Automated Optimum Design of Unstiffened Girder Cross Sections

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SINCE THE advent of the electronic digital computer, substantial change has occurred in the practice of structural engineering. This change has been particularly rapid since the second generation machines became available with their substantially reduced computation cost. The third generation machines have been even less expensive to use and much more accessible.

For the structural engineer, the greatest application has been in the elastic analysis of large structures and today almost every engineering firm makes routine use of some computer analysis capabilities. The impact on structural design has been less direct. Certainly, the availability of a good analysis has increased the capability and confidence of the designer, but, in general, application design methods have remained practically unchanged. Either the computer has been applied exclusively as an analysis tool or traditional design methods have been programmed for automatic computation. The first approach leads to serious problems with man-machine communications. Large amounts of *error-free* input data must be prepared and voluminous output digested. The second approach is successful in many cases as proven by the numerous, elaborate design programs which have been written in the last few years. However, for problems where the number of variables that must be selected is large or closely dependent, direct methods either fail or are extremely inefficient. It seems that programmed optimization methods have reached a stage where they can be useful in solving some of these problems. One purpose of this paper is to present an optimization technique which can be useful to the structural designer today.

The idea of structural optimization is a very old one and the problem has been approached in many different ways. Here, a rather narrow definition of the term will be used. Optimization will refer to design methods which select the best design that satisfies all predetermined limitations using automated search techniques.

It is fair to say that the major efforts in automated structural design began at Case Institute of Technology (now Case Western Reserve University) in the late 1950's under Prof. Lucien Schmit[1]. The primary applications were for minimum-weight design of aircraft and aerospace structures. Subsequently, attention was turned to civil engineering structures, using minimum weight or minimum cost as a design criterion. One of the most extensive applications of the automated design procedures has been the development of the GAD program for the selection of the minimum cost design of highway bridge plate girders[2,3]. It was developed on behalf of the Ohio Department of Highways and is used by scores of firms for their bridge design work. Recent versions of the program have included variable depth, composite sections and material strength selection. Other examples of optimization in engineering structures have been automated design of prestressed beams[4], grillages[5], selection of plate widths and thickness in ship decks and bulkheads[6], and transmission towers[7]. The mathematical techniques of automated design have included linear and dynamic programming and gradient methods. This paper presents an efficient technique based on unconstrained minimization. It is especially useful for the automated design of elements such as beams and columns with well defined code limitations. The technique has the special advantage that the formulation of the structural design can be separated from the automated optimization part of the program, which can be treated as a closed black-box procedure.

PROBLEM FORMULATION

Consider the design of a welded beam cross section made up of three plates assembled in a symmetrical wide flange shape. To define this cross section, four independent design variables are required:

b_f = flange width (symmetrical section)
t_f = flange thickness
b_w = web depth, measured from inside of flanges
t_w = web thickness

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In a particular application, it is assumed that the applied moment, shear, and unsupported length are known for the member. The section is to be proportioned to satisfy the requirements of the AISC Specification of February 12, 1969. This problem is similar to the one reported by Holt and Heithecker[8], except that the present paper includes the capability for designing laterally unsupported beams. It is assumed that the web is unstiffened. Many of the constraints change, depending on relations between the design variables. Therefore, they can best be stated under the broad subheadings of shear stress, bending stress, and geometrical constraints. All of the constraints are listed in the Appendix, with reference to the section number of the specification where the limitation is stated.

The objective selected for this problem is to find the minimum weight design. Since the member is defined to be prismatic, this goal reduces to minimum area cross section and can be stated as:

$$\text{minimize } f = b_w t_w + 2 b_f t_f \tag{1}$$

Thus, the design problem has been stated in a completely unambiguous fashion.

It has been implied that the design variables can vary continuously. This is rarely practical in the construction and fabrication of steel structures. Steel plates are available in discrete sizes, and plates must be selected from these sizes. No reference has been made to the solution of these problems. It is a relatively easy matter, however, to attach a simple grid search on available sizes at the end of the continuous optimization process. Also, a computer procedure has been developed[9] for the case of a structural system composed of many members, in which overall costs are reduced by the selection of members on the basis of optimum cost, including fabrication, rather than optimum weight.

