Applied Plastic Design of Unbraced Multistory Frames

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Many engineers assume that plastic design is too cumbersome when applied to unbraced multistory frames. Actually, a method exists that enables the average structural engineer to perform a plastic design of unbraced multistory frames easily. It is considered sufficiently accurate for the design of frames of limited height.

This paper describes this practical procedure. The method is suitable for the design office, since it can be readily adapted to tabular computations or for a computer.

DESIGN CRITERIA

There are three criteria for the structural design of an unbraced multistory building. In plastic design they may be stated as follows:

(1) The frame must have sufficient strength and stability under gravity loads only using a load factor, $F = 1.7$. This criterion would most likely govern the design of low rise structures with several bays.

(2) The frame may be subject to a limitation on drift due to wind or seismic loads under working loads, $F = 1.0$. This criterion might govern with taller and more narrow structures.

(3) The frame must be stable under combined gravity and lateral loads, $F = 1.3$. This criterion might also govern with taller and more narrow structures.

GRAVITY LOADS ONLY ($F = 1.7$)

The design procedure for unbraced frames ($F = 1.7$) is similar to that for braced frames. It is adequately described in available references[1,2] and, therefore, is not discussed in detail in this paper.

Essentially, the girders are designed as three-hinge mechanisms. The columns are designed using the appropriate interaction equation with an effective length greater than unity. (In this respect, it is anticipated that the procedures described by Yura[3] will result in significant economies.)

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WORKING LOAD DRIFT ($F = 1.0$)

Design for criterion 2 can be performed easily using a simplified procedure without the need for a complex elastic analysis.

From the theory of elasticity, the drift Δ in a single story multibay frame is

$$\Delta = h\theta \left[\frac{2\Sigma G_{\text{girders}}}{\Sigma G_{\text{columns}}} + 1 \right] \tag{1}$$

where

Δ = story drift, in.
h = story height, in.
θ = rotation of a particular joint, radians
G_{girders} = sum of all the stiffness factors in the frame, I/L in.[3]
G_{columns} = sum of all the stiffness factors of the columns in the frame, I/L in.[3]

In Eq. (1) it is necessary to determine θ in terms of the moment applied to the frame, $M_{\text{sway}} = Qh$. (Q is the total horizontal shear from wind or seismic forces.) For equilibrium, M_{sway} must not exceed the total of the resisting girder end moments, ΣM_g.

ΣM_g is determined by recognizing the relationship between the elastic girder end moment, its corresponding angle of rotation θ, and its stiffness factor, G. It has been shown that for practical purposes θ may be assumed equal for all girders.[4]

$$\Sigma M_g = 2 \times 6\, E\, \theta \Sigma G_{\text{girders}} = M_{\text{sway}}$$

$$\theta = \frac{M_{\text{sway}}}{12\, E \Sigma G_{\text{girders}}}$$

Substituting for θ in Eq. (1):

$$\Delta = \frac{h M_{\text{sway}}}{12\, E \Sigma G_{\text{girders}}} \left[\frac{2\Sigma G_{\text{girders}}}{G_{\text{columns}}} + 1 \right]$$

$$\Delta = \frac{h M_{\text{sway}}}{6E} \left[\frac{1}{\Sigma G_{\text{columns}}} + \frac{1}{2\Sigma G_{\text{girders}}} \right] \tag{2}$$

The drift computed in Eq. (2) should be less than that permitted by the specified drift index, Δ/h. The drift index is a design criterion and is selected on the basis of several variables, including the wind load

