Which Design Concept for Prestressed Steel?

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PRESTRESSED STEEL STRUCTURES are those in which, during manufacture, assembly, or exploitation, deliberate stresses are produced of precise magnitude, direction, and period of duration.

The most significant aims of prestressing are: enlargement of the elastic range in which the structure works; redistribution of internal stresses or forces; improvement of stability; increase of fatigue resistance; decrease in deformations; wider use of high strength steels.

Some examples of the many types of prestressed steel structures and methods of prestressing are: rigid basic structures (girders, trusses, frames, masts, towers, etc.) prestressed by high strength tendons; systems or networks of prestressed flexible strings (hanging roofs and walls, etc.); multi-layer and hybrid beams or vessels (simultaneous use of different materials as concrete and steel, carbon steel and quenched-tempered steel, etc.); statically indeterminate structures prestressed by enforced displacement of redundant restraints (usually by enforced shifting of some redundant supports or by compelled assembly of some elements fabricated with planned dimension "inaccuracies"); removal or exploitation of residual, secondary, or other "parasite" stresses (from welding, temperature treatments, mechanical operations with steel in cold state, unwanted constructional rigidity of some details, etc.). Prestressed structures utilizing tendons are the most widely used and most economical.

Prestressing could be utilized in many types of steel structures of civil, naval, aircraft, mechanical, and electrical engineering, both in designing new structures and strengthening old ones. In civil engineering, prestressing could be exploited mainly in: roofing, especially of great areas; structures of industrial platforms; cladding and wall panels; craneway girders and crane bridges; highway, railroad and transportation bridges from steel or of composite design (steel and concrete); stacks, towers, masts, and piers; large sheet steel vessels, tubes and pipelines; diverse structures for various special purposes.

In the United States, many interesting prestressed steel structures have already been designed, proportioned, and built.[1–6] In proportioning ordinary, nonprestressed steel structures, there are no special difficulties or discrepancies* in using the familiar concept of allowable stress; however, this is not so in designing prestressed steel structures. In this paper, the authors wish to discuss the distinct concepts which could be used in proportioning prestressed metal structures, and to recommend the best one for common usage.

PRINCIPAL CONCEPTS IN PROPORTIONING

During the design of a structure, two conditions must be considered to insure safety, to meet the service requirements, and to provide economy; these two conditions involve the state of stress and the state of deformation, respectively. Of the two, the state of stress inherently contains more problems and merits more consideration.

State of Stress

Concept of Allowable Stresses—The analysis of *non-prestressed structures* according to allowable stresses[7–9] is based on a unique safety factor for all stretched, compressed or bent** elements (Fig. 1a)

$$\nu = \frac{q_{lim}}{q_{all}} = \frac{\sigma_{lim}}{\sigma_{all}} \tag{1}$$

In Eq. (1), q_{lim} is the loading at which the stress in the material reaches the limit stress σ_{lim}. (For the majority of steel grades and structural shapes, this is the yielding point σ_{yp} or the yielding stress $\sigma_{0.2}$. But for some others, where it is difficult to determine even the yielding stress $\sigma_{0.2}$, as in some wire ropes, the ultimate

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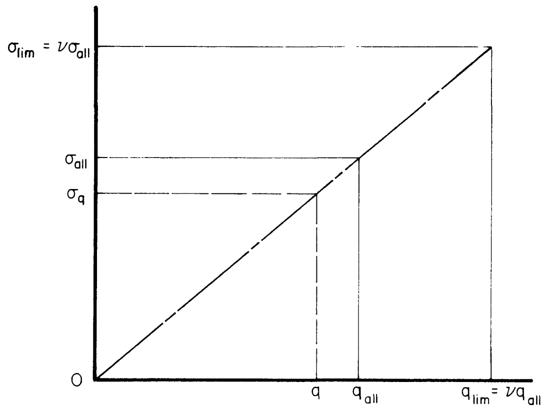
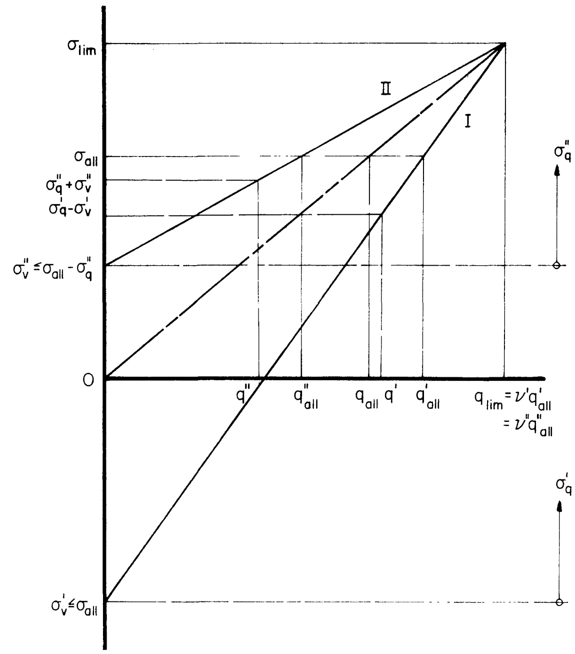
* *Nevertheless, the present specifications for ordinary steel structures are also subject to criticism, and revisions based on a new, more up-to-date base are being suggested.*[20,21]

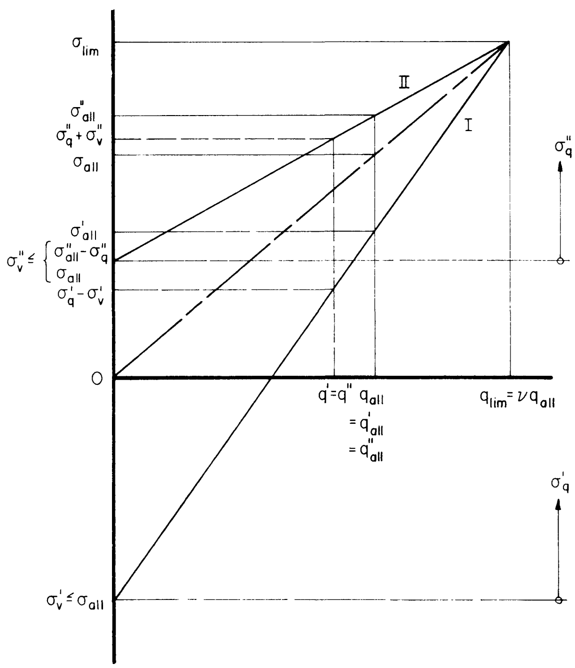
** *Assuming for the latter two that buckling or plastic behavior are not considered.*



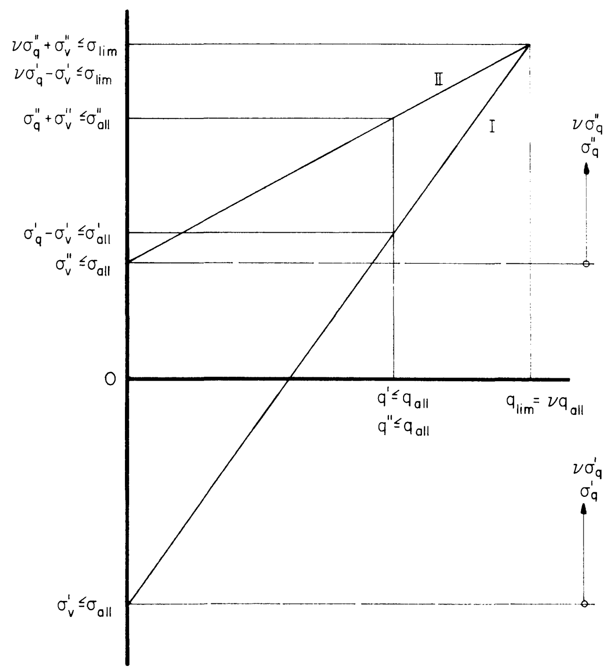
(a) ANALYSIS OF NON-PRESTRESSED STRUCTURES ACCORDING TO ALLOWABLE STRESSES.