DESIGN PROCEDURE

A recently developed technique of automated optimum design will be used. The general formulation of a constrained design problem, as described above, is:

$$\text{Minimize } f(x_1, x_2, \ldots x_n)$$

$$\text{such that: } g_j(x_1, x_2, \ldots x_n) \geq 0 \qquad j = 1, \ldots m \tag{2}$$

where $x_1, x_2, \ldots x_n$ are the design variables to be found. These are, in the girder design, the flange width and thickness and the web depth and thickness. The weight or cost that is to be minimized is $f(x_1, x_2, \ldots x_n)$. The constraints or limitations are, $g_j(x_1, x_2, \ldots x_n)$, based on the specifications. In this problem the constraints are from the AISC Specification and are given in this form in the Appendix.

This minimization problem will be put in the form of an unconstrained minimization, since many efficient mathematical tools are available in closed black-box-programs for minimizing such functions, i.e., minimize a sequence of functions:

$$P(x_1, x_2, \ldots x_n, r_k) = f(x_1, x_2 \ldots x_n) + r_k \sum_{j=1}^{m} \frac{1}{g_j(x_1, x_2, \ldots x_n)} \tag{3}$$

The function $P(x, r_k)$ is minimized successively for smaller values of r_k until $r_k \rightarrow 0$. At the end of this sequence, the minimum of $P(x, r_k)$ is the same as the minimum of the objective function $f(x)$. The reasoning behind this formulation is as follows: By starting with an acceptable design (and this is required for the technique), all the terms inside the summation, namely $1/[g_j(x)]$ will be positive. Therefore, in minimizing the function, $P(x, r_k)$, it is not possible to cross or even approach a design limitation or constraint, since at the constraint $g_j \rightarrow 0$ and $1/g_j$ would go to infinity, thereby increasing instead of decreasing the function. The term $1/[g_j(x)]$ may be viewed in the same way as a penalty term which is added to the cost function if a constraint is approached. It keeps the design in the acceptable region of the design space. The reason for a sequence of $P(x, r_k)$ functions with decreasing r_k values is that it has been found to be more efficient and to require less computation to approach the optimum by this sequence, rather than with a single minimization. The key to this sequence is that after each minimization the minimum point is used as the starting point for the next sequence with a reduced r_k value. At the constraint boundary, the function $P(x, r_k)$ goes to infinity. Any point outside the constraints in the unacceptable region is not considered. Further, for large r_k values, the constraints are felt some distance from the boundary and cause rather smooth contour lines for the function $P(x, r)$. For small r_k values, the contours of $P(x, r_k)$ approach those of $f(x)$, except near the constraint where the contour lines bunch together. Thus, the smaller the value of r_k, the closer the minimum approximates the true optimum design. The only technique needed by the user in this method is an estimate of the initial value of r_k that should be used and the rate at which r_k is decreased. However, a little experience or some trial and error testing usually gives, quite quickly, a set of reasonable r_k values for a particular problem.

Using the formulation given in Eq. (3) above, the automated structural optimization problem is converted to an unconstrained minimization problem. Reference 10 discusses the theory of minimizing unconstrained functions. It is usually possible to treat this minimization as a closed black box, with a function going in

