Research in Plastic Design of Multi-story Frames

GEORGE C. DRISCOLL, JR., AND LYNN S. BEEDLE

This paper was presented at the AISC National Engineering Conference, Omaha, Nebr. in May, 1964.

THE EIGHTH National Engineering Conference, held at Lehigh University in 1956, was devoted in its entirety to the subject of plastic design. At that time there were *no* plastically designed structures in the United States, or at least there were none that were a matter of record. Undoubtedly there must have been a number of structures designed on the basis of what has been described as a "plastic excuse for an elastic design"; and of course the provision included in the AISC Specification in 1945 that permitted a 20 percent increase of stress at points of interior support could only be justified on the basis of the same concepts that led to the current plastic design techniques.

The purpose of these remarks is to review briefly the advances made since 1956 and to describe in some detail the research being conducted at Lehigh University on the plastic design of multi-story building frames. The latter discussion is divided into two parts: braced frames and unbraced frames. In addition the results of recent pilot tests on a three-story structure are presented.

What have been the developments in plastic design since the 1956 conference? In the United States the technique was first used in 1957, the authorization being based on the Proceedings of the 1956 conference.[1] In 1958 the AISC issued a supplement to its basic specifications, these "supplementary rules" being the first codification of the concept in the United States.[2] The AISC manual *Plastic Design in Steel* followed very shortly in 1959;[3] it presented practical procedures for the plastic design of simple or continuous beams and one- or two-story rigid frames. By early 1960 plastic design was widely used throughout the United States, and had been adopted in most of the major building specifications. A detailed review of developments during this time of rapid expansion is available in Reference 4.

George C. Driscoll, Jr., is Research Associate Professor of Civil Engineering and Chairman, Structural Metals Division, Fritz Engineering Laboratory, Lehigh University, Bethlehem, Pa.

Lynn S. Beedle is Director, Fritz Engineering Laboratory, Lehigh University, Bethlehem, Pa.

The year 1961 saw the culmination of an effort, also begun in 1956, to give complete substantiation of this design technique. The ASCE issued its Manual No. 41, *Commentary on Plastic Design in Steel*, giving the theoretical background for the method, secondary design considerations, experimental verification of the theory, and design guides.[5] Later this same year the *Supplementary Rules* became (in updated form) "Part 2" of the new AISC Specification.[6] In addition, many of the provisions of allowable stress design (Part 1 of the Specification) were modernized based on knowledge of the plastic behavior of steel.

RESEARCH DEVELOPMENTS

What about the research work since 1956? Although a few specific details remained to be completed in connection with the earlier work, the principal effort at Lehigh University has been on two major extensions of the method:

1. Application to high strength steels
2. Application in a more general way to multi-story frames.

The work on high strength steels began in July of 1962 and is still underway. It constitutes, in the main, a major "checking" job (both theoretical and experimental) to see what modifications would be required in the application of plastic design to a steel of higher strength than A7 or A36. The results to date indicate that the same factors that were significant in A7 steel for plastic design are also significant in A441—namely, that the stress-strain diagram should have a flat plateau or plastic range followed by a positive strain-hardening characteristic. Some further experimental work remains to be done, but it appears that design recommendations should be available shortly. The work not only extends the previous design guides to higher strength steels, but also provides new information on local buckling, the deformation capacity of beams and beam-columns, and the bracing requirements of inelastically deformed members.[7]

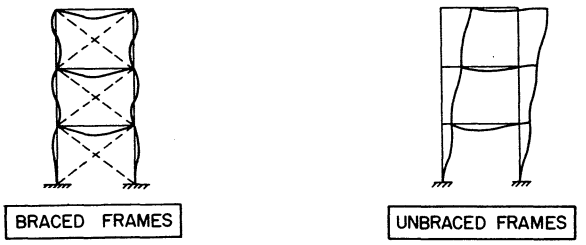


Fig. 1. Types of multi-story frames studied

Work on plastic design of multi-story building frames was underway to a modest extent at the time of the 1956 conference. In 1958 research began in earnest to provide a more complete application to tall buildings. Although the studies followed the lines of previous research, the problems and the approach were quite different. In the case of continuous beams and single-story frames, the method of analysis (the formation of mechanisms) was available almost from the beginning. For such structures the major effort that was needed to bring plastic design to the point that it could be used was to determine the plastic behavior of isolated members (beams, columns, connections); to learn the influence in the inelastic range of shear, axial force, repeated load, and instability; and to establish experimental confirmation of the theories developed.

