The Case for the Semi-Box Girder

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THE SEMI-BOX girder is that breed having a partly open and a partly closed cross section. As such, it is not as torsionally stiff as a rectangular box girder, but substantially stiffer in this regard than an open section. Its main attribute is its ability to resist failure by lateral torsional buckling and, as such, is useful in bridge work to facilitate erection, or in other instances when the girder is used in an isolated condition, such as in a monorail or crane girder.

This type of girder certainly did not originate with the author; it has been reported upon at least twice previously. Homer M. Hadley reported on the "Delta Girder"[1] at the same time that Edward Luss reported on the "Double Y Girder" on the following pages of the same reference.

The cross-sectional designs are illustrated in Fig. 1. In both instances (delta and double Y girders), the applications were in highway bridges, where the torsional strength lent itself to easy and safe erection, while the cross section, by bracing the web, eliminated stiffeners. The result was a simple and clean section, easy to look at and easy to maintain. In another article,[2] Mr. Hadley provided details of construction and photographs of certain bridges both during construction and following completion, in which he not only emphasized the above attributes, but also pointed out the additional advantages of a shorter effective roadway slab span and better bonding of the slab to the girder top.

The justification for this writing is, first, to describe an application somewhat different and involving a crane runway; secondly (and most important), to illustrate the ease of handling the design of this type of construction and thus encourage its use; and lastly, to suggest codification rules.

PROJECT DESCRIPTION

In 1965, the Morrison Railway Supply Corporation engaged the author's office to design their Columbus, Ohio, Cropping Plant. Included in this facility is a crane runway 1100 ft long and covered by two 80-ft span bridge cranes.

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Due to a combination of poor foundation conditions at the site involving cinder and loam fill, and a multitude of old foundations resulting from previous demolition, it was economical to increase the span of this type of "A" frame supported runway from the normal 20 or 25 ft to at least 40 ft. Conditions imposed by processes and track layout dictated 50-ft spans in several instances. As a result, 50 ft was selected as an economical typical span, which was used throughout the runway, except for one exception, where a process building dictated 63-ft and 37-ft spans. The "gull" cross section (Fig. 1) was used for all girders.

A general contract was let to the Columbus firm of Timmons, Butt and Head, who built the facility in 1966. International Steel Company of Evansville, Indiana, was the fabricator. The Columbus firm of Fling and Eeman provided engineering supervision during construction.

The work has been in constant service since its completion in December 1966. A recent inspection indicated no structural problems of any nature. Figure 2, photographed during this inspection, depicts the runway construction. The portion of the girder to the right of the foreground A-frame is the 63-ft span composed of a W36$\times$94 and two angles 8 x 6 x $\frac{3}{4}$, while the foreground girder, shown in its entirety, and all others are the typical section composed of W36$\times$135 and two angles 8 x 6 x $\frac{1}{2}$.

Of interest, several of the design features of the runway were based on recommendations in a paper by John E. Mueller[3]:

1. The runway was built in three sections, two 350-ft long and the other 400-ft, there being two expansion points provided by swing links from a short cantilever to the suspended spans. Each section has but one braced bay near the center, so that all other columns are free to rotate with temperature change.
2. All spans are simple, with bearing points designed for freedom of rotation. Figure 2 illustrates this. The sloping leg of the A-frame is inserted between the bearing stiffener of the girders with the joint pinned in a fashion that allows it to open or close at the top and still maintain alignnment.