Table 1. Results: Unstiffened Welded Steel Beam Design

	Light Loading		Heavy Loading	
	Laterally supported (Case No. 1)	No lateral support (Case No. 2)	Laterally supported (Case No. 3)	No lateral support (Case No. 4)
INPUT				
Unsupported length, in.	0.0	120	0.0	1020
Yield stress, ksi	36.0	36.0	36.0	36.0
Shear load, kips	9.5	9.5	102	102
Bending moment, in.-kips	285	285	26000	26000
OUTPUT				
Area, sq. in.	2.97	3.29	55.26	73.88
Flange width b_f, in.	2.47	6.07	12.28	36.40
Flange thickness t_f, in.	0.25	0.15	0.86	0.67
Web depth, b_w, in.	14.58	12.76	74.82	60.01
Web thickness, t_w, in.	0.12	0.11	0.46	0.42
Shear stress F_v, ksi	5.35	6.49	3.07	3.84
Controlling Design Limitation (See Appendix)	g_{1b}	g_{1b}	g_{1b}	g_{1b}
Bending stress F_b, ksi	21.60	19.26	21.60	15.20
Controlling Design Limitation (See Appendix)	g_{2e} and g_{2h}	g_{2c} and g_{2f}	g_{2h} and g_{2i}	g_{2c} and g_{2f}

one end of a special purpose computer program which produces a minimum value along with the component values of the optimum. Such routines are now becoming available from time sharing companies.

The method described above has been applied to a number of examples. The results given below are examples presented by Holt and Heithecker (Ref. 8) for laterally supported girders. Their results were obtained by evaluating all possible combinations of constraint equations as in a traditional design procedure. Such an approach would be prohibitive for laterally unsupported girders presented herein and hybrid beams. In any case, all calculations would have to be repeated for materials of different yield strengths or hybrid sections.

Table 1 shows the results of the design examples. These represent cases of light and heavy loadings, and both laterally supported and laterally unsupported conditions. Note that the material yield stress is an input to the program. Table 2 shows the sequence of r_k values used, which were arrived at after a few trial and error procedures by a relatively inexperienced user

Table 2. Results: Values of Unconstrained Function $P(x,r_k) = f(x) + r_k \Sigma 1/g$

r_k	Laterally supported (Case No. 1)	No lateral support (Case No. 2)	Laterally supported (Case No. 3)	No lateral support (Case No. 4)
1000.0			153.2	203.8
100.0	139.1	148.5	77.1	99.4
1.0	4.74	5.19		
0.1			56.58	75.75
0.01	3.08	3.41		
0.0001			55.27	73.90
0.00001	2.98	3.29		

of the minimization program. It should be noted that the first cycle of minimization may produce a heavier design than the initial value. Subsequent minimization cycles with reduced r_k values would bring the design to smaller values. The unconstrained function $P(x,r_k)$ is, of course, reduced in every cycle. It is important that the initial design not violate any constraints or even lie on the constraint boundary. An additional routine using minimization techniques can be added to the program, which locates an acceptable starting design.

The active constraints and allowable stresses for the optimum design are also shown in Table 1, along with the relevant AISC Specification section. For the laterally unsupported cases, the optima exhibit rather wide flanges. This is consistent with the new 1969 AISC Specification, which does not preclude such flanges. The stress penalty introduced for the wide flange is balanced by its additional strength against lateral buckling.

The actual computer program containing the specifications, input and output information, and the evaluation of the $P(x,r_k)$ function runs to about four pages of listing. The minimization routine, which is a separate package on magnetic tape, has five pages of listing. The execution time on the UNIVAC 1108 computer was about 2 seconds, which gives a cost of approximately $0.50 per girder. In these results no attempt has been made to improve the basic minimization program by specializing it for the girder design, since further computer cost reduction hardly seemed justified.

COMMENTS ON THE SPECIFICATION

A more detailed study of the type reported here can be useful in evaluating the problems associated with the use of a particular specification in automated design. Inadequacies of a code, which can be placed in two categories, can cause as many difficulties for computer programs using direct programmed design techniques as they do for optimization programs. One category is the case where a particular constraint (an allowable stress) is either not defined for some value of design variables, or else has more than one value. The current version of the AISC Specification is quite free of these conditions. The only area where question arose was for the case where more than one of the requirements of Section 1.5.1.4.1. was violated. It was assumed that the smallest resulting value of allowable stress was then to be used.