Not so with multi-story frames: The basic plastic analysis of the tall building had been explored for specific cases,[8,9] but remained undeveloped as a practical design technique. So the emphasis in the new research was different. Building on the knowledge of the behavior of structural components, it was necessary to take an intermediate step and study the behavior of a "subassemblage", an element of a structure consisting of a group of columns with beams attached. Under the heading of "subassemblages" one of the first detailed studies was of restrained columns. Other studies were of frame stability, sway deflections, and bracing. Also, in contrast to the earlier work, much more effort has been required in developing plastic analysis techniques and design procedures.

Fig. 1 gives two sketches symbolizing the principal types of frames being studied at Lehigh. These are frames of regular shape with relatively uniform column heights and bay spacings. The studies have been divided into two categories: "braced frames" and "unbraced frames". The former include any of those structures for which sway due to instability under vertical loads or drift due to lateral loads is resisted with the aid of bracing in the form of diagonal X-bracing, K-bracing, knee-braces, or shear walls. All of these are symbolized in these figures by panels with X-diagonals. "Unbraced frames" are those in which all resistance to lateral drift and sidesway under vertical loads is provided by the rigid frame action of the structural framework.

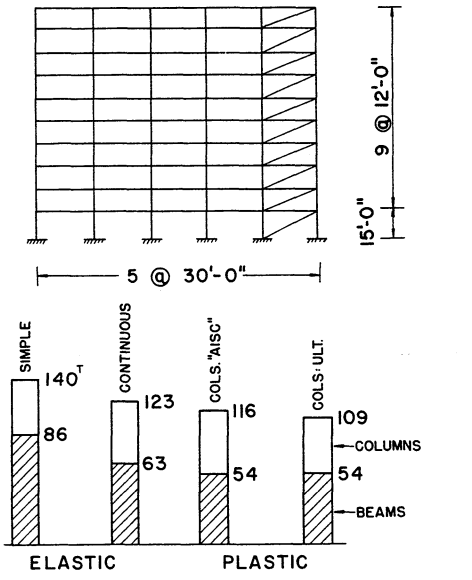


Fig. 2. Weight comparison for four design methods

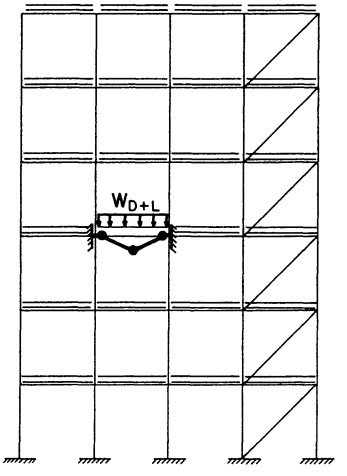


Fig. 3. Girder selection based on beam mechanism

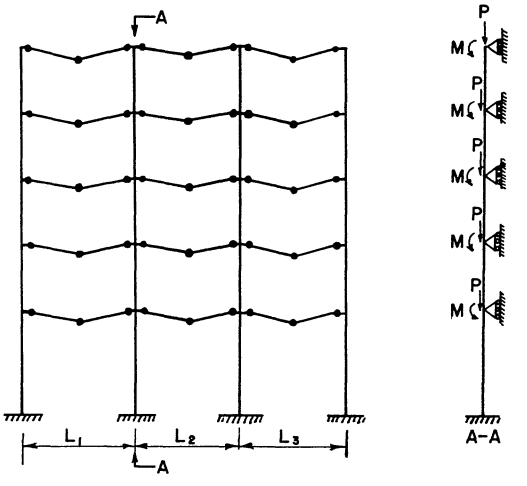


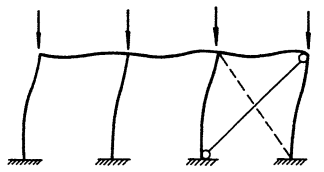
Fig. 4. Continuous column design in braced frame

BRACED FRAMES

Studies of the problem of braced multi-story frames under vertical loads and under vertical load combined with horizontal load were considered first. The work on this aspect is nearing completion and should appear in the form of reports containing design guides and design charts. Indications of possible savings in steel are illustrated in Fig. 2, for a structure consisting of a ten-story five-bay frame with diagonal bracing. In the lower portion are tabulated the weights of four different designs, presented in bar chart form. The shaded portion shows the weight of beams and the open part the weight of the columns.

