature in these areas; the references used here do contain citations of most of the relevant literature.

Human response is directly related to the characteristics of the vertical motion of the bridge. A typical record of the deflection-time history at midspan of a simple-span steel bridge with a composite deck traversed by a heavy vehicle is shown in Fig. 5. Most of the peak deflection arises from the static component of the loading as is indicated by comparison of the total deflection (Curve **b**) and the static component which is denoted as the crawl curve (Curve **a**). Although the nature of the variation of the dynamic component (Curve **c**) with time is complex, this part of the motion is dominated by oscillations occurring at the natural frequency of the bridge. This discussion of human response to bridge vibrations is based on a simplified, two-component, representation of the history of movement:

(1) The static component, or crawl curve, which has a peak amplitude dependent on flexibility of the bridge and weight of the vehicle and duration dependent upon bridge span and vehicle velocity.

(2) A simplified dynamic component which has a peak amplitude related most strongly to the amplitude of the static component, surface roughness, and vehicle speed, has a natural frequency corresponding to that of the bridge, and persists for a number of cycles of vibration because of the low damping ordinarily encountered in highway bridges. Procedures for evaluation of the peak amplitude of the dynamic component were discussed previously.

In the following discussion it is emphasized that available information indicates that these two major components of the bridge response do not affect users in the same way; it appears that peak total deflection due to live load plus impact is not a meaningful measure of the degree of reaction of the user to bridge vibration.

The ASCE Committee on Deflection Limitations of Bridges[3] pointed out that users perceive bridge vibrations only when standing or sitting on the bridge itself, or when sitting in stationary vehicles. No contradictions of this observation have been encountered during the present investigation. As recommended in recent studies of load factor design,[28] human response to bridge vibrations seems to be a factor for consideration in design only when a significant proportion of the users of a bridge will be standing, walking slowly, or seated in stationary vehicles. This category will not include most rural bridges or most bridges for controlled access highways.

Much of the literature on human response to vibrations applies to the physical safety and the performance abilities of physically conditioned young male subjects in a vibrating environment. Most research supported by aerospace and Department of Defense agencies is of this nature. Only limited information is available on the comfort of humans who do not expect to tolerate vibration as part of their duties. Comfort is a subjective human response that is not directly measurable; people report vibrations to be perceptible, unpleasant, or intolerable; the same vibration may elicit widely varying reactions among different subjects.

Wright and Green[29] present a survey of investigations of the response of humans to steady simple harmonic motion, and discuss the rather different criteria various investigators have established to predict the effect of such motion. They conclude that the results of Goldman,[30] which are most accessible in summary form in

Fig. 44.31 of Ref. 31, provide the best currently available measure for human response to steady sinusoidal vibration; the results are shown in Fig. 7. Conditions of constant peak velocity, peak acceleration, and peak jerk (rate of change of acceleration) are shown in the upper right hand corner of the figure. It is evident that no simple physical characteristic of the vibration completely describes the human response, but a peak acceleration criterion is a good fit in the frequency range of 1 to 10 cps. This is the range of the natural frequencies of multibeam highway bridges.

Tentative international standards for human exposure to vibration[32] also use an acceleration criterion in this frequency range. This evidence suggests acceleration is preferable to the velocity criterion appearing in recent British[33] specifications.

It is important to note that only the dynamic component of highway bridge vibrations (Curve **c** of Fig. 5) is at all comparable with the steady, simple harmonic motion considered for Fig. 7. Lenzen,[34] in an investigation of human susceptibility to building floor vibration found definite indication in actual experience that people are much less sensitive to vibrations that decay rapidly. He observed that people do not respond to vibrations which persist for fewer than 5 cycles. For susceptibility criteria comparable to those of Fig. 7, Lenzen suggests that the displacement tolerance be multiplied by a factor of the order of 10 if the vibration decays to less than 10 percent of its initial magnitude in 5 to 12 cycles. The static component of highway bridge deflection, therefore, has negligible effect on human response because it persists for just one-half cycle. The static component is neglected by Wright and Green[15] in their evaluations of human reaction to bridge motions and a similar permissible increase in amplitude for shorter durations appears in Ref. 32.