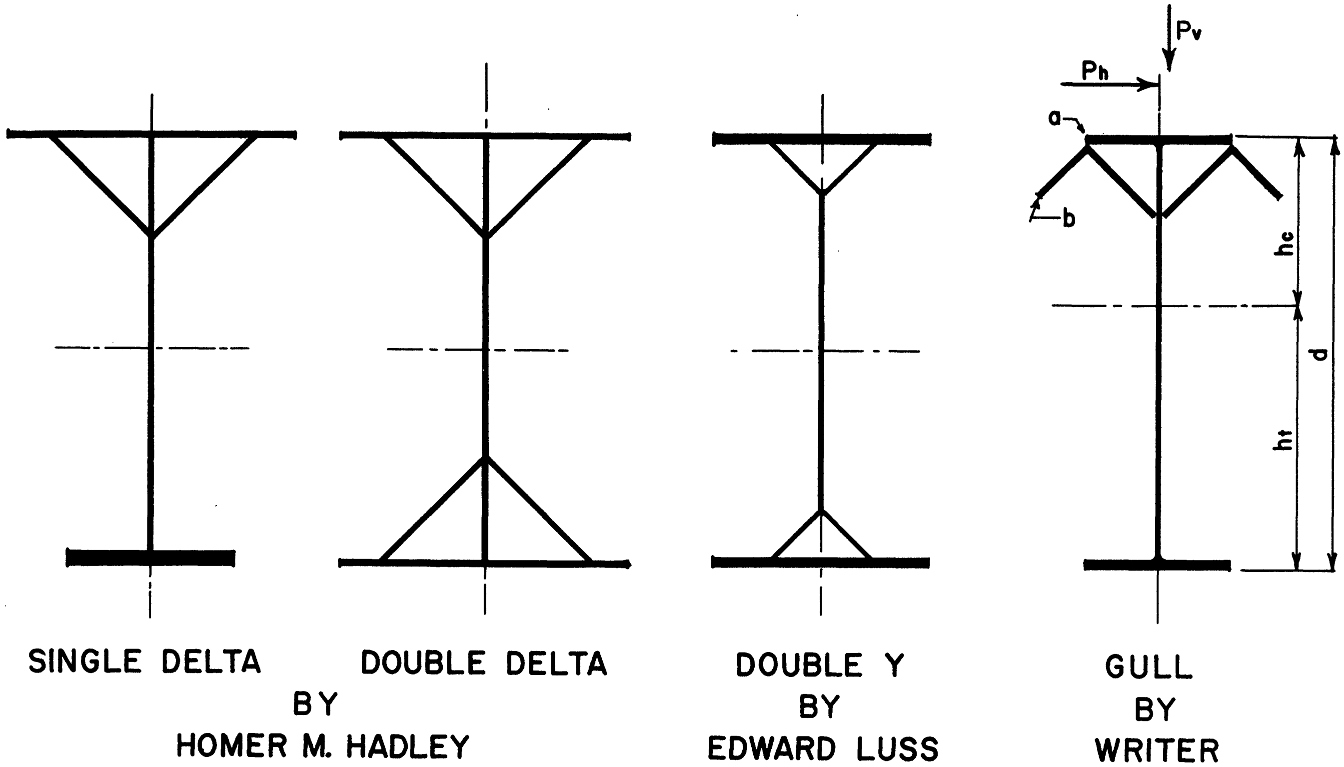


Fig. 1. Some semi-box girder types

3. The rail is of the "floating" type. The rail at the central braced bay is the only clamped section. Thus, freedom of longitudinal motion is allowed over each joint.

SPECIFICATION CONSIDERATIONS

The 1963 AISC Specification, Section 1.5.1.4.3, read as follows:

"Tension and compression on extreme fibers of box-type members, whose proportions do not meet the provisions of a compact section. but do conform to the provisions of Sect. 1.9,

$$F_b = 0.60\,F_y\text{''}.$$

The corresponding section in the Commentary read as follows:

"Box-type members, even those where width-thickness ratios are such that they cannot be classified as compact members, are torsionally very stiff. Hence, reduction of the full stress, as provided by Formulas (4) and (5) for the open top sections is not required."

This left the designer pretty much on his own to judge when or when not to classify a section as a box section, since no limits were stated.

Section 1.5.1.4.4 of the present 1969 Specification, which covers non-compact box girders, places the additional requirement that the compression flange be braced at intervals not to exceed $2500/F_y$ times the transverse distance out-to-out of the webs. The Commentary concerning this section indicates that the safe working stress value can be obtained by use of Formula (1.5-1), the column formula for short and medium length columns,

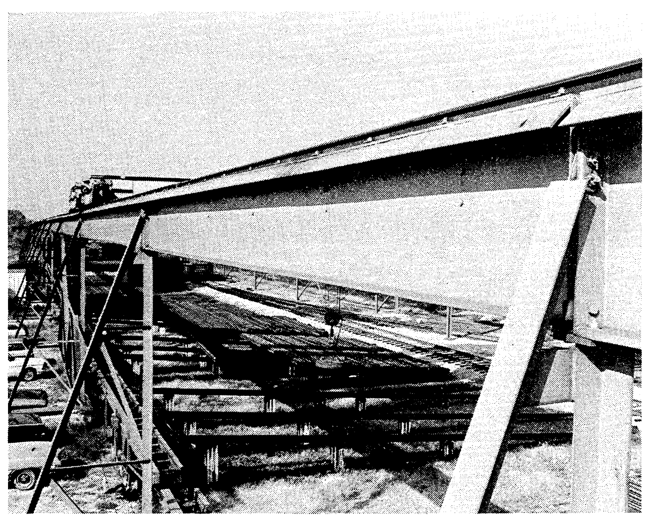


Fig. 2. Gull girders at Morrison–Columbus Plant

by computing an "equivalent" l/r value from the formula

$$\left(\frac{l}{r}\right)_{\text{equiv}} = \sqrt{\frac{5.1 S_x}{\sqrt{J I_y}}}$$

where J is the torsion of the section, and the other terms have their usual meanings.* The Commentary finally states:

> "It can be shown that when $d \leqslant 10b$ and $l/b \leqslant 2500/F_y$, the allowable compression flange stress indicated by the above equation will approximate $0.6F_y$. Beyond this limit, deflection rather than stress is likely to be the design criterion."