(b) ANALYSIS OF PRESTRESSED STRUCTURES ACCORDING TO ALLOWABLE STRESSES.



(c) ANALYSIS OF PRESTRESSED STRUCTURES ACCORDING TO MODIFIED ALLOWABLE STRESSES.



(d) ANALYSIS OF PRESTRESSED STRUCTURES ACCORDING TO FRITZ'S METHOD.

Fig. 1. State of stress checked according to different concepts.

tensile stress σ_{us} may be used.*) The term q_{all} is the loading which produces stresses in the material equal to the allowable value, σ_{all}.

The basic allowable stress—in tension, pure compression, or bending (from which the values for the other types of stresses could be derived, e.g., by the use of failure theories)—has been determined from the yielding point $\sigma_{lim} = \sigma_{yp}$ using the safety factor $\nu = 1.667$ (Sec. 1.5 in Ref. 7 and Table 1.3 in Ref. 9), so that

$$\sigma_{all} = \frac{\sigma_{yp}}{1.667} = 0.6\sigma_{yp}$$

This value corresponds[9] to the expected maximum deviations, -25% from the standard yield point, $+25\%$ from the standard loadings.

The term "a standard value" is hereby interpreted to mean the usual or most probable value as prescribed in specifications, codes, etc., and is explained more fully in the section regarding the Concept of Limit States Design.

For some specific wider combinations of loads (Sec. 1.5[7]) the safety factor could be reduced to

$$\nu = \tfrac{3}{4}(1.667) = 1.25$$

Related to ultimate stress $\sigma_{lim} = \sigma_{us}$, the considered safety factor is $\nu \geq 2.0$ (Sec. 1.5.1.1[7]), and for wire-ropes $\nu = 1.5$ through 3.0 (Sec. 10[3]).

In the concept of allowable stresses, a structural member is considered safe if the stress produced in it due to an external load q meets the criterion

$$\sigma_q \leq \sigma_{all} \tag{2}$$

Now let us examine the design safety of *prestressed structures*. To simplify the problem, stability considerations will not be taken into account.

In the following discussion, those elements in which the prestressing induces stresses of opposite direction to the stresses resulting from loading will be designated as elements of Group I; the majority of prestressed elements fall into this category. The quantities in this group will be denoted by a single prime superscript ($'$). On the other hand, those elements in which the prestressing induces stresses consistent with those caused by the loading will be considered as elements of Group II; these are in particular prestressing tendons. In this group, the quantities will be marked by a double-prime superscript ($''$).

* *Instead of "yielding stress" and "ultimate stress," the terms "yielding strength" and "ultimate strength" are frequently used, although the respective quantities are unit stresses. To the contrary, in publications concerning ropes, the term "breaking strength" designates a force. To prevent misunderstanding in this paper, the term "stress" will be employed for unit stresses (force per area), but the term "strength" for forces.*

When using the concept of allowable stress in its original appearance (that is, with unchanged values of σ_{all}) to design prestressed structures, the factor of safety ν as well as the allowable load q_{all} will vary (Fig. 1b). For elements in Group I, the safety factor will be smaller than for non-prestressed elements,

$$\nu' = \frac{q_{lim}}{q'_{all}} = \frac{\sigma_{lim} + \sigma'_v}{\sigma_{all} + \sigma'_v} < \nu \tag{3}$$

On the contrary, in the case of the elements of Group II, the safety factor will be greater,

$$\nu'' = \frac{q_{lim}}{q''_{all}} = \frac{\sigma_{lim} - \sigma''_v}{\sigma_{all} - \sigma''_v} > \nu \tag{4}$$

In Eqs. (3) and (4), σ'_v and σ''_v are the stresses due to prestressing. Their maximum possible values are obvious from Fig. 1b.

Assuming

$$\sigma_{lim} = \sigma_{yp} = \nu\sigma_{all} = 1.667\sigma_{all}$$

$$\sigma'_v = \sigma_{all}$$

$\sigma''_v = 0.667\sigma_{all}$ (a frequent value for high-strength prestressing tendons)

Equations (3) and (4) yield, respectively:

$$\nu' = \frac{1.667\sigma_{all} + \sigma_{all}}{\sigma_{all} + \sigma_{all}} = 1.333 < 1.667$$

$$\nu'' = \frac{1.667\sigma_{all} - 0.667\sigma_{all}}{\sigma_{all} - 0.667\sigma_{all}} = 3.0 > 1.667$$

Thus the safety factors are

$$\nu' = 80\%\,\nu \text{ (Group I)}$$

$$\nu'' = 180\%\,\nu \text{ (Group II)}$$

The curves in Fig. 2 are a plot of various ratios of the safety factors ν'/ν and ν''/ν (in percent) at various ratios of stresses σ'_v/σ_{all} and σ''_v/σ_{all}, respectively.

The condition of strength is

$$\sigma'_q - \sigma'_v \leq \sigma_{all} \tag{5}$$

and

$$\sigma''_q + \sigma''_v \leq \sigma_{all} \tag{6}$$

This concept with the unchanged values of allowable stress σ_{all} had been used by the pioneers in prestressing, for example by E. Freyssinet,[10] F. Dischinger[11] et al.