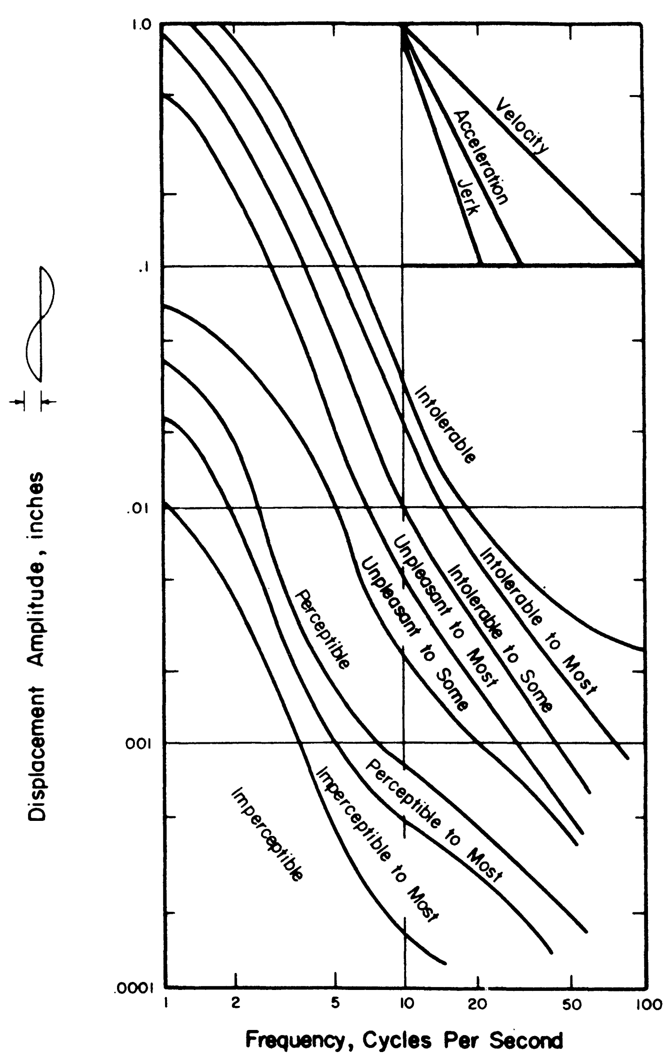

Fig. 7. Response to sustained harmonic vertical vibration

The dynamic component of motion does persist for a number of cycles even after the loading leaves the span. It can be shown by procedures described by Jacobsen and Ayre[35] to require 3 percent critical damping to meet Lenzen's criterion of decay to 10 percent amplitude in 12 cycles. This is more than the one-to-two-percent of critical damping observed on the average by Wright and Green.[15] However, normal service conditions would include a mix of traffic leading to minor vibration which would mask the "ringing" following passage of a heavy vehicle and have an effect similar to damping. Also, the basic vibration tolerances given by Goldman[30] are noted to apply for vibrations sustained for 5 to 20 minutes; a tenfold increase in tolerance is suggested for durations of less than one minute. Oehler[26] reports no persistence greater than 15 seconds for simple span and continuous bridges. Therefore, it seems consistent to evaluate human reaction by the amplitudes of acceleration represented in Fig. 7, but with a tenfold increase in transient acceleration required for a specific level of human response. The resulting acceleration-response relations are given in Table 1.

DESIGN STUDIES

These design studies illustrate the effects that current and proposed deflection criteria have on the proportions and behavior of multistringer steel highway bridges with reinforced concrete decks. A program[4] was used to provide minimum weight proportions for a specified web height and thickness. The use of the optimization procedure provides confidence that differences in designs and behavior result from changes in deflection criteria rather than differences in the amount of refinement among individual designs.

AASHO[1] bridge specifications are applied for all design criteria other than those governing stringer flexibility and slenderness. Designs employ both A36 and A441 steels, but hybrid designs with A441 flanges and A36 webs are not considered. Hybrid designs

Table 1. Acceleration Criterion for Human Response to Harmonic Vertical Vibration

Human Response	Amplitude of Acceleration (in./sec^2)	
	Transient	Sustained
Imperceptible		
	5	0.5
Perceptible to few		
	10	1
Perceptible to some		
	20	2
Perceptible		
	50	5
Unpleasant to few		
	100	10
Unpleasant to some		
	200	20
Unpleasant		
	500	50
Intolerable to few		
	1000	100
Intolerable to some		
	2000	200
Intolerable		

would require slightly more flange steel than the all-A441 designs, but there is no reason to anticipate different effects of deflection criteria for hybrid designs. Design live loading is the AASHO HS20-44; the special interstate loading does not control stringer proportions in the 60 to 150 ft span range of interest. The wheel load distribution to each stringer is taken as $S/5.5$, where S is the stringer spacing in feet. The impact factor is computed using AASHO 1.2.12(C). The influence of stringer flexibility on live load distribution and impact is noted in discussion of the bridge designs, but this influence is not incorporated in the designs presented.

