Plastic Behavior of Eccentrically-loaded Connections

CARL L. SHERMER

Structural connections are generally designed on the basis of their ultimate strengths. This has always been so, even though design methods might seem to suggest that stress is the criterion. However, there are probably very few connections of any kind in which stresses at some location have not reached the yield point during construction or under design loads. As a matter of fact, if connection material were not ductile and could not yield, connections and their method of design would have to be completely revised.

Present "elastic" design methods work because ductile behavior is assumed implicitly and design stresses are based upon the distribution of stress when local yielding occurs. For example, centrally-loaded connections are assumed to be stressed uniformly throughout the joint but distribution at working loads is not uniform even in the simplest of connections. For that matter, in a single rivet or bolt or in a single spot of weld there are steep stress gradients and stress concentrations. In addition, there is a great variation between individual fasteners or between different areas in the same weld. When the connection begins to yield, however, the stresses become quite uniform. "Elastic" analysis has been used as the basis for design because it involves reasonably simple calculations which lead to safe designs, even though it is known that the results in many cases are more conservative than necessary and that the calculated stresses are not representative of the actual stresses by a wide margin.

One type of connection in which present methods lead to unduly conservative designs is illustrated in Fig. 1. The usual method of analysis is to assume the eccentric load, P, as equivalent to a load, P, passed through the centroid of the fastener material* plus a moment, Pe. The centroidal force, P, is assumed to be uniformly distributed throughout the connection and the moment is assumed to cause stresses at any point which are proportional to the distance of that point from the centroid of the fastener material. The two components of stress at the point, illustrated in Fig. 2, are then added vectorially.

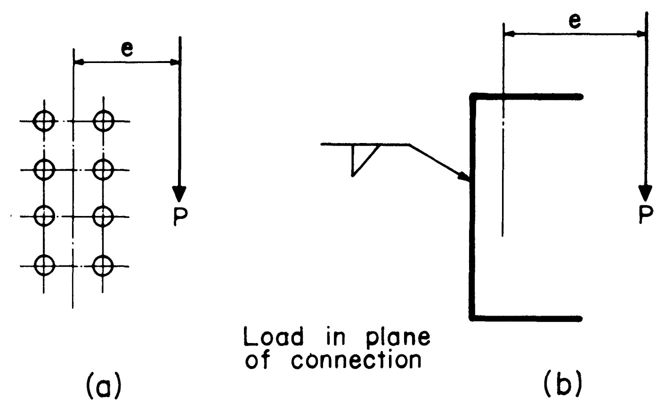


Figure 1

But such a connection does not fail when stress at the most highly stressed region reaches the yield point. Yielding at this point simply results in a redistribution of the stresses; the connection does not deform excessively until yielding occurs at all points. If the load is increased to a level which produces general yielding, the connection will rotate, relative to the connected member, about some center, 0, as shown in Fig. 3, and the stress at all points will be equal to F_{yv}. For any element of area, ΔA, of the fastener material, the force will be $F_{yv} \Delta A$. (With bolted or riveted connections, ΔA can be taken as the cross-sectional area of the fastener.) For welds, F_{yv} is the yield point stress in shear. In rivets or bolts, it is the stress at which the fastener is deemed to have failed by slip in friction-type joints or shear yielding of the fasteners (and perhaps the connected material) in bearing-type joints.

Equating to zero the sum of all forces in the horizontal direction, the sum of all forces in the vertical direction and the sum of all moments about the point 0,

Carl L. Shermer is Chairman, Department of Civil Engineering, Ohio University, Athens, Ohio.