Concept of Modified Allowable Stresses—Naturally, the factor of safety ν should not vary; this condition can be attained by introducing *modified allowable stresses* (Fig. 1c). From the following condition of constant safety factor ν (and/or constant allowable load q_{all}),

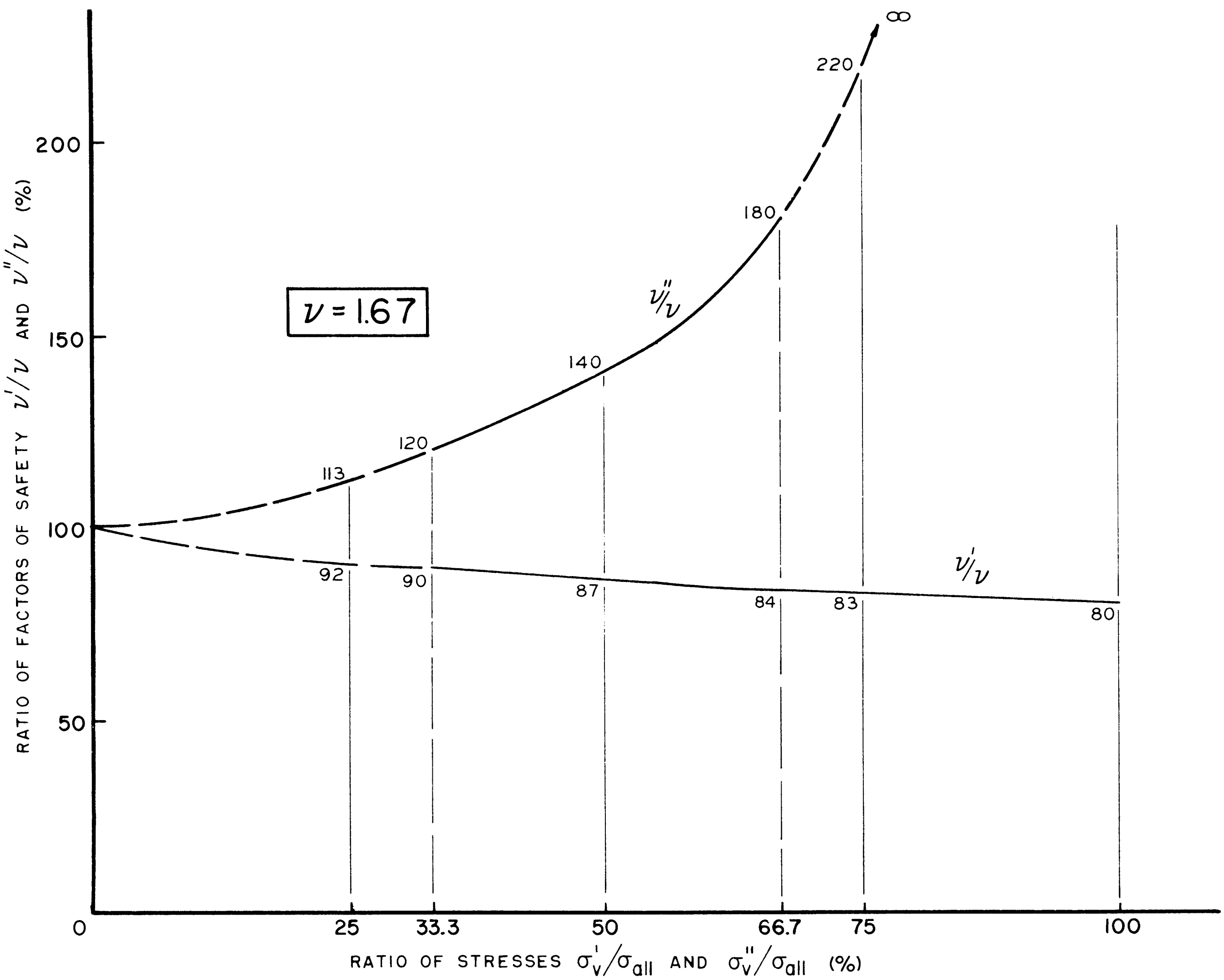


Fig. 2. Variation of safety factors ν', ν'' in prestressed structures designed according to allowable stresses.

$$\nu' = \nu = \frac{q_{lim}}{q_{all}} = \frac{\sigma_{lim} + \sigma'_v}{\sigma'_{all} + \sigma'_v}, \tag{7}$$

$$\nu'' = \nu = \frac{q_{lim}}{q_{all}} = \frac{\sigma_{lim} - \sigma''_v}{\sigma''_{all} - \sigma''_v} \tag{8}$$

an increase in the allowable stresses for Group I and a decrease in the allowable stresses for Group II would result. These modified allowable stresses are given by the following formulas:

$$\sigma'_{all} = \sigma_{all}\left[1 - \frac{\sigma'_v}{\sigma_{all}}\left(1 - \frac{1}{\nu}\right)\right] \leq \sigma_{all}, \tag{9}$$

$$\sigma''_{all} = \sigma_{all}\left[1 + \frac{\sigma''_v}{\sigma_{all}}\left(1 - \frac{1}{\nu}\right)\right] \geq \sigma_{all} \tag{10}$$

Limits for σ'_v and σ''_v from prestressing are shown in Fig. 1c and will be discussed at the end of this section.

As an example, for the most common carbon steel, ASTM A36, with the allowable stress $\sigma_{all} = 22$ ksi, and for the parameters mentioned under Eq. (4),

$$\sigma'_{all} = 22\left[1 - \frac{22}{22}\left(1 - \frac{1}{1.667}\right)\right] = 13.2 < 22 \text{ ksi}$$

$$\sigma''_{all} = 22\left[1 + \frac{0.667 \times 22}{22}\left(1 - \frac{1}{1.667}\right)\right]$$

$$= 27.9 > 22 \text{ ksi}$$

or
$$\sigma'_{all} = 60\%\, \sigma_{all} \text{ (Group I)}$$
$$\sigma''_{all} = 127\%\, \sigma_{all} \text{ (Group II)}$$