Bridge decks are $7\frac{1}{2}$-in. thick reinforced concrete slabs for cylinder strength $f'_c = 3000$ psi, modular ratio $n = 10$, and density is 150 pcf. The cylinder strength is conservative for consideration of strength and stiffness; however, specification of at least $f'_c = 4000$ psi is suggested[23] for deck durability in regions subject to freezing and thawing. In addition to the deck and steel stringers, dead load **1** includes a concrete haunch between the bottom of the top flange and the bottom of the deck. The haunch is the largest of the maximum top flange thicknesses or 2 in. Superimposed dead load **2** includes 10 psf for curbs and guardrails and 25 psf for future surfacing; these are considered in design for strength but omitted in consideration of dynamic response.

A stringer spacing of 80 in. is used for rolled beam stringers and a spacing of 100 in. is used for welded stringers. A roadway width of approximately 42 ft is obtained from 6 stringers at 80 in. or 5 stringers at 100 in. and is appropriate for recent standards.[36] AASHO Sect. 1.2.6 calls for three lanes of loading for this width. A uniform distribution of three lanes of loading is used without reduction in computation of deflection for criteria limiting the ratio of span to deflection. The $S/5.5$ distribution is more critical for strength than the uniform distribution of three lanes of loading.

A detailed view of the effects of slenderness and deflection criteria on material requirements, load distribution, and human response is obtained by comparison of designs for continuous bridges with two 90-ft spans. The stringers are welded from three A36 steel plates and are proportioned for composite action in positive moment regions. Slenderness ratios are $L/d = 20, 30, 40$ and deflection limitations are $L/\delta = 400, 1200, 1600$.

Effects of slenderness and flexibility on steel requirements, including web stiffeners, are shown in Fig. 8. The very slender bridge with $L/d = 40$ does require substantially more steel, but this cost might be balanced by reduction of embankment. The L/δ at which the horizontal dashed line becomes an ascending solid curve defines the conventional deflection limitation at which a price is paid for reduced flexibility. The price can be severe, the weight of steel required roughly doubles as L/δ is doubled from its initially effective value and the limitation becomes effective at smaller L/δ for higher strength steels.

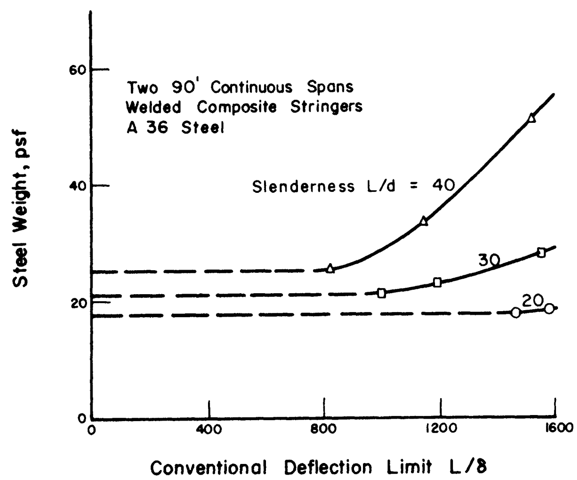

Fig. 8. Effect of flexibility criterion on steel requirement

The wide variations considered for L/δ and L/d produce no significant difference in human response as seen when the amplitudes of acceleration plotted in Fig. 9 are related to human reaction in Table 1. The more slender bridges for given L/δ are slightly more comfortable as a result of greater weight and reduced natural frequency.