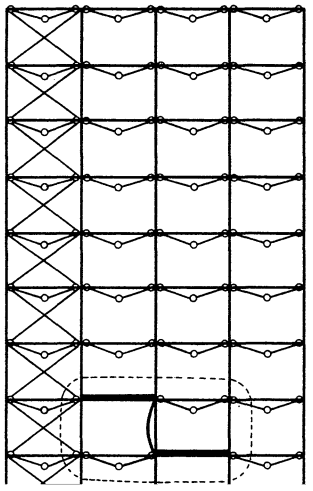
The four designs in Fig. 2 were as follows: Two were carried out according to allowable stress concepts and two according to plastic design concepts. The allowable stress designs were on the basis of simple beams and continuous beams, respectively. The difference in the plastic design is in the design of the columns, one method using allowable stress formulas, the other using maximum load techniques. The resulting weights of the four designs indicate that increased weight saving is possible as more and more utilization is made of plastic load-carrying capacity of the members. For this example, the weight saving for the complete plastic design is 22 percent compared with the allowable stress design using "simple beam" analysis.

The method for the design of braced frames starts with designing beams and girders to support their expected dead and live load multiplied by the appropriate



$$A_{br.} = \frac{I}{E} \frac{\ell^3}{hL^2} \Sigma P$$

Fig. 5. Bracing design to resist lateral displacement of vertical load



which causes single curvature bending in addition to high column thrust. In this condition some of the girders carry only dead load and therefore remain "elastic" at ultimate load, and these girders give added restraint to the columns. An important part of the research work has been an attempt to utilize this available restraint when proportioning columns. The design of a column can be improved by recognizing the help which it can receive from framing members which remain elastic.

One way to study such a restrained column is to consider a subassemblage formed of a column and the girders framing into it (Fig. 7). Moments equal in magnitude to the plastic hinge value in adjacent beams are applied to the stub ends at the same time that the thrust is applied to the column. These moments are the plastic hinge moments introduced as the result of full loading applied to the girders—moments that must be shared by the column and the restraining beams.

Fig. 8 shows a photograph of a test setup that was developed to simulate the condition just described.[13] The vertical column (whitewashed) is shown in the center of the photograph. The tension jack and tension dynamometer through which the plastic moment is applied are shown in front of the column. The restraining beams extend away from the column to a support point.

Fig. 9 shows symbolically the behavior to be expected of a typical restrained column in a subassemblage with