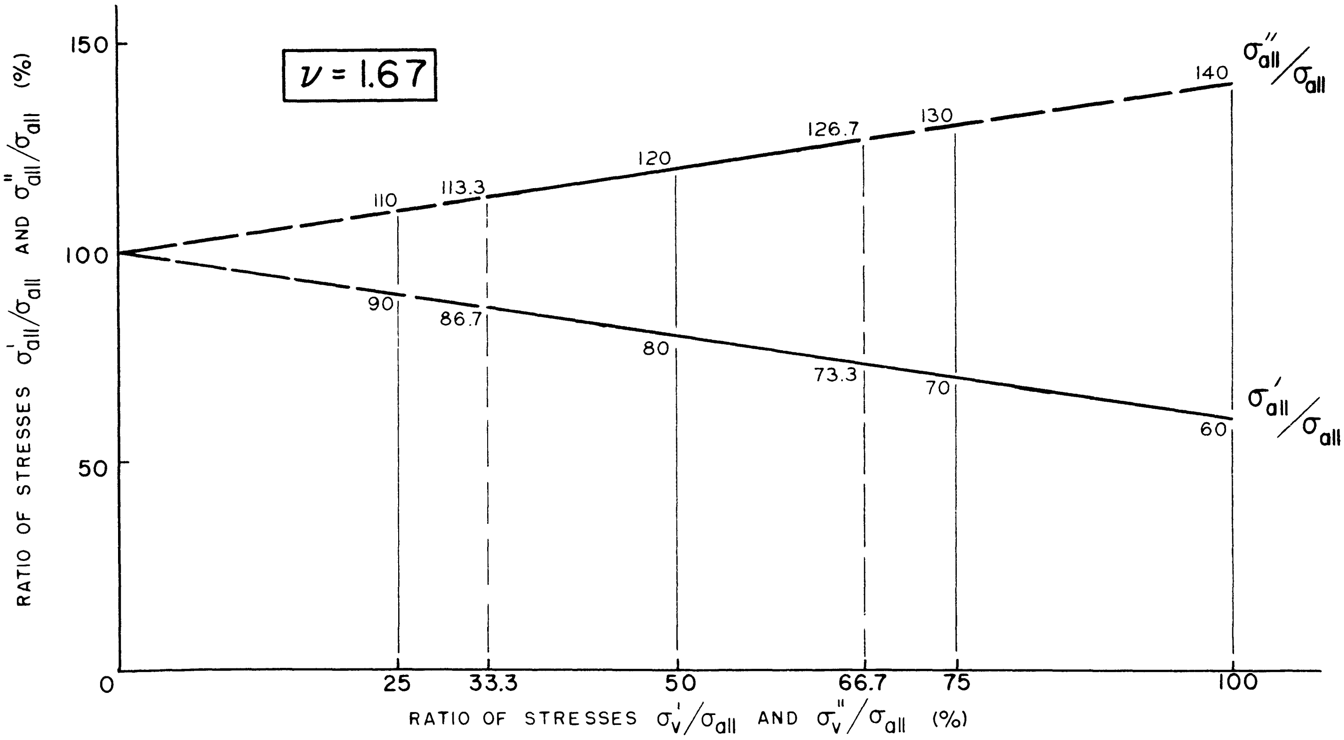


Fig. 3. Variations of allowable stress $\sigma''e_{all}$, σ''_{all} in prestressd structures designed according to modified allowable stresses.

The two latter results could be compared with those of Fig. 3.

The condition of safety then becomes

$$\sigma'_q - \sigma'_v \leq \sigma'_{all} \tag{11}$$

and

$$\sigma''_q + \sigma''_v \leq \sigma''_{all} \tag{12}$$

The concept of modified allowable stress was introduced by G. Magnel[12] for the elements of Group I, and since that time several other authors have utilized his approach.

It is noted that the variance in the values of the modified allowable stresses, which depends on the magnitude of the prestressing (Fig. 2b), destroys the simplicity and clarity of the analysis and might lead to errors.

In the case of the elements of Group II, the raising of the allowable stress σ_{all} to σ''_{all} may be questionable; they are stressed by prestressing and even so they are allowed to carry the same loading q_{all} (which increases the initial stress σ''_v due to prestressing) as a non-prestressed, non-stressed element of Fig. 1a. This is why some supporters of the concept of the modified allowable stresses have designed elements which fall into Group II with an allowable stress $\sigma''_{all} = \sigma_{all}$, reasoning that the increment of stress σ''_q due to the external loads is in most cases small and the character of the state of stress for these elements is therefore analogous to the state of stress in a non-prestressed structure under a dead load. This explanation, however, is valid only for slender high-strength prestressing tendons. From a similar consideration, that stress σ'_v due to prestressing does not increase under any loading $q' > 0$, the limitation $\sigma'_v \leq \sigma_{all}$ follows (Fig. 1c).

Concept Approaching Limit States Design—It was probably B. Fritz[13] who first attempted to do away with the liabilities of the previously mentioned concepts of analysis. The approach has also been utilized by several other authors such as W. Wrycza,[14] K. H. Schneider,[15] and M. Mortensen[16] et al. Fritz's method is a simplified and, unfortunately, an imperfect limit states analysis (as described in the next section). The fundamental condition of safety has the form

$$\nu\sigma'_q - \sigma'_v \leq \sigma_{lim} \tag{13}$$

$$\nu\sigma''_q + \sigma''_v \leq \sigma_{lim} \tag{14}$$

Here, the unique factor of safety ν for the external loading q', q'' is the same as above, Eq. (1). It is presumed that the stresses σ'_v, σ''_v from prestressing (of highest values according to Fig. 1d) could be induced nearly precisely, so that at the increased stress level $\nu\sigma'_q$ or $\nu\sigma''_q$ caused by the external loading, a safety factor for prestressing equal to unity is quite satisfactory. The inheritance from the preceding methods (and the mentioned inconsistency from the point of view of limit states design) is the unchanged condition for stresses due to prestressing

$$\sigma'_v \leq \sigma_{all} \tag{15}$$

$$\sigma''_v \leq \sigma_{all} \tag{16}$$

From the graphical interpretation of the above relations (Fig. 1d), it can be seen that Fritz's method in fact only expresses in another form the relationships of the modified allowable stress concept.

This can also be confirmed analytically: taking Eqs. (13) and (14) and dividing them by the safety factor ν, then adding and subtracting the respective stress σ'_v or σ''_v from the left hand side of the obtained relation, we have

$$\sigma'_q - \frac{\sigma'_v}{\nu} - \sigma'_v + \sigma'_v \leq \sigma_{all} \tag{17}$$

$$\sigma''_q + \frac{\sigma''_v}{\nu} + \sigma''_v - \sigma''_v \leq \sigma_{all} \tag{18}$$

After rearranging these expressions, we obtain

$$\sigma_q - \sigma'_v \leq \sigma_{all}\left[1 - \frac{\sigma'_v}{\sigma_{all}}\left(1 - \frac{1}{\nu}\right)\right] = \sigma'_{all} \tag{19}$$

$$\sigma_q + \sigma''_v \leq \sigma_{all}\left[1 + \frac{\sigma''_v}{\sigma_{all}}\left(1 - \frac{1}{\nu}\right)\right] = \sigma''_{all} \tag{20}$$

Equations (19) and (20) are obviously identical to the expressions for the modified allowable stress method, Eqs. (9) and (10), Fig. 2b.

*Concept of Limit States Design**—The previously mentioned difficulties and some others could be overcome by a consistent employment of limit states design principles.[17,18] Two limit states are decisive for steel structures:

First limit state of bearing capacity
Second limit state of deformations

The state of stress is dealt with in checking the *first limit state* where the greatest probable design effects from external loading and prestressing are compared with the smallest probable design bearing capacity of the considered element; expressed in terms of stresses

$$\sum_q c_q n_q \bar{\sigma}'_q - \sum_v n_v \bar{\sigma}'_v \leq mk\bar{R} \tag{21}$$

$$\sum_q c_q n_q \bar{\sigma}''_q + \sum_v n_v \bar{\sigma}''_v \leq mk\bar{R} \tag{22}$$

Symbols in Eqs. (21) and (22) have the following meaning:

* *A modification of this concept is known in the U.S.A. under the name "Load Factor Concept".*[20,21]

q = Subscript denoting effects of external loading
v = Subscript denoting effects of prestressing
c = Grouping factor (expressing improbability of simultaneous appearance of the most unfavorable loadings under the so-called wider or extraordinary load combinations—see Table 4). Used to reduce effects of short acting live loads ($c \leq 1.0$)
n = Load factor or possibly prestressing accuracy factor replacing (together with m and k) the safety factor ν. Expresses possibility of overloading or underloading ($n_{vl} \leq 1.0$, $n_{vu} \geq 1.0$)
$\bar{\sigma}$ = Stress due to standard effects
m = Factor of working conditions respecting special circumstances under which the structural element works (m can be less than, equal to, or greater than 1.0)
k = Factor of homogeneity considering probable deviations from the supposed physical or geometrical characteristics ($k < 1.0$)
$\bar{R}$ = Standard stress (standard yield point $\bar{\sigma}_{yp}$ or yield stress $\bar{\sigma}_{0.2}$ or standard ultimate stress $\bar{\sigma}_{us}$)