The more flexible designs for the continuous bridge of two 90-ft spans show reduced steel requirements (for a given design slenderness L/d), better load distri-

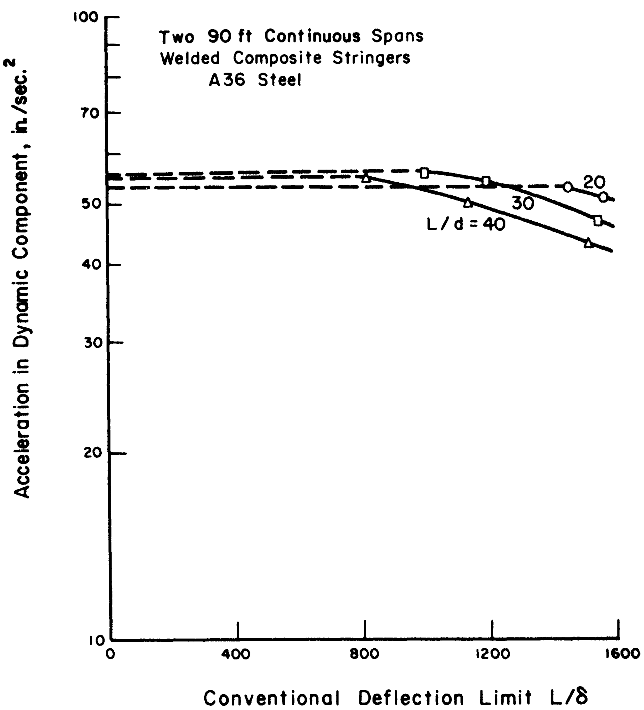

Fig. 9. Effect of flexibility criterion on human response

bution, and no significant change in the tendency to human reaction to bridge motion (as measured by the amplitude of acceleration in the dynamic component of motion). The more slender, more flexible designs do not appear to differ in susceptibility to other modes of failure either in terms of their predicted behavior or by current AASHO specifications. Designs which are not limited by deflection criteria generally are limited by resistance to yielding in flexure (AASHO Sect. 1.7.1 A). Local buckling (Sect. 1.7.70), lateral buckling (Sect. 1.7.1 B), and fatigue (Sect. 1.7.3) constraints are not systematically more active for these designs and the improved load distribution increases the margin of safety for response in these modes.

It is apparent that the general variation of steel requirements, load distribution, and human response with stringer flexibility and slenderness will be similar for other bridge configurations to those described above for continuous composite 90-ft spans using A36 steel. However, it is necessary to view the effects of relaxed deflection criteria over a wide range of design parameters to ascertain whether altered criteria may provide significant advantages or significant difficulties. Important parameters to be investigated are variations in span length, use of high strength steel, comparison of simple and continuous spans, and comparisons of composite and noncomposite designs. A view is given of the effects of extremes of deflection criteria with these parameters. Very stiff bridges are achieved by imposing the severe deflection limitation of $L/\delta = 1600$ for $L/d =$ 20 or 30 and using noncomposite design with A36 steel. However, composite behavior is assumed for these very stiff bridges—increasing the stiffness further—when considering human reaction to the serviceability loading of one heavy vehicle. Very flexible bridges are achieved by imposing no deflection limitation for $L/d = 40$ and using composite design with A441 steel.

Amplitudes of dynamic component of acceleration are shown in Fig. 10. The effect of greater span is to reduce the peak acceleration and, therefore, to increase predicted human comfort for both very stiff and very flexible bridges. For a particular span, the very stiff bridges are predicted to undergo less acceleration and therefore to be more comfortable. The increase in comfort, about half an increment of human response, is not great compared to the increase required in material to achieve the stiffness.