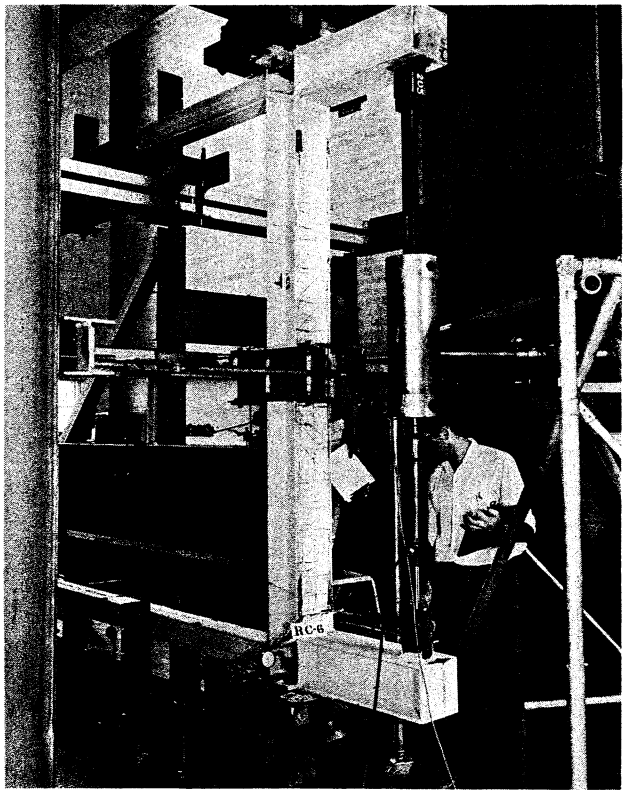


Fig. 8. Restrained column test setup

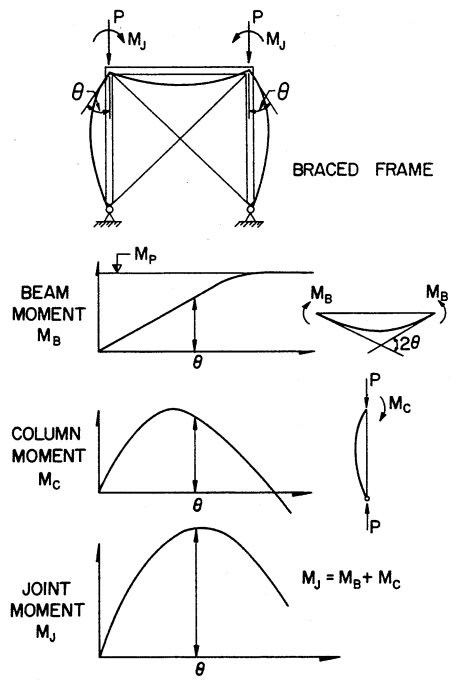


Fig. 9. Theoretical behavior of parts of a subassemblage

external moment and thrust applied at the column tops. The figure shows the moment vs. rotation behavior of the beam, the moment vs. rotation of the column, and the moment vs. rotation of the entire joint or assemblage. The joint moment is the sum of the beam and column moments for the same rotation. It is particularly significant that the joint assembly can reach its maximum moment even after the column moment has started to decrease. At the instant at which the joint itself has reached the maximum moment, the column is somewhat beyond and below its point of maximum moment; however the beam is still on the increasing portion of its moment-rotation curve. This is one of the new concepts of structural behavior that has been solved and explained, and recently it has been confirmed experimentally.[14]

Currently some tests of complete frames are being planned to help verify the procedures for the analysis of braced frames mentioned earlier. As shown in Fig. 10 the test setup involves 6-in. WF columns and 12-in. beams in a two-bay, three-story structure. The testing scheme will involve four different loading conditions: full gravity load, checkerboard gravity load, gravity load plus wind load, and checkerboard gravity load plus wind load.

The planning of a suitable test setup for these unbraced frames has proven to be quite a challenge. In

order to accommodate lateral motion, particularly in the case of unbraced frames, and to avoid restraint by the loading system, what is really needed is a floating high capacity testing machine (or a way to carry 100 tons in a 5-lb bag). After considering the use of dead weights, lever systems, block and tackle, hydraulic jacks, and testing machines, a system has been designed at Fritz Laboratory which makes use of a mechanism linkage which is able to apply vertical loads independently to each girder. It will permit sway of unbraced frames with no restraint. A pilot model of the "gravity load simulator" has been tested, and the results were so successful that the final arrangements for the first frame tests are now proceeding. Fig. 11 shows a diagrammatic sketch of the simulator with the mechanism in two different positions. Note that the direction of load application is always vertical and that the entire system can move freely as a unit in the plane of the test frame.

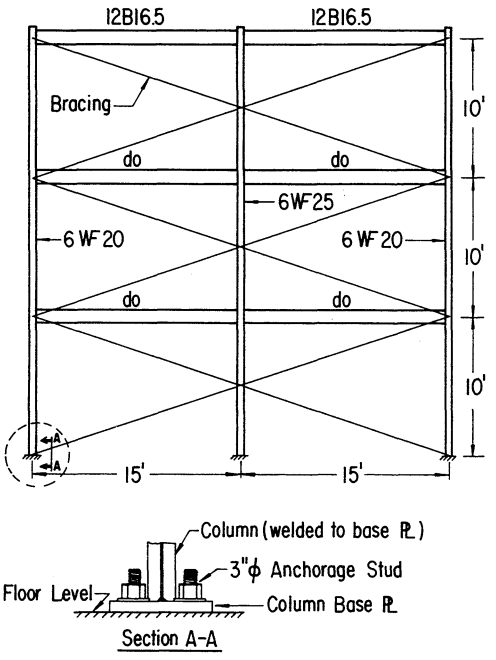


Fig. 10. Planned braced frame test setup

COLUMN DEFLECTION CURVES

A new concept which was developed in the multi-story frame research, is the "column deflection curve." It is one of the important "building blocks" in current studies of both braced and unbraced frames.[10] Referring to Fig. 12, the column deflection curve is the shape that a compressed member will take when held in a bent position by an axial thrust. These curves are obtained by solving the equilibrium equation for the member, and there are no stability considerations involved. Each curve is defined by the load P and its end rotation θ_0, with the length L being a function of the values chosen for P and θ_0.