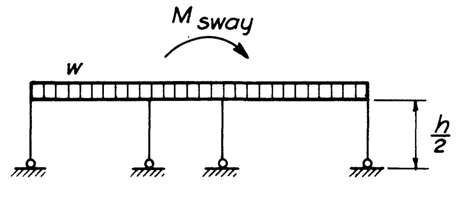

Figure 1

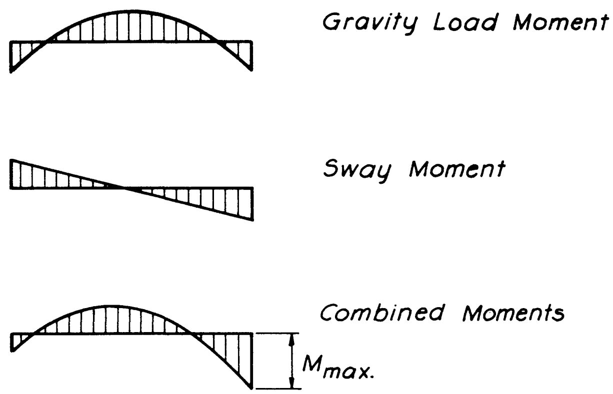

Figure 2

required by the applicable code. Elligator and Nassetta have published an excellent discussion of this subject.[5]

It should be noted that M_{sway} in Eq. (2) does not include the $P\Delta$ effect. This is because the drift indices traditionally used are based on the historical practice of ignoring this small additional moment.

COMBINED GRAVITY AND LATERAL LOADS (F = **1.3**)

A simplified design procedure for combined loads (criterion 3) is presented in this paper. It is based on the subassemblage method developed by the research team at Lehigh University.[6]

A multistory frame can be considered to be a series of single story frames stacked vertically. If each story is stable, the entire structure can be considered stable.

Figure 1 is taken from a single story of a multistory frame with uniformly loaded girders subjected to a moment M_{sway}. To remain stable, the frame must be able to develop a resisting moment M_R greater than the sway moment.

$$M_R > M_{\text{sway}} \tag{3}$$

The following assumptions are made for the computation of M_R:

(1) Points of contraflecture are assumed at mid-story level.
(2) The ends of all the girders in the story rotate through the same angle.
(3) No plastic hinges are permitted to form in the columns.

M_{sway} in Eq. (3) must include the $P\Delta$ effect because of the large drifts associated with the collapse mechanism.

Under combined gravity and lateral loads, the girders in a frame will develop their largest moments at their leeward end (see Fig. 2). Consequently the largest possible moment M' that a girder could have available to contribute to sway resistance is

$$M' = M_p - M_{FE} \tag{4}$$

where

M_p = plastic moment capacity of the girder, kip-ft
M_{FE} = fixed end moment of the girder under combined lateral and gravity loads (F = 1.3), kip-ft

The relationship between M' and end rotation θ is

$$\theta = \frac{M'L}{6EI} = \frac{M'}{6EG} \tag{5}$$

The first girder to develop a plastic hinge (at its leeward end) is the one which rotates through the smallest angle θ before reaching M'. This will be the girder with the smallest M'/G ratio and is termed the "α girder." Its M'/G ratio is designated α.

$$\alpha = \left(\frac{M'}{G}\right)_{\min} \tag{6}$$

Since all the girders are assumed to rotate through the same angle, the maximum moment M'_R which the frame can resist before the first hinge forms (in the α girder) is

$$M'_R = 2\Sigma\theta\,\frac{6EI}{L} = 12E\theta\Sigma G_{\text{girders}}$$

From Eqs. (5) and (6),

$$\theta = \frac{\alpha}{6E} \tag{7}$$

Therefore,

$$\begin{aligned} M'_R &= 12E\left(\frac{\alpha}{6E}\right)\Sigma G_{\text{girders}} \\ &= 2\alpha\Sigma G_{\text{girders}} \end{aligned} \tag{8}$$

For frame stability M'_R must exceed the total sway moment, M_{sway}, imposed on the story (Eq. 3).

M_{sway} in Eq. (3) is calculated as a function of the wind (or seismic) story shear Q and $P\Delta$.

$$M_{\text{sway}} = Qh + P\Delta \tag{9}$$

where

P = total of all the vertical loads above the story being analyzed, kips
Δ = story drift, in.