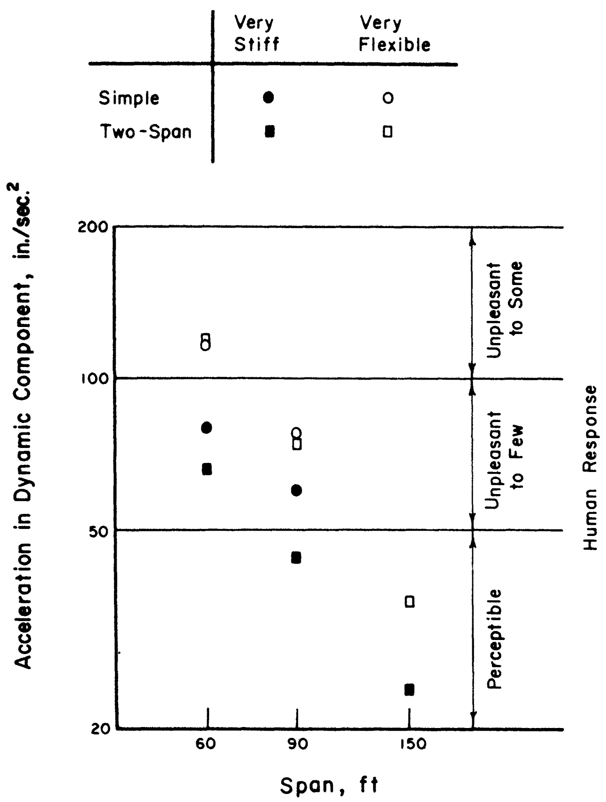

Fig. 10. Human response for very stiff and very flexible stringers

CONCLUSIONS AND RECOMMENDATIONS

The objective of this paper is to show the effects of the slenderness and flexibility of multistringer steel highway bridges on their strength and serviceability. Slenderness affects the amount of steel required for bridge stringers and has an associated secondary influence on the dead weight of the bridge. Flexibility affects responses to static (slowly moving) loadings and to dynamic loadings.

The effects of flexibility and their relationship to bridge performance in terms of deck durability and human response, as well as the influence of relaxed flexibility criteria have been reviewed in this paper.

A first conclusion is that a more flexible bridge is stronger in resistance to yielding of a stringer. This results from a marked improvement of static load distribution which is only partially counteracted by an increase of the impact factor. Fatigue susceptibility also is reduced by increased flexibility. The improvement in load distribution reduces both peak stress and stress range in the most heavily loaded stringer.

Durability of concrete bridge decks does not appear to be affected by stringer flexibility. Observed cracking in deteriorated decks cannot be related to live load stresses. Even if it were, increased flexibility would tend to reduce transverse cracking by virtue of better load distribution between stringers. Longitudinal cracking would be little affected—peak transverse negative moments tending to produce cracking of the wearing surface are slightly reduced by increased stringer flexibility.

Human reaction to bridge motions is rather indirectly related to stringer flexibility. Pedestrians and occupants of moving vehicles appear to respond primarily to the accelerations in the dynamic component of bridge motion. This acceleration is only slightly increased by increased stringer flexibility—the proportional increase in dynamic component of acceleration is about $\frac{1}{4}$ that in flexibility. Human response to acceleration, like that to sound, varies with the logarithm of the amplitude. Therefore, substantial variations in flexibility produce only modest effects on human response to bridge motions. The current deflection criterion, a limiting ratio of span to deflection due to live load plus impact, does not assure human comfort because it does not control the dynamic component of acceleration.

Current limits on flexibility and slenderness do affect bridge economy because they may prevent economical applications of high strength steels and of slender girders. Trial designs with relaxed flexibility and slenderness limitation demonstrate that substantial economies are possible and that predicted human reactions would not be substantially greater than those observed when design is in accord with current criteria for flexibility and slenderness.

Recent studies[5] of load distribution recommend a procedure for consideration of the beneficial effects of increased stringer flexibility on load distribution, so no detailed recommendations are necessary here. There is no evidence of bridge motions producing discomfort of occupants of moving vehicles, so there appears to be no need for limits on deflections or accelerations of bridges which do not carry pedestrian traffic under normal conditions. Example designs presented here for very slender, very flexible bridges do not exhibit undesirable side effects other than the mildly increased dynamic components of deflection from increased flexibility and increased amounts of steel with increased slenderness. Economic requirements are likely to prevent use of slenderness beyond the range considered here. An absence of limitation on the deflection of slenderness of steel bridges is consistent with modern European specifications and current U. S. specifications for other types of bridges.

Bridges designed for pedestrian traffic or stationary vehicles should be limited in motion by a serviceability criterion assuring human comfort. The level of limitation is still in doubt, because a tolerable motion is a question of psychology. Humans are not disturbed by clearly perceptible motions when they walk, dance, or ride in automobiles or elevators because the motions are anticipated. When a motion is unexpected and suspected to be a symptom of structural inadequacy, its perception alone is disturbing. It is suggested here that the amplitude of the dynamic component of acceleration in the fundamental mode of vibration be limited to 100 in./sec^2 (or approximately 0.25 gravity). For normal damping this is a clearly perceptible acceleration which will require that pedestrians be educated to expect to feel bridge motions.