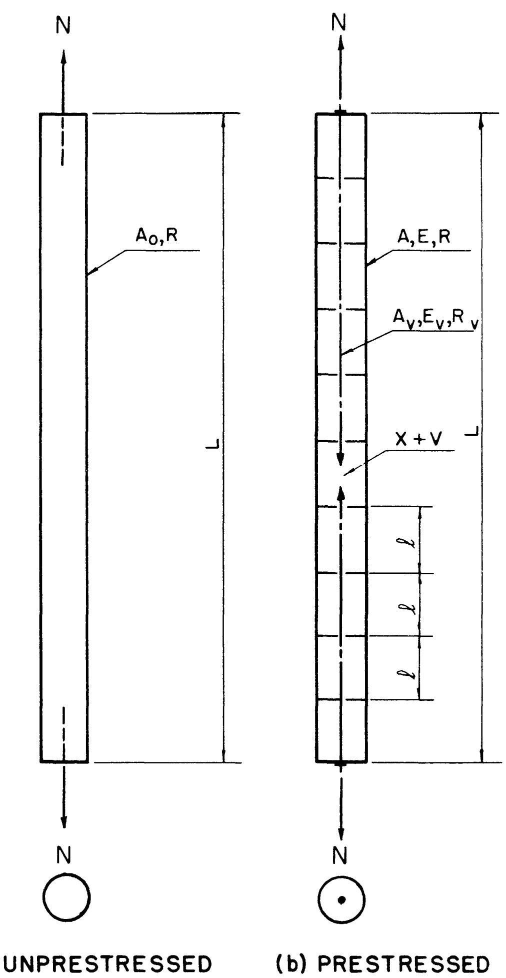


Fig. 4. Stretched bar.

Table 1. Design of a Prestressed Bar According to Fig. 4, General Formulas

	Concept of	Allowable Stresses	Limit States
Nonprestressed Bar	Strength conditions	$\bar{N} = \Sigma \bar{N}_i$ $\sigma^+ = \dfrac{\bar{N}}{A_O} \le \sigma_{all}$	$N = \Sigma n_{iu} \bar{N}_i$ $\sigma^+ = \dfrac{N}{A_O} \le R$
	Design formulas	$A_O \ge \dfrac{\bar{N}}{\sigma_{all}}$	$A_O \ge \dfrac{N}{R}$
Prestressed Bar	Strength conditions	$\bar{N} = \Sigma \bar{N}_i, \bar{X} = \Sigma \bar{X}_i$ $\sigma^- = \dfrac{\bar{V}}{A} \le \dfrac{\sigma_{all}}{\theta}$ $\sigma^+ = \dfrac{\bar{N} - \bar{X} - \bar{V}}{A} \le \sigma_{all}$ $\sigma_v{}^+ = \dfrac{\bar{X} + \bar{V}}{A_v} \le \sigma_{all,\, v}$	$N = \Sigma n_{iu} \bar{N}_i,\ X = \Sigma n_{iu} \bar{X}_i$ $\sigma^- = \dfrac{\theta n_{vu} \bar{V}}{A} \le R$ $\sigma^+ = \dfrac{N - X - n_{vl} \bar{V}}{A} \le R$ $\sigma_v{}^+ = \dfrac{X + n_{vu} \bar{V}}{A_v} \le R_v$
	Redundant force in tendon	$\bar{X} = \dfrac{\bar{N}}{1 + \epsilon \dfrac{A}{A_v}}$	$X = \dfrac{N}{1 + \epsilon \dfrac{A}{A_v}}$
	Design formulas	$A \ge \dfrac{\bar{N}}{\sigma_{all}} \dfrac{\epsilon\rho - \eta}{\eta \left(\epsilon\rho + \dfrac{1}{\theta} - \eta\right)}$ $A_v \ge \dfrac{\bar{N}}{\sigma_{all}} \dfrac{\epsilon}{\theta\eta \left(\epsilon\rho + \dfrac{1}{\theta} - \eta\right)}$ $\bar{V} = \bar{N} \dfrac{\epsilon\rho - \eta}{\theta\eta \left(\epsilon\rho + \dfrac{1}{\theta} - \eta\right)}$	$A \ge \dfrac{N}{R} \dfrac{\epsilon\rho - \eta}{\eta \left(\epsilon\rho + \dfrac{1}{\theta} - \eta\right)}$ $A_v \ge \dfrac{N}{R} \dfrac{\epsilon}{\theta\eta \left(\epsilon\rho + \dfrac{1}{\theta} - \eta\right)}$ $\bar{V} = \dfrac{N}{n_{vu}} \dfrac{\epsilon\rho - \eta}{\theta\eta \left(\epsilon\rho + \dfrac{1}{\theta} - \eta\right)}$
	Parameters	$\epsilon = \dfrac{E}{E_v};\ \rho = \dfrac{\sigma_{all,\, v}}{\sigma_{all}} = \dfrac{\bar{\sigma}_{lim,\, v}}{\bar{\sigma}_{lim}} \cdot \dfrac{\nu}{\nu_v};\ \eta = 1 + \dfrac{1}{\theta}$	$\epsilon = \dfrac{E}{E_v};\ \rho = \dfrac{R_v}{R} = \dfrac{m_v k_v \bar{\sigma}_{lim,\, v}}{mk\bar{\sigma}_{lim}};\ \eta = 1 + \dfrac{1}{\theta} \dfrac{n_{vl}}{n_{vu}}$

Standard effects (prescribed by specifications or codes and designated by a bar over the symbol) multiplied by the corresponding factors are called *design effects;* the right hand side of Eqs. (21) and (22) define the design stress $R = mk\bar{R}$.

There are other factors which could be considered and would appear on the left hand side of Eqs. (21) and (22). One example* would be the buckling coefficient $\theta > 1.0$; another example would be a dynamic coefficient $\kappa > 1.0$. On the right hand side of these formulas we might have a fatigue coefficient $\gamma < 1.0$, etc.**

Let us illustrate the difference between the designs according to allowable stresses and limit states, respectively, by a simple problem. Consider a stretched bar prestressed by a high-strength tendon as shown in Fig. 4. The governing relations are assembled in Table 1 and prestress is assumed to be introduced before any load acts on the bar. Effects of dead loads are denoted by subscript g and effects of live loads by subscript p;

* *Up to now, a different approach has been utilized in the United States to check stresses: instead of introducing coefficients $\theta > 1.0$ and $\kappa > 1.0$, in the first case the allowable stress has been reduced (Sec. 1.5[7]); in the second case, a further component of stress (expressed as a percentage of the live load stress) has been taken into account (Sec. 1.3[7]).*

** *The present American specifications (Sec. 1.7[7]), instead of decreasing the allowable stresses, increase the computed stresses.*

Table 2. Design of a Prestressed Bar According to Fig. 4 and Table 1, Numerical Results