What is the significance and usefulness of the column deflection curve? The usual column in a building is one loaded with thrust and end bending moments. Consider, as shown in Fig. 13a, a column loaded with equal and opposite moments and supporting axial load. This is the same condition as a member loaded with axial thrust and with equal end eccentricities (Fig. 13b). Such a member will assume the bent position shown in Fig. 13c as load is applied. The curve is not only the deflected shape of the column; since the bending moment is the product of applied load and distance to the deflected position, this curve is also the shape of the moment diagram. As shown in Fig. 13d, this curve can be extended to the point at which the moment equals zero, and the only loading now necessary is the axial thrust. Such a curve is half of the column deflection curve shown in Fig. 12. For the full column deflection curve, the important characteristics are the length L, the load P, and the end slope θ_0.

Now, what can be obtained from the column deflection curve? Fig. 14 shows some particular solutions of the equilibrium configuration of bent columns—

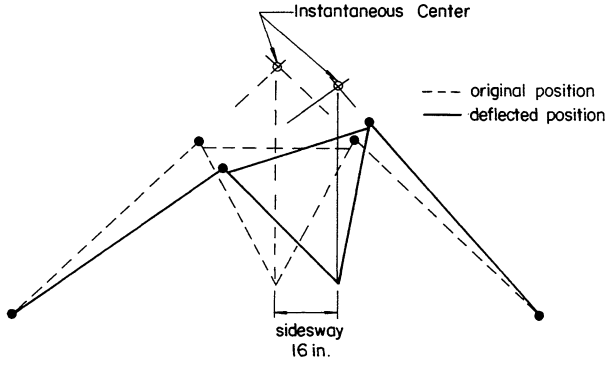
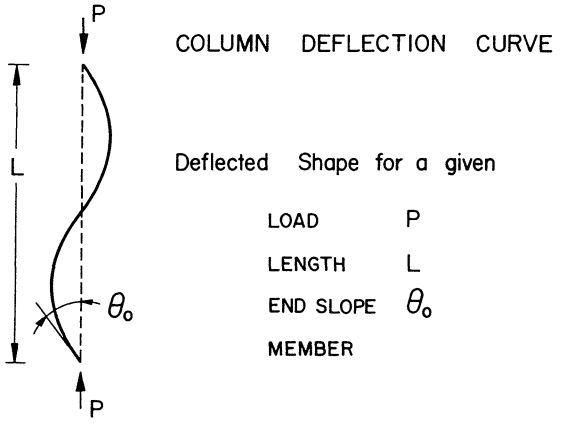


Fig. 11. Gravity load simulator. Load remains vertical despite sidesway of structure



In the studies of unbraced frames, restrained columns will again play a significant role. Additional information will be needed to use these curves. One of the questions now being examined is how to handle the boundary conditions which show the effect of other members in the structure upon the column under consideration. Procedures similar to plastic moment distribution and slope-deflection are being examined. At the present time computer programs are being developed which include the effects of residual stress and partial plasticity in the member.

FRAME TESTS

Preliminary pilot experiments have been conducted recently on a three-story unbraced frame. A photograph of the general setup is shown in Fig. 17. The structure was loaded with dead weights and a lever system. The test assembly was actually two parallel frames braced with diagonal bracing perpendicular to the plane of bending in order to prevent out-of-plane buckling. The girders of the frame were loaded with two concentrated loads distributed across the beam. Large concentrated loads would be involved in the lower columns—a situation which was simulated by the earlier tests on single story frames that preceded the current experiments.[17] The model was designed with "fence post" sections, with a depth of 2⅝ in. and flange width of 1⅝ in.

Fig. 18 shows a sketch of the three-story frame and the results of the first test in terms of deflection under load. To the left is shown the relationship between applied concentrated load on the beams vs. vertical deflection at the center of the top beam. The elastic slope approximates fairly well the predicted value shown by the dashed line. It is evident from the figure that the load approached very closely to the predicted ultimate load. At this point the test was stopped because a beam mechanism was developing in the upper beam and it was evident that the whole assembly was being restrained. (The top beam would fail first because it is subjected to the least restraint by the columns.)