The model used to predict bridge motions for design is as important as the direct limitation placed on the predicted motions. A rather simple, yet accurate, formulation is possible even though a multitude of parameters do influence the peak value of the dynamic component of bridge acceleration. Because the critical motions for serviceability arise from the relatively frequent passage of a single heavy vehicle and vehicle velocity tends to increase with distance from the sidewalk, a distribution factor of 0.7 wheel loads may be used without consideration of stringer spacing (in the range 4 ft to 12 ft) or flexibility. The dynamic component of acceleration is related to the basic static deflection computed for this load distribution, the natural frequency of bridge vibration, and the design vehicle speed by an approximate procedure. The relations in summary are:

(1) Static Deflection δ_s, computed conventionally for a wheel load distribution factor = 0.7, defining the loading on one stringer acting with its share of the deck

(2) Natural Frequency f_b, for simple or equal spans given by Eq. (6):

$$f_b = \frac{\pi}{2L^2}\sqrt{E_b I_b g/w}$$

(3) Speed Parameter α, Eq. (3):

$$\alpha = \frac{v}{2f_b L}$$

(4) Impact Factor DI, Eq. (5):

$$DI = 0.15 + \alpha$$

(5) Amplitude of Dynamic Component of Acceleration a, Eq. (7):

$$a = DI\delta_s(2\pi f_b)^2$$

Because reliable evidence on human reaction to bridge motions is so severely limited, it is suggested that the recommended acceleration criterion receive empirical confirmation prior to any adoption. Actual bridges could be used in the investigation under circumstances that permit discontinuance of pedestrian traffic if discomfort is pronounced. Because greatest potential of discomfort is expected for short spans, the cost of such testing would be reasonable.