The second class of difficulty is the presence of a discontinuity in the value of a constraint. The portions of the AISC Specification covered here contain discontinuities in two places. One of these occurs at the value of d/t_w equal to $412/\sqrt{F_y}$. For values of d/t_w less than or equal to this value, the allowable bending stress is specified as $0.66F_y$, while for greater values the bending stress is limited to $0.6F_y$. It was a simple matter to provide the linearized transition discussed in the Appendix. This modification is similar to the allowable stress given in Section 1.5.1.4.2. for flange width-thickness ratio.

A more serious difficulty occurs in dealing with lateral buckling. Two limitations are placed on L_b if an allowable bending stress of $0.66F_y$ is to be used. These limitations are placed on L_b/b_f and $L_b d/A_f$. If they are violated, a third limitation, L_b/r_t, is introduced (or since the more exact approach was used here, L_b/r_e). In the AISC Specification, Formula (1.5-6a) is used when this value lies within a specified range. It was assumed (and this is not clearly stated in the Specification) that if L_b/b_f and $L_b d/A_f$ limits are exceeded, but L_b/r_t is less than the specified limit, $0.6F_y$ should be used as the allowable stress. Obviously, a discontinuity exists at the point where the allowable stress jumps to $0.66F_y$. No difficulties were experienced in the limited number of examples tested. This portion of the Specification would be substantially simplified for computer applications if limits on L_b/r_t were used in both Sections 1.5.1.4.1 and 1.5.1.4.6a, with a continuous transition between $0.66F_y$ and $0.6F_y$.

It was found that the current version of the AISC Specification for girder design was quite amenable to computer application. Since it must be usable for hand computation, simplified allowable stresses must be given. The availablity of alternate limits given in the Appendices or the Commentary makes it much easier to use in computer applications.

CONCLUSIONS

A method has been presented for automating the selection of a girder cross section, subject to the limits of a rather complex specification. This method has the advantage that it need not be completely reprogrammed for a new set of specification limits. It is only necessary to drop, modify, or add the appropriate limitations (g-expressions). All of the remainder of the program is unchanged. Furthermore, it is unnecessary to have an extensive knowledge of optimization techniques, since unconstrained minimization programs are readily available and can easily be used.

No claims of cost saving are made for the optimum design, since obviously the same design could be found by the designer by direct design procedures. However, due to time limitations usually facing the designer, it is likely that the automatically obtained designs will be less expensive both in design time and structural cost. The required input data is very brief, making teletype input easily feasible. It should be possible to obtain the design of a single section for approximately one dollar in computer rental costs.

APPENDIX

Shear Stress Constraints (g_1)

If $C_v/2.89 \geq 0.4$

where $$C_v = \frac{240{,}300}{(b_w/t_w)^2 F_y} \quad \text{if } C_v \geq 0.8$$

and $$C_v = \frac{190}{(b_w/t_w)}\sqrt{\frac{5.34}{F_y}} \quad \text{if } C_v > 0.8$$

$$g_{a1} = 0.4F_y - f_v \quad \text{(AISC Sect. 1.10.5.2)}$$

If $C_v/2.89 < 0.4$

and C_v is as defined above,

$$g_{1b} = (F_y/2.89)C_v - f_v \quad \text{(AISC Sect. 1.10.5.2)}$$

Bending Stress Constraints (g_2)—This constraint is selected as the smallest of all of the following:

(a) If (i) $L_b \leq \dfrac{76.0 b_f}{\sqrt{F_y}}$

(ii) $L_b \leq \dfrac{20{,}000 b_f t_f}{dF_y}$

(iii) $\dfrac{d}{t_w} \leq \dfrac{412}{\sqrt{F_y}}$

(iv) $\dfrac{b_f}{2t_f} \leq \dfrac{52.2}{\sqrt{F_y}}$

$$g_{2a} = (0.66F_y - f_b) \quad \text{(AISC Sect. 1.5.1.4.1)}$$

(b) If (i), (ii), and (iii) are as for Case a,

but (iv) is $\dfrac{52.2}{\sqrt{F_y}} \leq \dfrac{b_f}{2t_f} \leq \dfrac{95.0}{\sqrt{F_y}}$

$$g_{2b} = F_y\left[0.733 - 0.0014\left(\frac{b_f}{2t_f}\right)\sqrt{F_y}\right] - f_b$$