To the right in Fig. 18 is shown the lateral deflection at the top of the first floor column. This deflection, after an initial displacement, did not increase very rapidly—nor was it expected to increase until the point of frame instability had been reached. Even so, it was found that unexpected restraint was introduced as a result of misalignment of the loading system. It was because of the sensitivity of such experiments to very small restraints that the test was stopped and the loading system was rearranged.

In the revised setup, wire ropes were used instead of the solid round rods that had been employed in some parts of the loading system in the first test. The previously loaded frame was then realigned and subjected to a second test. In this test the restraints were very small indeed, and eventually an instability type of failure occurred.

Fig. 19 shows the load deformation behavior of the "second test" on a basis similar to that described for the first test. The load vs. vertical deflection behavior in the "initial" region is similar to that obtained in the first test, the deflection being somewhat greater because a deformed frame was being tested.

The deflection of the structure horizontally under the action of loads applied in the second test also reflects the fact that the frame had been plastically deformed in the first experiment. Eventually the frame failed due to frame instability, but even so, the load increased significantly above an approximate critical load predicted by Merchant's formula[18] and to a load

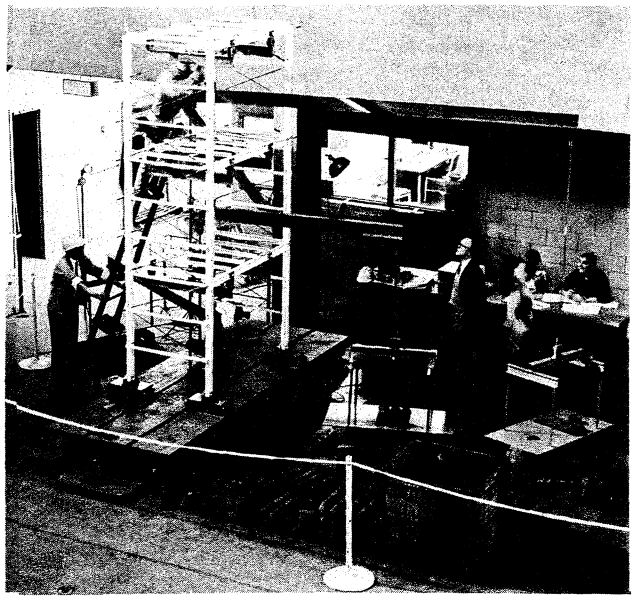


Fig. 17. Model three-story unbraced frame test

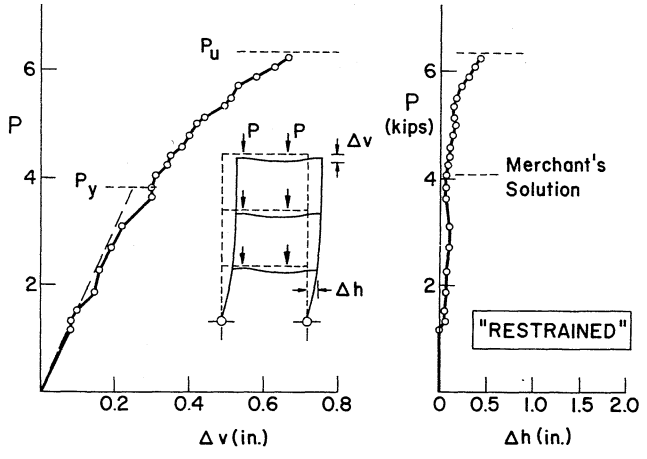


Fig. 18. Load versus vertical and horizontal deflection of model three-story frame with accidental restraint

hat began to approach the ultimate load predicted by lastic theory. The upper portion of the curves are otted because failure occurred while the loading ystem was being adjusted to preclude the development f restraints. The behavior of the structure indicates that ome restraining force might have been present. Fig. 20 hows a photograph of the second test of the three-tory frame after failure. The bottom story had a 'sudden" sway failure and it is the lower story which would be the most sensitive to such failure because of he proportionately higher axial loads present in the olumns.

The results of the tests, although of a pilot nature, ;ave valuable information on experimental techniques. Even more important, they indicate that the load at vhich frame instability would occur may be much higher han previously considered possible. Although the vidence is incomplete this suggests that frame in-tability may not be as serious a limitation as had previously been supposed. Further tests on full-scale unbraced multi-story, multi-bay frames are needed to nvestigate this point. Such tests are being planned and vill involve frames similar to that shown in Fig. 10, but with the bracing members removed. (The gravity oad simulator described earlier will also be used to apply load to the test frames.)