An expression for the drift can be derived as follows:

From Eq. (1),

$$\Delta = h\theta\left[\frac{2\Sigma G_{\text{girders}}}{\Sigma G_{\text{columns}}} + 1\right]$$

From Eq. (7),

$$\theta = \frac{\alpha}{6E}$$

Therefore,

$$\Delta' = \frac{h\alpha}{6E}\left[\frac{2\Sigma G_{\text{girders}}}{\Sigma G_{\text{columns}}} + 1\right] \tag{10}$$

where Δ' is the story drift associated with the first hinge to form.

The sway moment, M_{sway}, can now be calculated by substituting Δ' from Eq. (10) into Eq. (9). *It should be noted that the total M_{sway} for a particular story is the M_{sway} computed for that story by Eq. (9) plus the sum of all the sway moments above the story.* For this reason design procedure is to start at the top story.

If M_{sway} exceeds M'_R, it indicates that a hinge has formed at the leeward end of the α girder. However, it does not necessarily mean that the frame is unstable.

The behavior of the frame at this stage can be visualized by a typical plot of M_R vs. Δ, as in Fig. 3. The frame is entirely elastic until the first hinge forms in the α girder at point 1. In this state the frame may still be stable, but the M_R vs. Δ relationship changes as indicated by the change in the slope of the line between points 1 and 2. At point 2, a plastic hinge has formed at a second location in the frame. The girder in which this second hinge forms is termed the β girder. At point 3, a third girder (γ girder) has formed a plastic hinge. More plastic hinges may possibly form (δ, ϵ, etc.), but eventually the curve reaches a peak indicating unlimited sway Δ and instability. Therefore, if Eq. (3) is not satisfied prior to the formation of the first hinge, the analysis continues to point 2 or beyond as explained below.

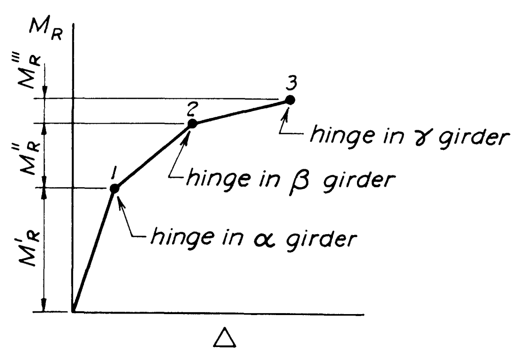

Figure 3

The second hinge in the frame may form at the leeward end of a particular girder other than the α girder, or it may form at the windward end of the α girder. The smallest M''/G ratio for either case defines the location. M'' is the additional moment which would result in a plastic hinge.

The smallest M''/G ratio for all girders *except the α girder* will be for the leeward end of the girder with the second lowest M'/G ratio. Since the M'/G ratios are known, this girder can be immediately identified and its M''/G ratio can be readily determined from Eq. (12) as derived below:

$$M'' = M_p - M_{FE} - \alpha G \tag{11}$$

$$M'' = M' - \alpha G$$

$$\frac{M''}{G} = \frac{M'}{G} - \alpha \tag{12}$$

In Eq. (12), both M'/G and α have, of course, been previously calculated.

The M''/G ratio for the α girder must be calculated on a slightly different basis. This is because the next hinge to form in this girder will be at its windward end or within the span. Also the stiffness of this girder has been reduced due to the existence of the hinge at its leeward end.

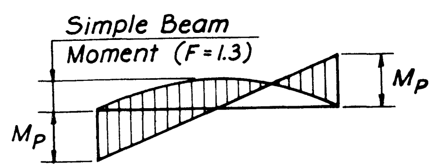

Figure 4

The moment diagram of the α girder is shown in Fig. 4. For a uniformly loaded girder, it can be shown that for a second hinge to form, the moment at the windward end, M_e is