* *In this paper, fastener material means rivets, bolts, and welds.*

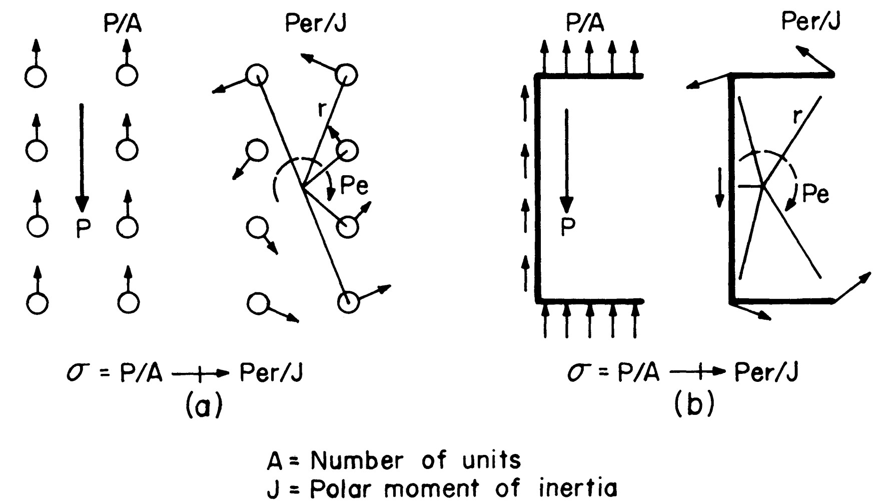


Fig. 2. Elastic analysis

$$\int F_{yv}(\Delta A) \sin \theta - P_x = 0 \qquad (1)$$

$$\int F_{yv}(\Delta A) \cos \theta - P_y = 0 \qquad (2)$$

$$\int F_{yv}(\Delta A) r - P(m + e) = 0 \qquad (3)$$

These three equations of equilibrium can be used to determine the coordinates of the center of rotation and the value of P which will cause failure.

They look simple, but their solution must be accomplished by trial and error. A location of the center of rotation, 0, is assumed, as shown in Fig. 3, distance m from the centroid of the fastener group. Then the values or r and θ are computed for each point in the fastener material in terms of m and the geometry of the group, and substituted in the equations to determine if they are satisfied. If the equations are not satisfied, a new location of 0 is assumed and the process is repeated. For riveted or bolted connections, the integrals in the equations become simple summations. If the weld pattern is difficult to integrate, it can be divided into a dozen or so segments with each segment treated as a spot of weld whose area is equal to the throat area of the weld segment. A simple summation of these segments will give quite accurate results. Fortunately, where the connection has an axis of symmetry, as is often the case, and where the load is normal to that axis of symmetry, the center of rotation must lie on the axis to satisfy Eq. (2) and the amount of computation is reduced considerably. This is the case with the connections illustrated.

Where connections are very large, one might question the validity of the plastic design method, because material far removed from the center of rotation might rupture before the yield stress is reached at those points near the center. Such a condition could not occur in a friction-type bolted connection, because slipping could not occur under one bolt unless slipping occurred under all bolts. In a very large riveted, welded, or bearing-type

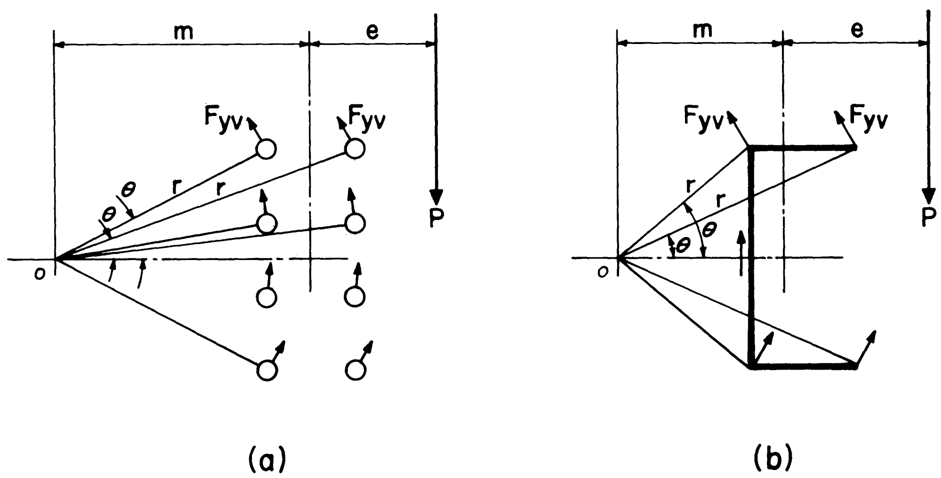
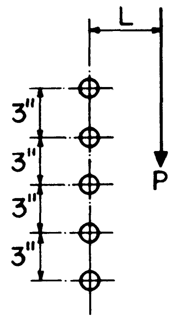