Concept of	Allowable Stresses		Limit States	
Assumed values of parameters	$\bar{N}_g = 0.5\bar{N}; \bar{N}_p = 0.5\bar{N}; \bar{N} = \bar{N}_g + \bar{N}_p$; $\epsilon = 1.1; \theta = 1.25$[a]			
	$\eta = 1.8$ $\nu = \nu_v = 1.667$ $\rho = 5; \sigma_{all} = \frac{\bar{\sigma}_{yp}}{1.667}$		$\eta = 1.655$ $n_{gu} = 1.1; n_{pu} = 1.3; n_{gl} = n_{pu} = 0; n_{vu} = 1.1; n_{vl} = 0.9$[b] $\frac{k_v}{k} = 0.9$[c]$; \rho = 0.9 \times 5 = 4.5; R = \frac{1.5}{1.667} 0.9\bar{\sigma}_{yp} = 0.81\bar{\sigma}_{yp}$[d]	
Results according to formulas from Table 1	$A_O = 1.667 \frac{\bar{N}}{\bar{\sigma}_{yp}}$	(113%)	$A_O = 1.48 \frac{\bar{N}}{\bar{\sigma}_{yp}}$	(100%)
	$A = 0.761 \frac{\bar{N}}{\bar{\sigma}_{yp}}$	(106%)	$A = 0.720 \frac{\bar{N}}{\bar{\sigma}_{yp}}$	(100%)
	$A_v = 0.181 \frac{\bar{N}}{\bar{\sigma}_{yp}}$	(94.2%)	$A_v = 0.192 \frac{\bar{N}}{\bar{\sigma}_{yp}}$	(100%)
	$A + A_v = 0.943 \frac{\bar{N}}{\bar{\sigma}_{yp}}$	(103.4%)	$A + A_v = 0.912 \frac{\bar{N}}{\bar{\sigma}_{yp}}$	(100%)
	$\frac{A + A_v}{A_O} = 0.566$	(91.9%)	$\frac{A + A_v}{A_O} = 0.616$	(100%)
	$\bar{V} = 0.365\,\bar{N}$	(93.8%)	$\bar{V} = 0.389\,\bar{N}$	(100%)

Notes: [a] Buckling coefficient $\theta > 1.0$ could be easily controlled by arranging stabilizing diaphragms; their distance l equals the buckling length, Fig. 4.
[b] Cf. with Table 3.
[c] Homogeneity factors for high strength steels have lower values than for carbon steels, cf. with Table 5.
[d] Difference between the U. S. requirements of safety ($\nu = 1.667$) and the continental ones ($\nu = 1.5$) expressed by a reduction factor of 1.5/1.667.

subscript i is a general notation for any type of load. Subscripts u and l at prestress accuracy factors n_v designate their upper or lower value, respectively: $n_{vu} > 1.0$; $n_{vl} < 1.0$.

The assumed values of parameters and the numerical results are indicated in Table 2. Obviously, the limit states design brings material savings to the non-prestressed structures (A_0), especially because of low values of safety factor $n_{gu} = 1.1$ for dead loads. When the limit states design is used in the prestressed structures ($A + A_v$), such an economy is nearly lost. This is caused chiefly by the necessity of considering both values $n_{il} < 1.0$, $n_{iu} > 1.0$ of load factors and prestress-accuracy factors, for the sake of safety. Material is differently distributed than in the allowable stresses analysis; prestressing force $\bar{V}$ is of a greater value, economy from prestressing $(A + A_v)/A_0$ is a little less. However, the safety of the prestressed structure has grown larger due to all discrepancies being removed.

State of Deformations—The aforementioned problems are not encountered in the analysis of deformations of prestressed structures. No matter which concept is used to design the prestressed structure, the conditions limiting the deformation are the same.

For elements falling into Group I:

$$\sum_q \bar{\delta}'_q - \sum_v \bar{\delta}'_v \leq \bar{\Delta}' \tag{23}$$

For elements falling into Group II:

$$\sum_q \bar{\delta}''_q + \sum_v \bar{\delta}''_v \leq \bar{\Delta}'' \tag{24}$$

The formulas have been written in the symbols of the limit state analysis: $\bar{\delta}_q$ is the deformation due to the load; $\bar{\delta}_v$ is the deformation due to prestressing; $\bar{\Delta}$ is the standard deformation (a limit prescribed by specifications).

The bars over the symbols emphasize that the deformations are computed for standard loads as requested by rules of limit states design. Also the dynamic coefficients $\kappa > 0$ are not considered when checking deformations. The reason is that the deformations are checked for the frequent conditions of service and not for an extraordinary stage when the structure is shortly overloaded to the greatest possible extent (such a possibility is studied in the analysis of the state of stresses).

Difficulties encountered in the state of stress disappear, as neither the safety factor ν, nor the load coefficient n_q, nor the prestress accuracy factor n_v influence Eqs. (23) and (24). However, another problem arises for elements of Group I resulting from service requirements or dictated by esthetic aspects: that is, are the conditions of Eqs. (23) and (24) really suitable for prestressed structures, or should we satisfy a stricter condition:

$$\sum_q \bar{\delta}'_q \leq \bar{\Delta}' \tag{25}$$

Let us compare the analysis according to both conditions in an example of a simply supported beam (Fig. 5), where the deformation δ represents the deflection f.

The supporters of the stricter conditions, Eq. (25), consider the deflection from prestressing (Fig. 5a) as an initial camber resulting from the manufacture of the girder (Fig. 5b); and in structures with a camber, the design deflection $\bar{\Delta}$ is usually measured from the cambered position (Fig. 5d).

On the other hand, the advocates of Eq. (23) treat the prestressing effects as a special kind of external loading. They compare, for example, the girder prestressed by a straight tendon (Fig. 5a) with one having loaded overhanging ends (Fig. 5c). This load could be regarded as a dead load of the usual type or as a special means of prestressing—ballast. Naturally, when checking deformations, a unique standpoint should be employed for both cases (Figs. 5a and 5c); i.e., to measure the limit deformation $\bar{\Delta}$ from the horizontal axis (Fig. 5e).

Our recommendation is to consider the upward deflection from prestressing as a camber and to sum it up with constructional camber, and then to disregard the deflection due to the dead load, if the total camber is greater than this deflection (Fig. 6a). If they are equal, take into account only the difference between the values due to the dead load and the total camber in a case when the total camber is less than the deflection due to the dead load (Fig. 6b). Formulas appearing in Fig. 6 are then in force.