One concept which appears to be quite promising or the plastic design of multi-story frames is the com-bined use of high strength steel columns and carbon teel girders, proportioned according to restrained col-umn theory. High strength steel columns that are short and stout can carry considerably more load than struc-ural carbon steel columns having the same unbraced ength. Frequently the carrying capacity of a column :an be increased considerably by increasing the size of the girders framing into it.

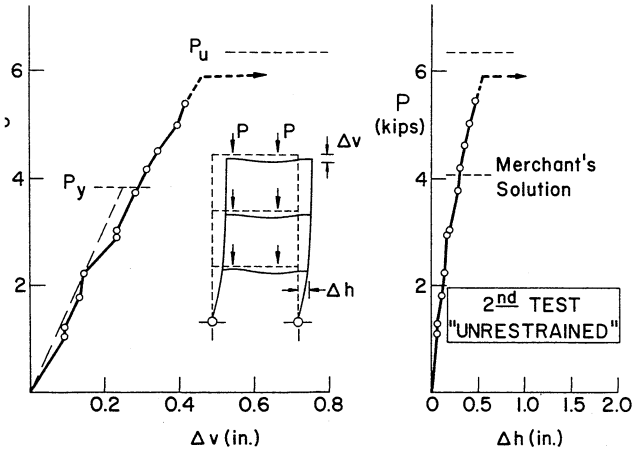


Fig. 19. Load versus vertical and horizontal deflection of unrestrained model three-story frame

Recent studies of a two-story portion of a multi-story building have been made in which the sizes of A36 girders were increased to permit a reduction in the size of A441 columns. The net decrease in weight was 1.2 percent, and the net decrease in cost of steel was 2.5 percent; but the most dramatic result was the net decrease of 43 percent in drift of each story. The results of these and other studies show that the drift due to bending of the girders is about 80 percent of the total drift. Increasing the girder sizes in less expensive structural carbon steel permits a reduction in the size of the more expensive high strength steel columns. It also provides relative column and girder stiffnesses that are favorable for the resistance of lateral deflection and frame instability.

SUMMARY

In recent years it has been possible to make considerable progress in research leading to the extension of plastic design to high strength steels and to multi-story frames. For the former, the analysis and design concepts are quite similar to those for structural carbon steels, and specific provisions should be available shortly.

In "braced" multi-story frames it has been shown that additional economies are possible in comparison with methods currently being used. Methods of proportioning the bracing members have been developed. By

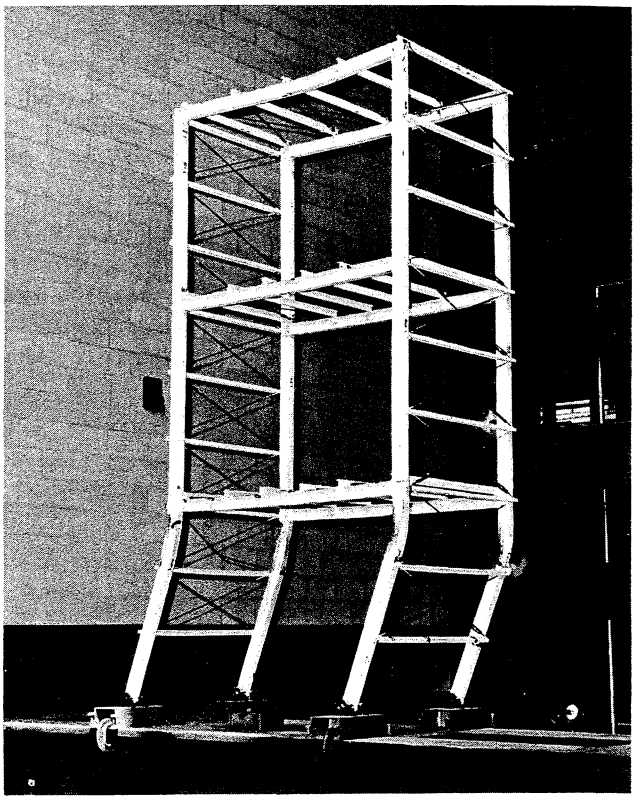


Fig. 20. Model three-story frame after sway failure of lowest story

considering a "subassemblage", an elementary unit of a larger structural framework, it has been shown that a more complete utilization of the strength of columns and of the restraining influence of beams will be possible in plastic design of multi-story frames.