Fig. 3. Plastic analysis

Table 1



L	n = 2	3	4	5	6	7	8	9	10	11	12
1.5	1.42	2.54	3.64	4.71	5.75	6.79	7.81	8.84	9.85	10.86	11.88
2	1.20	2.29	3.41	4.51	5.58	6.64	7.68	8.71	9.74	10.76	11.78
2.5	1.03	2.04	3.17	4.28	5.37	6.45	7.50	8.55	9.60	10.63	11.66
3	0.90	1.82	2.92	4.04	5.14	6.23	7.31	8.38	9.43	10.48	11.52
3.5	0.79	1.63	2.69	3.79	4.90	6.00	7.10	8.18	9.25	10.30	11.36
4	0.70	1.46	2.47	3.55	4.65	5.76	6.86	7.96	9.04	10.11	11.17
4.5	0.63	1.32	2.28	3.32	4.41	5.52	6.63	7.73	8.82	9.90	10.97
5	0.57	1.19	2.11	3.11	4.18	5.28	6.38	7.48	8.59	9.68	10.76
5.5	0.53	1.09	1.96	2.91	3.95	5.04	6.14	7.24	8.35	9.45	10.54
6	0.48	1.00	1.83	2.73	3.74	4.81	5.89	7.00	8.11	9.21	10.31
6.5	0.45	0.92	1.71	2.57	3.54	4.58	5.66	6.67	7.86	8.97	10.07
7	0.42	0.86	1.60	2.42	3.36	4.37	5.43	6.52	7.62	8.73	9.83
7.5	0.39	0.80	1.51	2.29	3.19	4.17	5.21	6.29	7.38	8.48	9.59
8	0.37	0.75	1.42	2.16	3.04	3.99	5.00	6.06	7 14	8.24	9.35
8.5	0.35	0.71	1.35	2.05	2.89	3.81	4.80	5.84	6.91	8.00	9.10
9	0.33	0.67	1.28	1.95	2.76	3.65	4.62	5.63	6.68	7.77	8.85
10	0.30	0.60	1.16	1.77	2.52	3.36	4.27	5 24	6.26	7.32	8.39
11	0.27	0.55	1.06	1.62	2.32	3.10	3 96	4.89	5.87	6.89	7.94
12	0.25	0.50	0.98	1.49	2 15	2.88	3.69	4.57	5.52	6.50	7.52
14	0.21	0.43	0.84	1.28	1.86	2.50	3.23	4.03	4.89	5.80	6.76
16	0.19	0 37	0.74	1.12	1.64	2.21	2 87	3.59	4.38	5 22	6.11
18	0.17	0 33	0 66	1.00	1.47	1.98	2.58	3.23	3.95	4.72	5.55
20	0.15	0.30	0.59	0.90	1.33	1.79	2 33	2.93	3.59	4.31	5.07
22	0.14	0.27	0.54	0.82	1.21	1.63	2.13	2 68	3.29	3.95	4.67
24	0.12	0.25	0.50	0.75	1.11	1.50	1.96	2.47	3.04	3.65	4.32

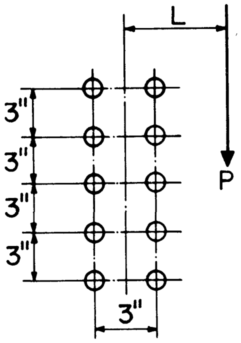
bolted connection, where the center of rotation was within or close to the connection centroid, there could be rupture at points distant from the center before centrally-located material was fully developed; however, the capacity of the connection would not usually be affected greatly for the following reasons. A center of rotation located within or close to the connection itself must be associated with a large eccentricity, e, for the load, and the connection is primarily a moment-resisting one. The moment resisting capacity is usually most significantly developed by resisting forces farthest removed from the center, and only to a small degree by forces close to the center. If the center of rotation is some distance from the connection, as it would be for a load with a small eccentricity (for pure shear the center would be at an infinite distance), then no part will be so close to the center that it will not be fully stressed before other parts fail completely.