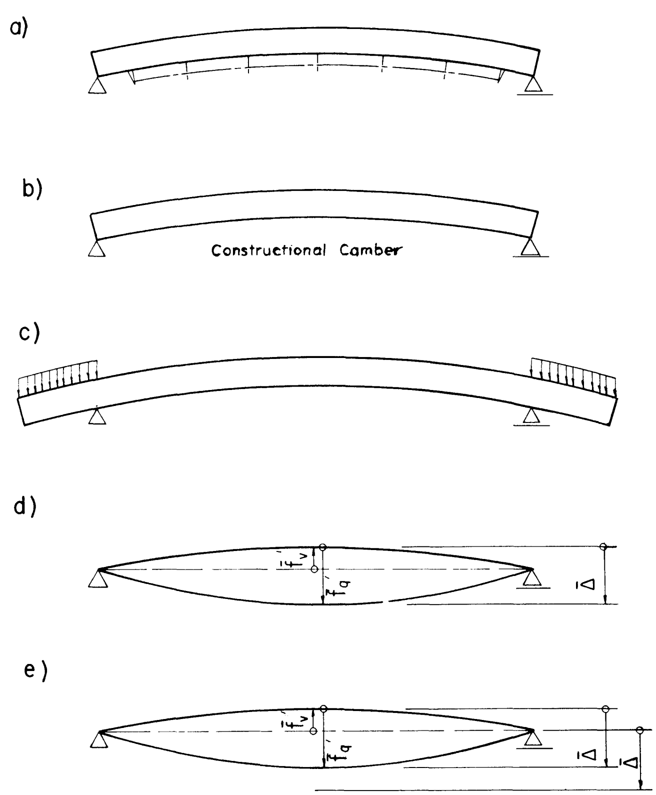


Fig. 5. Diagrams explaining Eqs. (23) and (25) for checking deformations of prestressed structures.

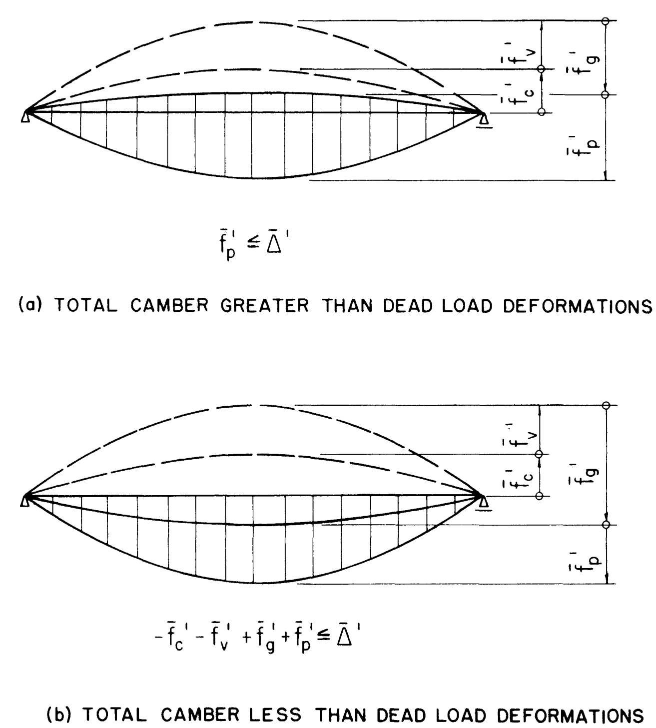


Fig. 6. Checks of deflections of a loaded prestressed beam: (a) Total camber (sum of constructional camber f_c^o and of camber due to prestressing f_v^o) is greater than deflection from dead load: $f_c^o + f_v^o > f_g^o$; (b) total camber is less than deflection from dead load: $f_c^o + f_v^o < f_g^o$. In (a) and (b) f_p^o means deflection from live load.

Additional Information about Limit States Design—The concept of limit states design was introduced first in the Soviet Union[17,18] in the 60's, and later in other East European countries. For the present, the limit states analysis is utilized only for civil engineering structures other than bridges, because not enough experimental data have been gathered to specify all needed coefficients and parameters for the latter.* The research work on the fundamentals for design of bridges according to limit states is in progress, so that specifications and standards for bridges could be expected to be issued in a few years. In recent years, the limit states design concept has been studied seriously in Western European states as well.[22] There are signs that the United States will also follow in the use of limit states design eventually, Sec. 10[3]; Art. 1.6[9]; p. 1377.[6,20,21]

Two limit states are distinguished in metal structures:

First limit state—when checking strength and stability.
Second limit state—when checking deformations.

When investigating the *first limit state*, the so-called design loading is to be used; with the *second limit state*, the standard loading. If required by the character of loading, the fatigue of materials is accounted for in the computations according to the first limit state; in this case the standard loading is considered.

In checking the second limit state, there are no substantial alterations from present concepts. There are, however, major *changes in the calculations* according to the first limit state.

The unique safety factor ν, Eq. (1), has been replaced by three groups of factors as they appear in Eqs. (21) and (22):

(a) the load factors n_q or the prestress accuracy factor n_v**
(b) the working condition factor m
(c) the homogeneity factor k

The improbability of the simultaneous appearance of the most critical load combination is handled by a decreasing grouping factor $c < 1.0$.

Coefficients and parameters prescribed by specifications for the limit states design have been obtained by a statistical analysis of numerous sets of data and test results. The arrangement of coefficients and computations not only results in a safe and more scientific design, but frequently leads to savings in both material and cost, especially in non-prestressed structures.

* *On the contrary, in the U.S.A. tentative criteria for the load factor design were first prepared for bridges.*[20]

** *Furthermore, coefficients n_q and n_v are differentiated (Table 3) according to: character of loading ($n_{qu} = 1.1$ for self-weight; $n_{qu} = 1.4$ for snow); critical combinations of loading (where more severe, lower values of coefficients $n_{ql} < 1.0$ or $n_{vl} < 1.0$ are to be used rather than the upper values $n_{qu} > 1.0$ or $n_{vu} > 1.0$).*

Table 3. Load Factors n_q and Prestress-Accuracy Factors n_v

Type of Loading	Lower Value n_l	Upper Value n_u
Self-weight	0.9	1.1[a]
Snow	0	1.4
Wind	0	1.2[b]
Movable and/or moving loads	0	1.2–1.4[c]
Hydrostatic pressure	1.0	1.1
Pressure of loose materials	0.9–0.8	1.2–1.3[c]
Prestressing	0.9	1.1[d]

[a] For concrete or brick-structures or for elements with great productional tolerances . . . $n_{qu} = 1.2$.
[b] For slender structures with height-width ratio $H/B \geq 5 \ldots n_{qu} = 1.3$.
[c] Values according to the reliability of information about loading.
[d] At less reliable prestressing techniques and insufficient check of prestressing . . . even more unfavorable values. At extraordinarily accurate prestressing and check . . . until $n_{vl} = n_{vu} = 1.0$.

Principal information concerning the factors encountered in limit states design are presented in Tables 3 through 7, predominantly according to Czechoslovak sources.[19] At the present time, Czechoslovakia is the only country having specifications for designing prestressed steel structures.†

Table 3 is comprised of the common values of the *load factors* n_q and of the *prestress accuracy factors* n_v. The standard loads whose effects are to be multiplied by the mentioned coefficients are, in principle, the same as those used in the design according to allowable stress.

The values of the *grouping factors* c appear in Table 4 along with some basic information concerning three *groups of loading* (basic, wider and extraordinary combinations).

The *homogeneity factors* k (Table 5) account for the possibility that the actual limit stress (yield point σ_{yp}, yield stress $\sigma_{0.2}$, or ultimate stress σ_{us}) could be smaller than that assumed by specifications and denoted by bars over the symbols. The more complex (e.g., either in chemistry or production method) the material, the lower the value of k. By the aid of the homogeneity factor, another danger is also guarded against: the deviations of the actual profile of the element from that stated in the manufacturer's catalog.