It is then apparent that the entire spectrum of unbraced flexural sections which are box-like or incorporate a box, but fall beyond the specified limit such that their safe working stress may or may not be $0.6F_y$, are not presently covered by the current AISC Specification. While this spectrum extends all the way to the open section, it is the purpose here to confine the discussion to those sections whose strength when twisted, or stability when bent, is due primarily from torsional strength rather than warping strength.

REVIEW OF THEORY OF ELASTIC FAILURE

Timoshenko,[5] in his classical solution, showed that the critical elastic moment for a simple unsupported beam subjected to constant moment along its length, with ends free to rotate about either axis but restrained from twisting, is given by the relationship:

$$M_{cr} = \frac{\pi}{l}\sqrt{EI_y GJ\left(1 + \frac{E}{G}\frac{C_w}{J}\frac{\pi^2}{l^2}\right)} \qquad (1)$$

where

$$G = \text{the shear modulus of elasticity} = \frac{E}{2(1+\nu)} \qquad (2)$$

J = the torsion constant of the section
C_w = the constant of warping

Likewise,

$$F_{bcr} = \frac{\pi}{S_x l}\sqrt{EI_y GJ\left(1 + \frac{E}{G}\frac{C_w}{J}\frac{\pi^2}{l^2}\right)} \qquad (3)$$

If C_w is small enough to be neglected,

$$F_{bcr} = \frac{\pi}{S_x l}\sqrt{EI_y GJ} \qquad (4)$$

* *The derivation of this equivalent l/r relationship and examples of its use are given in Ref. 4.*

Timoshenko also determined, in the same reference, the critical top flange loading for both a single central load and a uniform load. When warping is neglected and the relationship shown in terms of F_{bcr}, the equation is of the same form as Eq. (4), except that a value slightly different from π is required. For the centered concentrated load, a value of approximately 3.7 is required, and for the uniform load, a value of approximately 3.22 is required. Hence, the elastic strength for the concentrated load is about 18 percent greater than for a constant moment and only slightly greater for a uniform load. Thus, if π is used for all ordinary conditions, Eq. (4) is conservative.

Substituting in Eq. (4),

$$Z = \sqrt{\frac{I_y J}{S_x}} \qquad (5)$$

E = 29,000 ksi and $G = E/2.6$ (derived by substitution of $\nu = 0.30$ in Eq. (2)), we obtain

$$F_{bcr} = \frac{56{,}500}{(l/Z)} \qquad (6)$$

If, further, the normal factor of safety 1.67 is used,

$$F_b = \frac{34{,}000}{(l/Z)} \qquad (7)$$

within the range of elastic behavior.

The term Z is then a length to the first power and will be in inches. It should not be thought of as $\sqrt{I_y J/S_x}$ any more than r is thought of as $\sqrt{I/A}$. Thus, Z is to lateral torsional buckling (without warping) as r is to compressive buckling. If one thinks in terms of l/Z, he will have the same feel for the general beam as he does when he thinks of l/r with regard to a column. It is significant that such a simple relationship exists under these circumstances. In observing the parameter, it is realized that for a significant value, both J and I_y must be sizable. *One is as important as the other when only stability is of concern.*

A comparison may be made with the well-known equation developed by Karl de Vries[6], still very much in use as AISC Formula (1.5-7). The modern version has adopted a coefficient to take into account the shape of moment curve. The original read as follows:

$$F_b = \frac{12{,}000}{ld/bt} \qquad (8)$$

If approximate values of I_y, J, and S_x for a rolled beam, whose web is neglected, is represented by

$$J = \tfrac{2}{3} bt^3$$

$$I_y = \tfrac{1}{6} b^3 t$$

and $$S_x = btd$$

all as suggested by George Winter[7], it is found that

$$Z = \frac{1}{3}\frac{bt}{d} \tag{9}$$

If Eq. (9) is substituted in Eq. (7),

$$F_b = \frac{11{,}300}{ld/bt} \tag{10}$$

While the de Vries formula was developed partially by statistical means, variations in stated values of E and G could make almost as much difference.