REFERENCES

1. Standard Specifications for Highway Bridges *American Association of State Highway Officials, Washington, D. C., Tenth Edition, 1969.*
2. *Bresler, B.; T. Y. Lin; and J. B. Scalzi* Design of Steel Structures *John Wiley and Sons, Inc., New York, Second Edition, 1968, p. 472.*
3. Deflection Limitations of Bridges *Journal of the Structural Division, ASCE, Vol. 84, No. ST 3, May 1958.*
4. *Wright, R. N. and W. H. Walker* Criteria for the Deflection of Steel Bridges *American Iron and Steel Institute, New York, 1971.*
5. *Sanders, W. W., Jr. and H. A. Elleby* Distribution of Wheel Loads on Highway Bridges *National Cooperative Highway Research Program Report 83, Highway Research Board, Washington, D. C. 1970.*
6. *Lightfoot E. and F. Sawko* Grid Frameworks Resolved by Generalized Slope Deflection *Engineering, Vol. 187, London, 1959.*
7. *Rowe, R. E.* Concrete Bridge Design *John Wiley and Sons, Inc., New York, 1962.*
8. *Fenves, S. J.; R. D. Logcher; and S. P. Mauch* Stress—A User's Manual *MIT Press, Cambridge, 1964.*
9. *Newmark, N. M. and C. P. Siess* Moments in I-Beam Bridges *Bulletin No. 336, University of Illinois Engineering Experiment Station, Urbana, 1942.*
10. *Siess, C. P. and I. M. Viest* Tests of Continuous Right I-Beam Bridges *Bulletin No. 416, University of Illinois Engineering Experiment Station, Urbana, 1953.*
11. *Fenves, S. J.; A. S. Veletsos; and C. P. Siess* Dynamic Studies of Bridges on the AASHO Road Test *Publ. 968, Highway Research Board, 1962.*
12. *Walker, W. H. and A. S. Veletsos* Response of Simple-Span Highway Bridges to Moving Vehicles *Bulletin No. 486, University of Illinois Engineering Experiment Station, Urbana, 1966.*
13. *Veletsos, A. S. and T. Huang* Analysis of Dynamic Response of Highway Bridges *Journal of the Engineering Mechanics Div., ASCE, Vol. 96, No. EM 5, October 1970, p. 593–620.*
14. *Nieto-Ramirez, J. A. and A. S. Veletsos* Response of Three-Span Continuous Highway Bridges to Moving Vehicles *Bulletin No. 489, University of Illinois Engineering Experiment Station, Urbana, 1966.*
15. *Wright, D. T. and R. Green* Highway Bridge Vibrations, Part II, Ontario Test Programme *Report No. 5, Dept. of Civil Engineering, Queen's University, Kingston, Ontario, 1964.*
16. *Problem Statement for NCHRP Project 6–9* Potential Accelerating Effects of Chemical Deicing Damage by Traffic and Other Environmental Induced Stresses in Concrete *Highway Research Board, 1964.*
17. *Creskoff, J. J.* Crushed Stone vs. Gravel in Bridge Decks *Pennsylvania Triangle, Vol. 52, January 1965.*
18. *Axon, E. O.; T. T. Murray; R. M. Rucker* A Study of Deterioration in Concrete Bridge Decks *Missouri State Highway Dept. Jan. 1969.*
19. Durability of Concrete Bridge Decks *State Highway Commission of Kansas, Bureau of Public Roads, Portland Cement Association, 1965.*
20. Durability of Concrete Bridge Decks *Michigan State Highway Department, Bureau of Public Roads, Portland Cement Association, 1965.*
21. Durability of Concrete Bridge Decks *California Division of Highways, Bureau of Public Roads, Portland Cement Association, 1967.*
22. Durability of Concrete Bridge Decks *Missouri Division of Highways, Bureau of Public Roads, Portland Cement Association, 1967.*
23. Effects of De-Icing Chemicals on Structures *Bulletin 323, Highwav Research Board, Washington, D. C., 1962.*
24. *Larson, T. D.; P. D. Cady; and J. T. Price* Review of a Three-Year Bridge Deck Study in Pennsylvania *Highway Research Record, No. 226, Highway Research Board, Washington, D. C., 1968, p. 11–25.*
25. *Callahan, J. P.; J. LaLott; and C. E. Kesler* Bridge Deck Deterioration and Crack Control *Journal of the Structural Division, ASCE, Vol. 96, No. ST 10, October 1970, pp. 2021–2036.*
26. *Oehler, L. T.* Vibration Susceptibilities of Various Highway Bridge Types *Journal of the Structural Division, ASCE, Vol. 83, ST4, July 1957, 1318-1-41.*
27. *Oran, C. and A. S. Veletsos* Analysis of Static and Dynamic Response of Simple-Span, Multi-Girder Highway Bridges *Civil Engineering Studies, SRS No. 221, University of Illinois, Urbana, 1958.*
28. *Vincent, G. S.* Tentative Criteria for Load Factor Design of Steel Highway Bridges *American Iron and Steel Institute, New York, February 1968.*
29. *Wright, D. T. and R. Green* Human Sensitivity to Vibration *Report No. 7, Department of Civil Engineering, Queen's University, Kingston, Ontario, February 1959.*
30. *Goldman, D. E. A.* A Review of Subjective Responses to Vibration Motion of the Human Body in the Frequency Range 1 to 70 cps *Naval Medical Research Institute, Report NM-004-001, Washington, D. C., 1948.*
31. *von Gierke, H. E. and David Goldman* The Effects of Shock and Vibration on Men *Chapter 44, Shock and Vibration Handbook, C. M. Morris and C. E. Crede, Eds., McGraw-Hill Book Co., 1961.*
32. Guide for the Evaluation of Human Exposure to Whole-Body Vibration *ISO/TC 108 WG 7, International Organization for Standardization, June 1970.*
33. Composite Construction in Structural Steel and Concrete *Part 2, Beams for Bridges, British Standard Code of Practice, CP 117, British Standards Institution, 1967, Appendix A.*
34. *Lenzen, K. H.* Vibration of Steel Joist-Concrete Slab Floors *Engineering Journal, AISC, Vol. 3, No. 3, July 1966, 133–136.*
35. *Jacobsen, L. S. and R. S. Ayre* Engineering Vibrations *McGraw-Hill Book Company, New York, 1958, p. 201.*
36. Highway Design and Operational Practices Related to Highway Safety *Special AASHO Traffic Safety Committee, American Association of State Highway Officials, Feb. 1967.*