The problem has been solved with the aid of a computer for some of the common patterns of rivets and welds and, for comparison with an elastic solution, the results are given in Tables 1, 2 and 3. These tables, respectively, cover the same connections as Tables X, XI and XVI in the AISC Manual, 7th Edition, and the tabulated coefficients are used in exactly the same way. They give safe design values based upon ultimate strength, whereas the tables published in the Manual give safe design values based upon computed stress using the conventional "elastic" method.

The Manual Tables X through XIII use an "effective" length of the moment arm and the coefficients are computed from an elastic analysis using this effective length. The effective length is less than the actual length and the use of this reduced length is a concession to the plastic behavior of the connection at failure; however, no direct account is taken of plastic action in Tables XIV through XXI of the Manual. Analysis of the plastic behavior of the connection, as used in calculation of the coefficients of Tables 1, 2 and 3, is rational, sound from an engineering standpoint, affords a uniform factor of safety, and improves economy in field connections. Therefore, the values given in these tables are considered by the author to be preferable to the values in the AISC 7th Edition of the Manual.

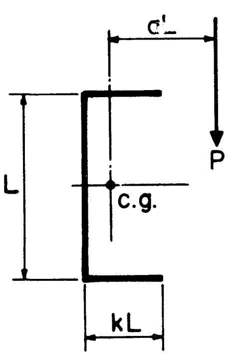
A number of specimens, both bolted and welded, were tested in the laboratories of Ohio University and good correlation between theoretical and experimental values was obtained.

Table 2



L	n: 1	2	3	4	5	6	7	8	9	10	11	12
1.5	1.00	3.04	5.16	7.30	9.42	11.51	13.58	15.63	17.67	19.70	21.73	23.75
2	0.86	2.69	4.70	6.87	9.04	11.17	13.28	15.35	17.42	19.48	21.52	23.56
2.5	0.75	2.39	4.26	6.41	8.59	10.76	12.90	15.02	17.11	19.20	21.26	23.33
3	0.67	2.15	3.86	5.96	8.13	10.32	12.48	14.63	16.76	18.87	20.96	23.04
3.5	0.60	1.94	3.50	5.53	7.66	9.85	12.04	14.21	16.37	18.50	20.61	22.71
4	0.55	1.77	3.18	5.13	7.20	9.37	11.56	13.75	15.93	18.08	20.23	22.36
4.5	0.50	1.62	2.91	4.78	6.77	8.90	11.09	13.28	15.47	17.66	19.81	21.96
5	0.46	1.49	2.69	4.45	6.36	8.45	10.61	12.80	15.01	17.20	19.37	21.54
5.5	0.43	1.38	2.50	4.17	5.99	8.02	10.15	12.33	14.53	16.72	18.92	21.09
6	0.40	1.28	2.33	3.90	5.63	7.60	9.69	11.85	14.04	16.25	18.44	20.64
6.5	0.37	1.20	2.19	3.67	5.32	7.23	9.26	11.39	13.57	15.76	17.96	20.17
7	0.35	1.12	2.06	3.46	5.03	6.88	8.86	10.94	13.10	15.27	17.48	19.69
7.5	0.33	1.06	1.94	3.28	4.76	6.55	8.47	10.51	12.64	14.80	16.99	19.20
8	0.32	1.00	1.84	3.11	4.51	6.24	8.11	10.10	12.18	14.34	16.52	18.72
8.5	0.30	0.95	1.75	2.95	4.29	5.96	7.76	9.71	11.77	13.89	16.04	18.24
9	0.29	0.90	1.67	2.81	4.09	5.70	7.44	9.34	11.35	13.44	15.58	17.76
10	0.26	0.81	1.52	2.56	3.73	5.23	6.85	8.66	10.58	12.59	14.69	16.83
11	0.24	0.75	1.40	2.36	3.43	4.83	6.35	8.06	9.89	11.83	13.86	15.94
12	0.22	0.69	1.30	2.18	3.18	4.49	5.90	7.52	9.25	11.12	13.07	15.10
14	0.19	0.59	1.12	1.89	2.77	3.91	5.15	6.61	8.18	9.89	11.69	13.60
16	0.17	0.52	0.99	1.67	2.45	3.47	4.56	5.89	7.30	8.86	10.54	12.30
18	0.15	0.47	0.89	1.49	2.20	3.11	4.10	5.30	6.58	8.01	9.55	11.19
20	0.14	0.42	0.80	1.35	1.99	2.87	3.72	4.81	5.98	7.30	8.71	10.25
22	0.13	0.38	0.73	1.23	1.82	2.57	3.40	4.41	5.47	6.70	8.01	9.43
24	0.12	0.35	0.67	1.13	1.68	2.37	3.13	4.06	5.04	6.19	7.40	8.73