DESIGN COMPUTATIONS

Figure 3 summarizes section properties and resulting stresses for the design example. Values of gravity center, moment of inertia, and section modulus are computed in the ordinary fashion. Values of J for open beams may be taken from Ref. 8 or 9, where they are designated as K, or may be neglected. The design example incorporated these values primarily to indicate the relationship of "open" to "closed" J. Values of C_w, the warping constant, distance a, and ratios a/l and a/d are in no way required for analysis, but do serve to help define the semi-box girder, and will be discussed below. It is to be noted that the full value of I_y is used for stability in computing Z, but only I_{yc}, the moment of inertia at the top chord, is used for computing section modulus and stress resulting from horizontal loading. Stress at the toe of the bottom flange due to the net effects of horizontal loading less rotation of the cross section is small and may be neglected in the semi-box girder. This procedure is conservative since this strength is neglected.

The stress summary indicates two design conditions. The second condition was considered to be sufficiently remote that an overstress of the order indicated was acceptable. It is to be noted that AISC Formula (1.6-2) is suggested for the stress addition, rather than a direct numerical addition. The top chord value of allowable bending stress F_{bx}, indicated as 18.3 ksi, was taken from a design curve developed later (see Fig. 5).

MAXIMUM SLENDERNESS WITHOUT STRESS REDUCTION

In order to approach the upper limit of the slenderness ratio l/Z, where failure in the inelastic range will take place at or very close to F_y, refer to the present AISC rule for a full box girder, stated previously.

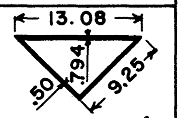
Referring to Fig. 4 for a simple rectangular box girder, the values

$$I_y \approx b^2\left(\frac{A_w}{2} + \frac{A_f}{6}\right)$$

$$J \approx \frac{2b^2d^2A_wA_f}{d^2A_f + b^2A_w}$$

$$S_x \approx d\,(A_f + \tfrac{1}{3}A_w)$$

DESIGN EXAMPLE PROPERTIES

Section	W36X135 2-Ls 8x6x½	J-Closed $= \frac{4A^2}{\Sigma \frac{\Delta s}{t}}$	 137. in.^{4}
Weight	181 lbs/ft.		
Area	53.2 in.^{2}		
h_c	14.1 in.	J-Total	145.6 in.^{4}
h_t	21.4 in.	$Z = \frac{\sqrt{I_y J}}{S_x}$	.450 in.
I_x	9890 in.^{4}		
S_{xc}	701 in.^{3}	ℓ/Z	1333
S_{xt}	462 in.^{3}	C_w	99,200 in.^{6}
I_{yc}	579 in.^{4}	$a = \sqrt{\frac{C_w E}{J G}}$	42.0 in.
I_{yt}	103 in.^{4}		
I_y total	682 in.^{4}	$\frac{a}{\ell}$	.070
S_{yc}	57.8 in.^{3}		
J-Open= $\Sigma\frac{1}{3}bt^3$	Bm 7.5 in.^{4}	$\frac{a}{d}$	1.18
	Ls 1.1 in.^{4}		

DESIGN EXAMPLE STRESSES

Office practice analysis Unit Stress at pts. indicated	1 crane on span M_v=762 ft.-kips M_h= 30 ft.-kips including impact		2 cranes on span M_v=996 ft.-kips M_h= 40 ft.-kips excluding impact	
TOP CHORD	PT. A	PT. B	PT. A	PT. B
f_{bx}	−13.1	−8.4	−17.1	−11.1
f_{by}	−3.7	−6.2	−4.9	−8.3
$f_{bx}/F_{bx}=f_{bx}/18.3$	.71	.46	.93	.60
$f_{by}/F_{by}=f_{by}/21.6$	.17	.29	.23	.38
SUM	.88	.75	1.16	.98
BOT– f_b	+19.8		+25.9	
TOM F_b	+21.6		+21.6	
CHORD Over	—		20%	

Figure 3

may be substituted in Eq (5). Then,

$$Z_{box} \approx \frac{b^2\sqrt{[A_wA_f(A_w + \frac{1}{3}A_f)]/(d^2A_f + b^2A_w)}}{A_f + \frac{1}{3}A_w} \tag{11}$$

This relationship is simplified if $K = A_w/A_f$. Then,

$$Z \approx \frac{b^2}{\sqrt{Kb^2 + d^2}}\left\{\frac{\sqrt{K(K + \frac{1}{3})}}{1 + \frac{1}{3}K}\right\} \tag{11A}$$

Curves were plotted (Fig. 4) to determine the values of Z/b for various values of d/b and A_w/A_f. A lower limit of $Z/b = 0.125$ was established on the following basis: An actual practical girder with 50 x ¼-in. webs and a 5 x 1⅜-in. flange ($d/b = 10$) was considered as one which would meet the AISC Specification and would not be subjected to lateral buckling. For this girder,