The *working condition factors* m are used to account for peculiarities of behavior of some exceptional or unusual structural detail or element where the frequent

† *M. Tochacek had the distinction to be the main author of them.*

Table 4. Load Combinations and Grouping Factors c

Load Combination	Components: Dead Loads	Components: Live Loads, Long Acting[a]	Components: Live Loads, Short Acting[b]	Components: Extraordinary Loads[c]	Grouping Factor c
Basic	1 or more	1 or more	1	More	1.0
Wider			More		0.9
Extraordinary				1	0.8
			↑ multiply		↑

Examples of loads:

[a] Movable loads, long-lasting temperature effects, effects of mining subsidence and supports settlements.
[b] Snow, wind, moving loads, short-lasting temperature effects.
[c] Earthquake, explosions, defect-loads.

Table 5. Homogeneity Factors k

Material			Standard Stress $\bar{R}$	Homogeneity Factor k
Constructional steels	Carbon steels		$\bar{\sigma}_{yp}$	0.9
	High-strength low alloy steels		$\bar{\sigma}_{0.2}$	0.85–0.8[a]
	Heat treated high-strength carbon steels			0.8–0.75[a]
	Heat treated alloy steels			0.75
Reinforcing bars (as used in concrete engineering)			$\bar{\sigma}_{0.2}$	0.85–0.75[a]
	Light alloys		$\bar{\sigma}_{0.2}$	0.85–0.8[a]
Single wires	Uncoated		$\bar{\sigma}_{us}$	0.65
	Zinc-coated (class A)[c]			0.60
Strands and ropes with helically laid wires. Wires are:	Uncoated	Unbrazed	$\bar{\sigma}_{us}$[b]	0.70
		Brazed		0.65
	Zinc-coated[c] (class A)	Unbrazed		
		Brazed		0.60
Ropes with straight wires. Cables. Wires are:	Uncoated		$\bar{\sigma}_{us}$[b]	0.70
	Zinc-coated[c] (class A)			0.65

[a] Lower values for more complex materials with higher yielding stress $\bar{\sigma}_{0.2}$.
[b] The ultimate stress of a wire-rope or of a cable

$$\bar{\sigma}_{us} = \frac{\Sigma \bar{\sigma}_{us,1} A_1}{\Sigma A_1}$$

is determined as the minimum breaking strength—approximate metallic area ratio. The minimum breaking strength $\Sigma \bar{\sigma}_{us,1} A_1$ (given in catalogs or specifications) is the sum of minimum ultimate tensile strengths of all bearing wires in a rope, if the wires are tested individually; the total approximate metallic area ΣA_1 (given also in specifications or catalogs) is the sum of areas of all bearing wires. Subscript 1 denotes the respective quantities for one wire. Obviously, the term "minimum breaking strength" is not quite precise (the Continental nomenclature uses the term "nominal ultimate strength") because the actual strength at failure is smaller and could be approximately determined by multiplying the specified breaking strength by homogeneity factor $m < 1.0$ from Table 6.

[c] The higher value of k, that value for the uncoated wires, is to be used for the zinc-coated wires, if the minimum breaking strength for the zinc-coated wires is already considered in the formula in note[b] and similar ones. Class A coating is the most frequent one for structural purposes. If heavier coatings are utilized, the homogeneity factor k need not be decreased when the specified minimum breaking strengths are adequately reduced.

Table 6. Working Condition Factors m for Prestressing Tendons

Type of Tendon					m
Strands and ropes with helically laid wires	"Bridge" strand Locked coil strand	Single strand, multiple wire, zinc-coated Z- or H-O-locked strand			0.80
	"Bridge" rope	Multiple strand, wire strand core, zinc-coated	Number of wire lays in a strand	1	0.90
				2	0.85
				3	0.82
	Other ropes	Constructions with more than 222 wires. Lang lay wire ropes (Seale, Warrington, etc.)			0.75
Ropes with straight wires. Cables					0.92
Constructional steels. Reinforcing bars					1.00

methods of analysis are not quite reliable or suitable. For example, stress concentrators (grooves, notches, cavities, holes, etc.) increase stresses locally above the average value; then $m < 1.0$. In another case, due to the plastic reserve of ductile materials, a detail or a section may have a greater bearing capacity than supposed by the "elastic" analysis; then $m > 1.0$.

In prestressed steel structures, the working condition factors of tendons (Table 6) are of a special interest: for instance, in tendons composed of several elements (wire ropes, cables, etc.), stress is distributed non-uniformly among the elements and, therefore, overstressing of some elements could occur; hence, it is reasonable to introduce $m < 1.0$. Other working condition factors m for prestressed steel structures are listed in Table 7.

It is necessary to draw the reader's attention to the fact that in the United States, a *higher safety factor* (ν = 1.667) for constructional steels is specified than was in Europe ($\nu = 1.50$). The mentioned coefficients and parameters of the limit states design have been determined to correlate with the concept of allowable stress and a safety factor $\nu = 1.50$. To employ the concept of limit states and not deviate too much from the American safety factor $\nu = 1.667$, it is reasonable to take into consideration, in addition to the other working condition factors, a *supplemental factor* $m = 1.5/1.667 = 0.9$.

The supplemental working condition factor $m = 0.9$ normally need not be employed for tendons of wire-ropes, cables or wires. Limit states design using factors from Tables 3 through 6 gives approximately equivalent results as the usual U. S. design (Sec. 10[3]).

If *more than one* of the working condition factors m would be employed, just their product is used. However, if more than one of them is greater than 1.0, for the sake of greater safety, it is better to consider in this product only the greatest factor $m_{max} > 1.0$.

Table 7. Working Condition Factors m for Prestressed Steel Structures

Special detail or special type of service	m
Parts of anchoring	0.8
Prestressing tendons in zones of curvatures if stresses are figured out according to elementary elastic theory	1.15[a]
Short-acting overstressing of elements caused especially by: (a) prestretching ropes or wires; (b) increasing prestressing effects to reduce successive prestress-losses; (c) unfavorable transport or erection effects	1.10

[a] The value could be raised if it is reasoned by results of tests.

CONCLUSIONS

The brief analysis of principal concepts of proportioning prestressed steel structures presented has demonstrated that the concept of allowable stress is not suitable at all for these structures. Special character of these structures requires the use of more ingenious concepts of which the limit states design seems to be the most suitable.

The usage of the allowable stress concept for non-prestressed structures could be tolerated as long as such large safety factors like 1.667 (related to yielding point) or 3.0 (related to ultimate stress) are utilized. However, future efforts for a higher economy and lower costs of structures will lead to reductions of safety factors. Since the appropriate safety is to be maintained, there will probably be no other way than to use the limit states concept for non-prestressed structures too.

For the present, designers in the United States could profit from the experience of the Europeans and their data, coefficients, and parameters might be adapted (see Tables) to American standards for the design according to the limit states. Nevertheless, the time is approaching to think about preparations of the American specifications. The first achievements[3,20,21] in this direction should be highly appreciated.

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