$$M_e = 4\sqrt{3M_{FE}M_p} - 6M_{FE} - M_p$$

From Fig. (4),

$$M'' = M_e + M_{FE} - \alpha G \tag{13}$$

Therefore,

$$M'' = 4\sqrt{3M_{FE}M_p} - 6M_{FE} - M_p + M_{FE} - \alpha G \tag{14}$$

Multiplying the right hand side of Eq. (14) by M_p/M_p

$$M'' = \left[6.94\sqrt{\frac{M_{FE}}{M_p}} - 5\left(\frac{M_{FE}}{M_p}\right) - 1\right]M_p - \alpha G \tag{16}$$

or

$$M'' = KM_p - \alpha G \tag{17}$$

where

$$K = 6.94\sqrt{\frac{M_{FE}}{M_p}} - 5\left(\frac{M_{FE}}{M_p}\right) - 1 \qquad (18)$$

For the convenience of the reader, Fig. (5) plots K vs. M_{FE}/M_p for use in Eq. (17). It is noted that the M_{FE}/M_p ratio varies between 0.33 and 1.0. At 0.33 a hinge will have formed at the windward end and, of course, M_{FE} can not exceed M_p.

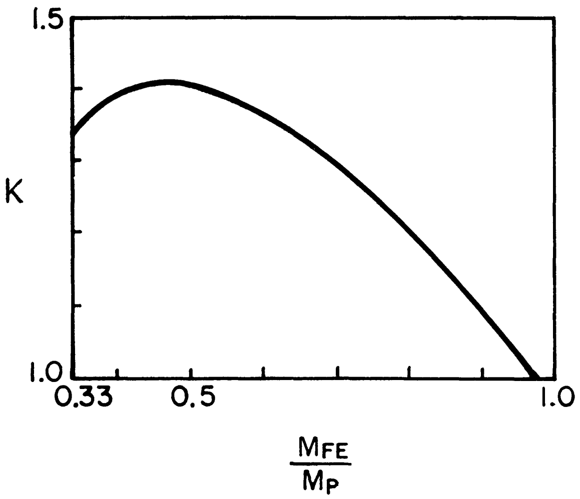

Figure 5

In Eq. (17), G is the stiffness I/L of the α girder, but to compute the M''/G ratio of the α girder at this stage, use $G = I/2L$. This is because the girder has a hinge (at the leeward end) with consequent loss of stiffness.

The smallest M''/G in the frame can now be determined, and is designated β. The β girder is identified.

The additional frame resisting moment M''_R is

$$M''_R = 2\beta\Sigma G_{\text{girders}} \qquad (19)$$

In Eq. (19), G for the girder with a plastic hinge is reduced. For instance, at this stage G for the α girder would be $I/2L$. If a girder develops a hinge at both ends, G for that girder would be zero.

The total resisting moment is

$$M_R = M'_R + M''_R \qquad (20)$$

At this stage a new value of Δ is also calculated by determining the additional drift Δ'':

$$\Delta'' = \frac{\beta h}{6E}\left[\frac{2\Sigma G_{\text{girders}}}{\Sigma G_{\text{columns}}} + 1\right] \qquad (21)$$

The total Δ is

$$\Delta = \Delta' + \Delta'' \qquad (22)$$

As in Eq. (19), G_{girders} in Eq. (22) is computed on the basis of the appropriate reduction in girder stiffness.

Using Δ from Eq. (22), M_{sway} is recalculated and compared to M''_R. If instability is still indicated, the analysis proceeds to the point where another hinge forms in the structure. This process continues to the point where the frame resisting moment, M_R, will either exceed M_{sway} or will not continue to increase. If the latter is the case, stability is impossible and the girders must be stiffened.

In practical design it would seem that analysis beyond the first several hinges would not be necessary. This is because of the following reasons:

(1) After several leeward hinges have developed, the M_R vs. Δ curve tends to flatten, indicating an unacceptable decrease in frame stiffness. At this stage, large deflections are required to achieve a small increase in moment resistance and imminent instability becomes apparent.

(2) Frames with several bays are more likely to be controlled by criterion 1, gravity loads only.

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