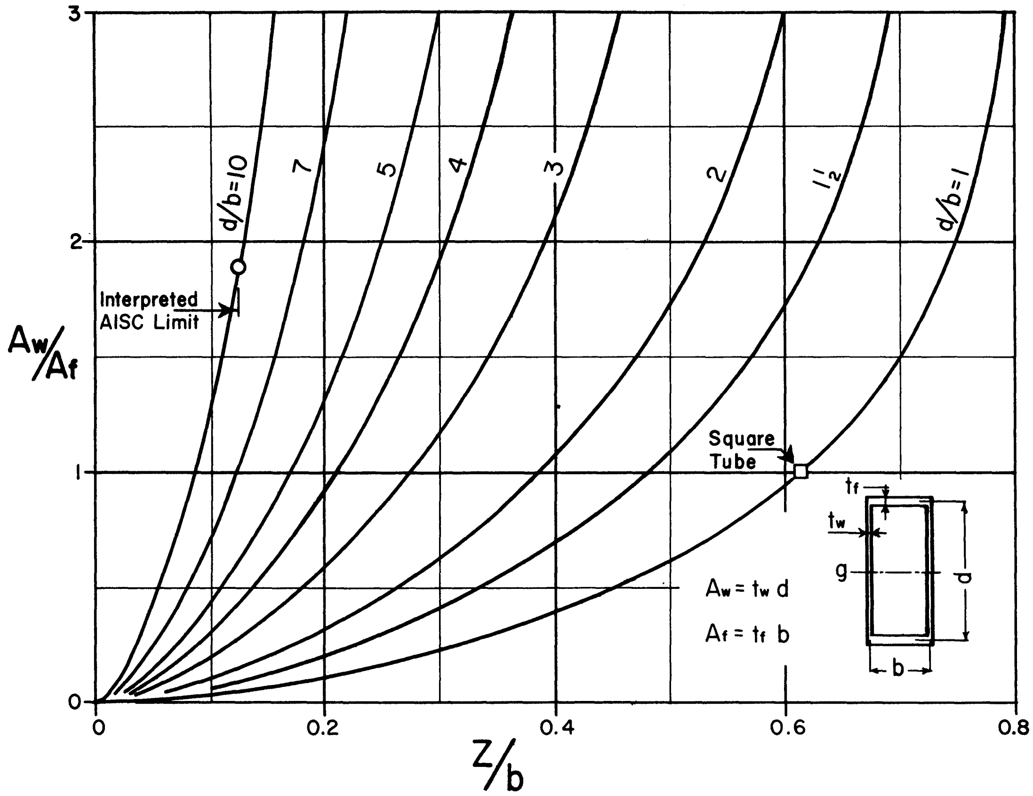


Fig. 4. Z Related to Box Girder Proportions

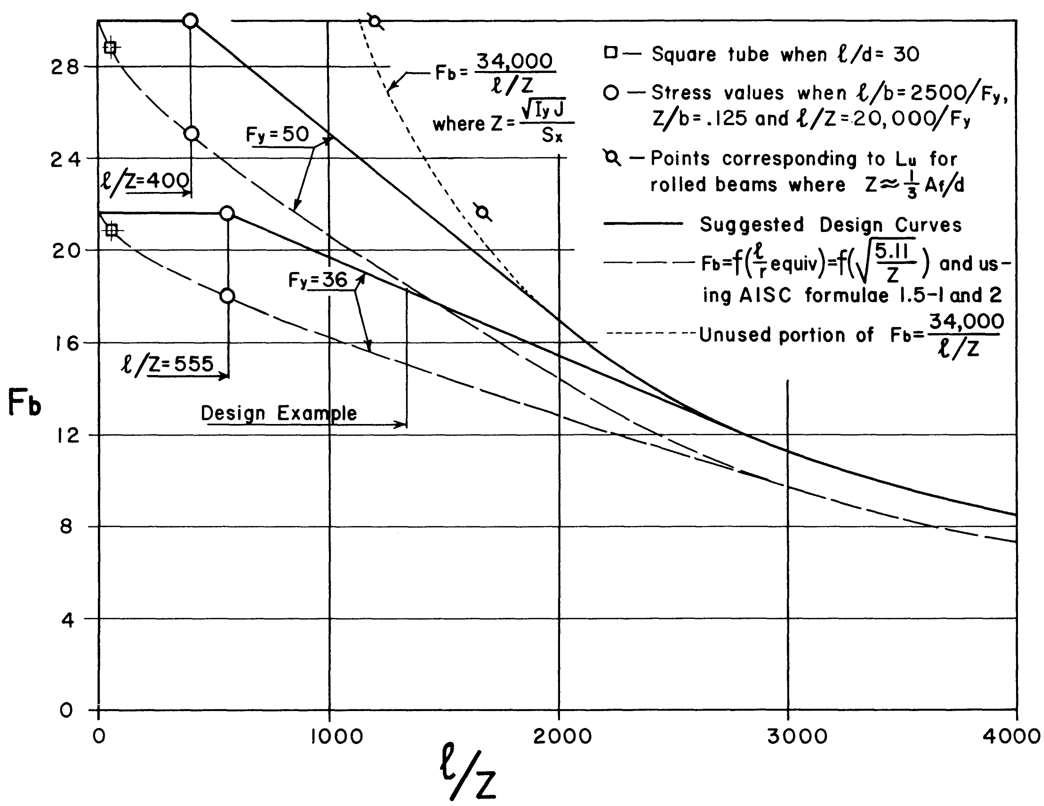


Fig. 5. Working stress vs. slenderness ratio for simple beams

Z/b was found to be 0.125. Likewise, a girder with $d/b = 7$ and with web and flange area about equal will also result in a value of $Z/b = 0.125$. A girder such as this may be built with webs of 49 x ¼-in. and flanges of 7 x 1¾-in., again a practical girder. Thus, the value of $Z/b = 0.125$ appears to the writer to be well within the limit suggested in the AISC Commentary.

The unit stress $F_b = 0.60F_y$ is allowable when $l/b \leqslant 2500/F_y$. The substitution of $Z/b = 0.125$, results in

$$l/Z \leqslant 20{,}000/F_y \qquad (12)$$

under the same circumstances. This establishes quite well the upper limit of bending stress (without reduction), whatever the configuration of the girder, so long as it is symmetrical about its vertical (y-y) axis, since Z is a general term.

COLUMN ANALOGY METHOD AND ITS APPLICATION

Figure 5 depicts the curves resulting from plotting, as allowable values of bending stress, the allowable compressive stresses dictated by AISC Formulas (1.5-1) and (1.5-2) for l/r values determined by the equation $(l/r)_{equiv} = 2.26\sqrt{l/Z}$ for $F_y = 36$ and $F_y = 50$ and as recommended in Ref. 4. It is apparent that the spread between the "allowable" stresses at $l/Z = 555$ and $l/Z = 400$ is great enough that one cannot say one approximates the other, since the values at $0.6F_y$ are 20 percent greater than those dictated by the curves.

Also shown near the upper end of the curves are the points representing the square tube shown on Fig. 4, when the unsupported length is 30 times the depth. The points are found in a rational fashion as follows:

From Fig. 4, $Z/b = 0.613$

When $l/d = 30, \quad l/b = 30$

Therefore, $l/Z = 30/0.613 = 49$

According to this equivalent l/r method, the allowable stress for such a tube is reduced about 3½ percent. Without the aid of any theory, and relying only upon ordinary logic, it is apparent that a beam whose strength is equal in *all* directions can only fail in the direction in which load is applied. In other words, it cannot fail by general buckling. Initial inspection of this curve, which appeared unnatural, combined with the above, caused the writer to question its accuracy when applied to box members.

Putting the basic solution from Eq. (3) into the form

$$F_{bcr} = \frac{\pi}{lS_x}\sqrt{EI_yGJ + \frac{\pi^2E^2I_yC_w}{l^2}}$$

When C_w is either small or neglected, Eq. (3) reduces to

$$F_{bcr} = \frac{\pi}{lS_x}\sqrt{EI_yGJ} = \frac{\pi\sqrt{EG}}{S_x(l/Z)} = 56{,}500/(l/Z) \qquad (6)$$

Also,

$$F_{bcr} = \frac{\pi^2E}{S_xl^2}\sqrt{I_yC_w} = \frac{286{,}000}{l^2}\left(\frac{\sqrt{I_yC_w}}{S_x}\right) \qquad (13)$$

when the torsional constant J is either small or neglected.

Since

$$C_w = \left(\frac{I_{yc}I_{yt}}{I_y}\right)d^2 = KI_yd^2$$

where I_{yc} is the moment of inertia of the compression flange about the y-y axis, I_{yt} is the same for the tension flange for unlike flanges, and K is a constant whose value for flanges which are alike will be ¼. Substitution in Eq. (13) gives

$$F_{bcr} = \frac{\pi^2E}{l^2}\left(\frac{(I_y)\sqrt{K}}{S_x/d}\right) \qquad (14)$$

It is recalled

$$F_{acr} = \frac{\pi^2E}{l^2}\left(\frac{I}{A}\right) \qquad (15)$$

for an Euler Column.