(AISC Sect. 1.5.1.4.2)

(c) If (i), (ii), and (iii) are as for Case a,

but (iv) is $\frac{95.0}{F_y} < \frac{b_f}{2t_f} \leq \frac{176}{\sqrt{F_y}}$

$$g_{2c} = 0.6F_y\left[1.415 - 0.00437\left(\frac{b_f}{2t_f}\right)\sqrt{F_y}\right] - f_b$$

(AISC Appendix Sect. C2)

(d) If (i), (ii), and (iii) are as for Case a,

but (iv) is $\frac{b_f}{2t_f} > \frac{176}{\sqrt{F_y}}$

$$g_{2d} = \frac{20{,}000}{(b_f/2t_f)^2} - f_b \quad \text{(AISC Appendix Sect. C2)}$$

(e) If (i), (ii) and (iv) are as for Case a,

but (iii) is $\frac{412}{\sqrt{F_y}} \leq \frac{d}{t_w} \leq \frac{760}{\sqrt{F_y}}$

$$g_{2e} = F_y\left[0.7309 - 0.000172\left(\frac{d}{t_w}\right)\sqrt{F_y}\right] - f_b$$

[This limitation was added to avoid a discontinuity in allowable bending stress. Such discontinuities sometimes cause difficulties with optimization techniques. While this modification allows designs which are actually unsatisfactory, it is similar to the change occurring in the AISC Specification, listed here as constraint (2b). It is felt that this change does not violate the spirit of the Specification. It should, however, be emphasized that the particular values used above were arbitrarily selected.]

(f) If $\frac{L_b}{r_e} \leq \sqrt{\frac{510 \times 10^3 C_b}{F_y}}$

where $r_e^2 = \frac{C_b I_y}{2S_x}\sqrt{d^2 + \frac{0.156 L_b^2 J}{I_y}}$

and $J = \frac{2b_f t_f^3}{3} + \frac{d t_w^3}{3}$

$$g_{2f} = F_y\left[\frac{2}{3} - \frac{F_y(L_b/r_e)^2}{1530 \times 10^3 C_b}\right] - f_b$$

(AISC Sect. 1.5.1.4.6a)

(g) If $\frac{L_b}{r_e} > \sqrt{\frac{510 \times 10^3 C_b}{F_y}}$

$$g_{2g} = \frac{170 \times 10^3 C_b}{(L_b/r_e)^2} - f_b \quad \text{(AISC Sect. 1.5.1.4.6a)}$$

(h) If $\frac{760}{\sqrt{F_y}} < \frac{d}{t_w} < \frac{760}{\sqrt{F_{bi}}}$

where F_{bi} is the smallest value of allowable bending stress, $i = a, b, \ldots h$

$$g_{2h} = 0.6F_y - f_b \quad \text{(AISC Sect. 1.5.1.4.5)}$$

(i) If $\frac{d}{t_w} > \frac{760}{\sqrt{F_{bi}}}$

$$g_{2i} = (F_{bi})\left[1.0 - 0.0005\left(\frac{b_w t_w}{b_f t_f}\right)\left(\frac{b_w}{t_w} - \frac{760}{\sqrt{F_{bi}}}\right)\right] - f_b$$

(AISC Sect. 1.10.6)

Geometrical Constraints (g_3)

$$g_{3a} = \frac{14{,}000}{\sqrt{F_y(F_y + 16.5)}} - \frac{b_w}{t_w} \quad \text{(AISC Sect. 1.10.2)}$$

$$g_{3b} = 260 - (b_w/t_w) \quad \text{(AISC Sect. 1.10.5.3)}$$

In addition simple side constraints to be input by the user were applied to all design variables.

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