Table 3



a	k: 0.0	0.1	0 2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4	1.6	1.8	2.0
0.2	.754	.957	1.127	1.305	1.481	1 657	1.831	2.006	2.180	2.356	2.531	2.881	3.233	3.585	3.942	4.297
0.3	.622	.818	.979	1 144	1.308	1.476	1.640	1.804	1.970	2.137	2.191	2 640	2.982	3.324	3.669	4.015
0.4	.513	.690	.838	.993	1.144	1 297	1.451	1.604	1.762	1.921	2.078	2.403	2.730	3.062	3.397	3.735
0.5	.429	.588	.724	.862	1.002	1.143	1.285	1 428	1.575	1 725	1.873	2.182	2.497	2.820	3.144	3.476
0.6	.368	.506	.628	.757	.882	1 013	1.143	1.279	1.416	1.553	1.697	1.990	2.293	2.600	2.917	3.243
0.7	.320	.445	.553	.667	.786	.902	1 025	1.150	1.277	1.407	1.544	1.822	2.111	2.409	2.718	3.030
0.8	.283	.394	494	.598	.706	.815	.926	1.041	1.161	1 285	1.413	1.678	1.955	2.239	2.537	2 835
0.9	.253	.352	.443	.540	.638	.740	.845	.951	1.065	1.180	1.302	1.552	1.814	2.085	2.369	2.656
1.0	229	.320	.403	.491	.582	676	.772	.876	.981	1.090	1.203	1 441	1.686	1.944	2.213	2.491
1.2	.140	.267	.340	.415	.494	.576	.661	.752	846	.943	1.039	1 248	1.468	1.702	1.947	2.203
1.4	.164	.230	.293	.360	.429	.500	.578	.658	.740	.821	.909	1.092	1.294	1.504	1.728	1.964
1.6	.144	.202	.258	.317	.378	.443	.511	.581	.652	.727	.804	.970	1.150	1.343	1.547	1.766
1.8	.128	.181	.230	.283	.338	.397	.457	.519	.582	.652	.720	.871	1.033	1.209	1.399	1.598
2.0	.116	.162	.208	.256	.306	.358	.412	.468	.527	.588	.652	.788	.937	1.098	1.272	1.473
2.2	.108	.147	.190	.233	.279	.327	.375	.426	.479	.534	.593	.718	.857	1.005	1.167	1.340
2.4	.096	.136	.173	.215	.256	.300	.344	.391	.440	.491	.545	.661	.788	.926	1.075	1.237
2.6	.082	.125	.161	.198	.238	.276	.318	.361	.406	.454	.503	.610	.729	.857	.999	1.149
2.8	.079	.116	.150	.184	.221	.256	.295	.335	.377	.422	.468	.567	.678	.798	.930	1.073
3.0	.076	.108	.139	.173	.205	.239	.275	.312	.352	.392	.437	.530	.632	.746	.869	1.004