A striking similarity is observed. So much so that one would fully expect such a beam to follow a pattern of failure in the inelastic range much as a column. The equivalent formula,

$$\left(\frac{l}{r}\right)_{equiv} = l\sqrt{\frac{S_x}{\sqrt{I_yC_w}}} \qquad (16)$$

may be obtained, by substitution, in a fashion similar to that referred to above for a beam with torsional strength. It is apparent that the plot of this relationship will produce a curve identical in shape to the column curve, since for any section l/r and l are now proportional. This shows that a beam type, diametrically opposite to that considered, falls most nicely into the column analogy, where the factors of stability are almost identical, whereas this sort of resemblance is not found in the relationships applicable to the subject.

In order that the beam possessing entirely torsional stiffness lose stability to the same degree as either a beam possessing entirely warping stiffness or a column, it is apparent that the loss in the product $\sqrt{EI_y}\cdot\sqrt{GJ}$ for the former must be the same as EI_y for the latter. In other words, as the fibers of the torsion beam plasticize and reduce lateral stiffness, the torsional stiffness must fall off at the same rate, such that the product of the losses will be the same as that for the beam or column dependent only on lateral stiffness. If this does not occur, there is no point in attempting to compare a torsion beam to a column. Some treatises, including that by Timoshenko on pg. 272 of Ref. 5, assume that torsional rigidity diminishes in the same proportions as flexural rigidity. This seems reasonable for an open H-section.

In the instance of the configuration of a box with an overhanging flange, such as the Hadley Delta, it becomes apparent that initial loss of flexural stiffness at the overhang will have no effect on the torsional stiffness provided by the box. The degree of this isolation of flexural rigidity from torsional rigidity is considerably greater in the Luss Y or the Gull, where the overhang indicated is greater. Finally, in the Gull an added isolation is provided by the sloping tips, such that an initial loss in flexural stability will not occur as readily, since the tips are closer to the neutral axis than the flange. The result of what may appear as a design inefficiency is adequately compensated by this added section toughness.

Thus, the writer believes that the column analogy method should provide good correlation with inelastic buckling strength of members whose stability is mostly due to warping strength, such as deep plate girders, but that it is unduly conservative when applied to the inelastic buckling strength of the members considered.

REMAINDER OF SUGGESTED DESIGN CURVE

A straight line drawn from the upper limit derived from present codification to tangency with Eq. (7) appears to be the most reasonable approach to tie the two segments together. The point of tangency occurs at about 0.55 maximum fiber stress rather than 0.50, which has been used for the basic column curve and as explained on pg. 14 of Ref. 4. While the straight line might be changed to a curve to force tangency at the 0.50 level, it is not considered necessary.

In his dissertation, Lambert Tall[10] showed that the residual stresses in a welded box column are greater than in a rolled section, and thus welded columns are weaker than rolled columns. Since any box or semi-box section will require some welding, residuals will tend to be higher and perhaps suggest a point of tangency with the elastic curve below the 0.50 level, were there not other considerations.

The writer presumes that the isolation referred to above is sufficient to allow the point of tangency suggested. For example, if isolation were complete, the residual stress at extreme fiber could be as large as $0.7F_y$ and still allow the tangency suggested, since

$$\sqrt{1 \times (1 - 0.7)}\ F_y = 0.55F_y$$

In order to provide a degree of validation to these suggested curves, results of tests of unsupported rolled sections made by Hechtman, et al[11] are plotted on Fig. 6, along with the suggested curve. It must be explained that the tests used are for sections supported on knife edges, whose top flanges were supported in a fashion which kept the webs plumb at the support, without restraining the flanges. They did find validation for the use of L_u and Formula (1.5-7) when connection angles