In "unbraced" frames, those which depend on the rigid frame itself to resist lateral loads and frame instability, the plastic analysis of the frame is a complex problem primarily because the formulation of equilibrium conditions must consider the deformed shape of the structure. Computer programs are being developed for "precise" solutions against which simpler approximate design procedures can be tested.

The results of pilot tests of three-story frames tend to indicate that the problem of frame instability may not be as serious as previously supposed. Further evidence is needed on this point, toward which tests are currently being planned. Also it has been shown that the substitution of high strength steel for structural carbon steel in the columns, but using structural carbon steel girders (and with due consideration of economy) can decrease the drift in a story height by as much as 40 percent or more.

Additional tests of large multi-story frames will be made in the near future to correlate with current theoretical studies. These tests will be facilitated through the recent development of a "gravity load simulator" which permits the application of jack-induced gravity loads without restraining the frame from lateral motion.

ACKNOWLEDGMENT

The research reviewed in this paper is being conducted at Fritz Engineering Laboratory, Department of Civil Engineering, Lehigh University, in a program sponsored by the American Institute of Steel Construction, the American Iron and Steel Institute, The Bureau of Ships, and the Welding Research Council. This support is gratefully acknowledged, as are the major contributions being made by members of the research project staff at Fritz Laboratory.

REFERENCES

1. Proceedings National Engineering Conference, *American Institute of Steel Construction, New York, N. Y., 1956.*
2. Supplementary Rules for Plastic Design and Fabrication, *American Institute of Steel Construction, New York, N. Y., Dec. 4, 1958.*
3. Plastic Design in Steel, *American Institute of Steel Construction, New York, N. Y., 1959.*
4. *Beedle, L. S.* On the Application of Plastic Design, *Second Symposium on Naval Structural Mechanics, Brown University, 1960. Fritz Engineering Laboratory Report 205.70.*
5. Commentary on Plastic Design in Steel, *WRC-ASCE Joint Committee, ASCE Manual No. 41, 1961.*
6. Manual of Steel Construction, Sixth Edition, *American Institute of Steel Construction, New York, N. Y., 1963.*
7. *Lay, M. G.* The Static Load-Deformation Behavior of Planar Steel Structures, *Ph.D. Dissertation, Lehigh University, 1964.*
8. *Baker, J. F.* The Plastic Method of Designing Steel Structures, *Proceedings, A.S.C.E., Vol. 85 (ST4), 1959, p. 57.*
9. *Heyman, J.* An Approach to the Design of Tall Buildings, *Proc. Inst. Civil Engineers, 17, Dec. 1960, p. 431.*
10. *Ojalvo, M.* Restrained Columns, *Proc. ASCE, Vol. 86 (EM5), 1960, p. 1.*
11. *Levi, V.* Plastic Design of Braced Multi-Story Frames, *Ph.D. Dissertation, Lehigh University, 1962.*
12. *Galambos, T. V.* Lateral Support for Tier Building Frames, *AISC Engineering Journal, Vol. 1, No. 1, Jan. 1964, p. 16.*
13. *Lay, M. G., Aglietti, R. A., and Galambos, T. V.* Testing Techniques for Restrained Beam Columns, *Fritz Engineering Laboratory Report 278.7, Oct. 1963.*
14. *Lay, M. G. and Galambos, T. V.* The Experimental Behavior of Beam and Column Subassemblages, *Fritz Engineering Laboratory Report 278.10, July 1964.*
15. *Lay, M. G.* The Mechanics of Column Deflection Curves, *Fritz Engineering Laboratory Report 278.12, July 1964.*
16. *Lu, L. W.* Inelastic Buckling of Steel Frames, *Fritz Engineering Laboratory Report 276.7, June 1963.*
17. *Yen, Y. C., Lu, L. W., and Driscoll, G. C., Jr.* Tests on the Stability of Welded Steel Frames, *Welding Research Council Bulletin No. 81, Sept. 1962.*
18. *Merchant, W.* The Failure Load of Rigid Jointed Frameworks as Influenced by Stability, *The Structural Engineer, Vol. 32, No. 7, July 1954, p. 185.*