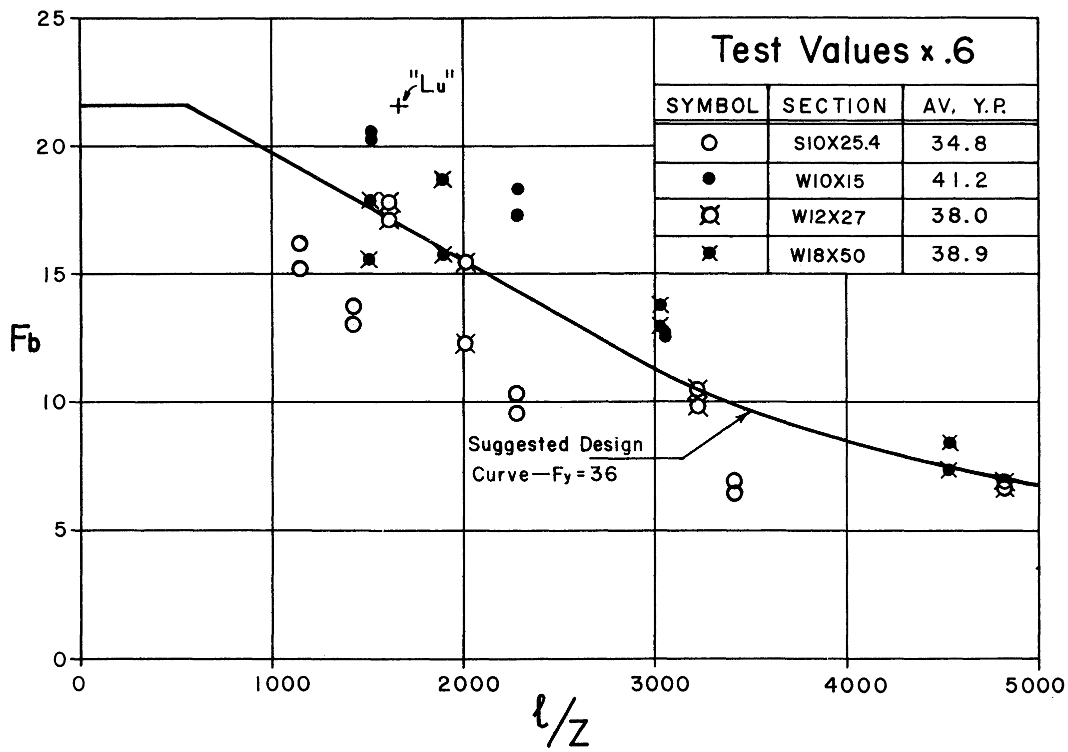


Fig. 6. Comparison of tests with design curve

or seated connections with clip were used. The equivalent points for L_u are plotted in Figs. 5 and 6 as a convenient point of reference, familiar to the reader. The fit of these tests appear quite good considering the number of tests available.

MATHEMATICAL REPRESENTATION

Equations resulting from the above and for the two yield points are then:

$$F_y = 50: \quad \left.\begin{array}{l} l/Z \leqslant 400; \quad F_b = 30 \\ 400 < l/Z \leqslant 2045; \quad F_b = 30 - 0.00813\ (l/Z - 400) \\ l/Z > 2045; \quad F_b = 34{,}000/(l/Z) \end{array}\right\} \tag{17}$$

$$F_y = 36: \quad \left.\begin{array}{l} l/Z \leqslant 555; \quad F_b = 21.6 \\ 555 < l/Z \leqslant 2840; \quad F_b = 21.6 - 0.0421\ (l/Z - 555) \\ l/Z > 2840; \quad F_b = 34{,}000/(l/Z) \end{array}\right\} \tag{18}$$

OTHER PROPERTIES

The value of C_w for the design example is shown in Fig. 3. When it and other required values are substituted into the relation

$$\sqrt{1 + \frac{EC_w}{GJ}\left(\frac{\pi^2}{l^2}\right)}$$

which is part of Eqs. (1) and (3), it is found that the value of the radical is 1.024, which means that the warping stiffness contributes another 2.4 percent to the critical moment or critical stress in the elastic range. Hence, it is negligible and its computation is not required.

The fact that the ratio a/l is well under 0.1 is another indication that warping will have but little effect on member behavior. This value is worthy of consideration when defining usage of a section as a semi-box girder.

The ratio a/d describes the section. Its value will probably be less than 1.75 to qualify. Most rolled beams have a ratio of a/d well over 3, as may be observed in Ref. 8. The smallest value is for a W14×426, which has a value of 1.77.

The torsional strength of the section should not be overlooked. As is indicated in Fig. 1, the vertical loading may be mis-centered due to a sweep in the beam, a rail mis-centered on the flange, or a combination of the two. Likewise, the horizontal load is applied well above the center of twist (which may be taken as the c.g of the box). These torsional moments are for the most part resisted by the torsional stiffness of the box. The resulting shear stress will be found to be small, and the twist of the section negligible, for the condition pictured.

The angle of twist, θl, may be approximated from relationships stemming from a cantilever twisted at the free end, which is defined by

$$\theta l = (M_t/GJ)l \tag{19}$$

If an exact solution is required, the term l must be replaced by $(l - a)$.

The value of the shear stress due to an applied moment M_t may be readily computed by the relationship

$$f_v = M_t/2At \tag{20}$$

where A is the area of the box and t the thickness of the box wall in question.

Not only are sections of this type quite strong and stiff in torsion, but also lend themselves to this simple analysis. Compare the above with the relationships found in the Appendix of Ref. 9.

FINAL COMMENTS

In the event others see in this type of construction the many virtues apparent to the writer, and described above, it should become sufficiently popular to warrant codification. Perhaps the sort of research indicated by Lynn S. Beedle, et al.[12] will provide additional information lacking at present, and listed as a future need. In the interim, the writer trusts that the suggestions made herein will assist those who wish to consider the semi-box girder.

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