Design of Steel Bearing Plates

RUSSELL S. FLING

BASE PLATES for columns and bearing plates for beams resting on masonry are details associated with the design of all steel structures. Given the load and allowable bearing pressure, the area of the plate is easily computed. From this, the overhanging cantilever span can be determined. If the yield point of the steel is known, the thickness of the plate required can be determined by the procedures outlined on pages 2-44 or 3-75 of the AISC Manual.[1]

The required plate thickness is given by:

$$t = \sqrt{3F_p n^2/F_b} \tag{1}$$

This solution is time consuming if more than an occasional bearing plate is encountered. Fortunately it can be easily presented in tabular or graphical form.

Design procedures presented in the AISC Manual are silent on two important questions. No limit is placed on plate deflections. Column base plates nearly the same size as the column cannot be properly designed by the AISC procedures.

PLATE DEFLECTION

As the size of the plate increases, the deflection becomes larger and portions of the plate which deflect most cannot distribute the assumed uniform loading to the supporting material. Thus portions of the plate which deflect the least must carry a higher load and may overstress the underlying support. Therefore some limit should be placed upon the deflection of bearing plates. Ideally, this limit should be a function of the deformability of the supporting material.

The equation for deflection of a fixed ended cantilever under uniform load can be restated to express the required thickness of a bearing plate as a function of the deflection at the edge of the plate.

thus,

$$t = \frac{n}{10}\sqrt[3]{\frac{F_p n}{20a}} \tag{2}$$

where E is assumed to be 30,000 ksi.

Russell S. Fling is President of Fling and Eeman, Inc., Consulting Engineers, Columbus, Ohio.

It is beyond the scope of this paper to present a theoretical analysis of the deflection that should be allowed for various supporting materials. However, it is believed that 0.01 in. of upward deflection at the plate edge is a reasonable and practical limitation for most bearing plates.

It is also useful to know the cantilever span length of plate below which strength considerations govern and above which deflection considerations govern.

Equation (1) can be restated, $F_p = F_b t^2/3n^2$ and used to replace F_p in the familiar equation for deflection of a cantilever beam under a uniform load, $a = 3n^4/2Et^2 \times F_p$. After rearranging, the following equation results:

$$t = n^2 F_b/60{,}000\ a \tag{3}$$

where E is assumed to be 30,000 ksi.

To simplify the chore of selecting bearing plate thicknesses, Eq. (1), (2) and (3) can be combined on one graph (see Fig. 1). For example, what thickness would be required for a 16″ x 16″ base plate under a 10W̅F49 column? The overhanging cantilever span would be $(16 - 0.80 \times 10)/2 = 4.0$ in. For a bearing stress of 750 psi, a thickness of 1¼ in., can be read from Fig. 1. Similarly, a 10″ x 10″ base plate under the same column has an overhanging span of one inch and a required thickness of less than ½ in. However, as the next section indicates, this thickness may not be adequate.

MINIMUM COLUMN BASE PLATE THICKNESS

As the load on a column diminishes, the required base plate approaches the size of the column itself. By the AISC method of analysis, the overhanging cantilever span and therefore the plate thickness approach zero. Obviously, the AISC method of analysis does not apply in such situations.

For example, an 8W̅F24 column of A36 steel would usually be used to carry 90 kips if $KL = 12$ ft. With a bearing pressure of 1125 psi, a base plate 10″ x 8″ in size would be required. The overhanging span is 1.4″ requiring a thickness of ½ in. by Eq. (1). Similarly, a 14W̅F87 column with a base plate 20″ x 1½″ x 1′–8″ would be required to carry 450 kips if $KL = 15$ ft. The subsequent analysis shows that these thicknesses

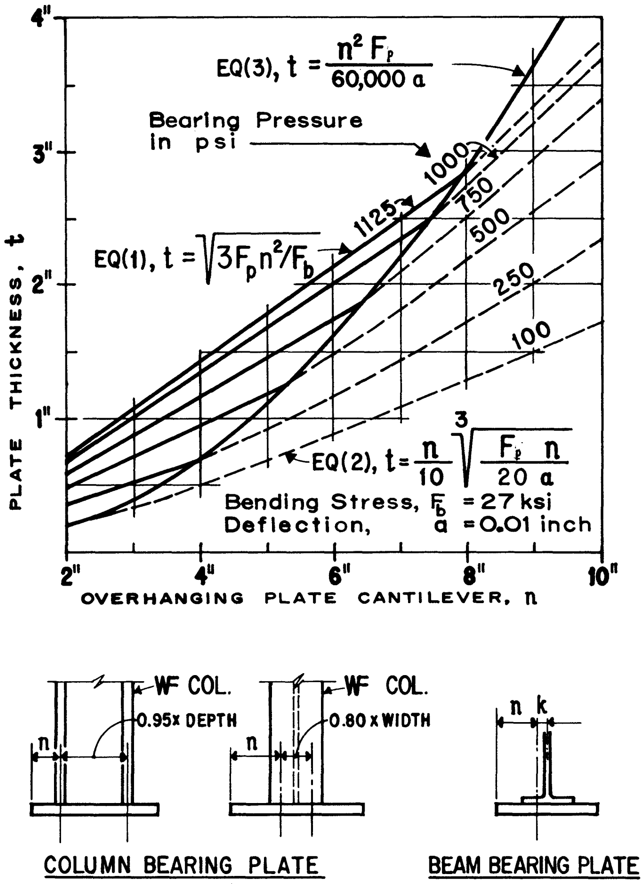

Fig. 1. Bearing plate thickness

are not adequate and that the plate thickness should be $\frac{7}{8}$ and $1\frac{3}{4}$ in., respectively, for these two examples.

This problem can be most easily solved by the yield line theory. The development of this theory can be found in standard textbooks on Structural Design.[2] Its application to the design of bearing plates for columns is as follows:

Referring to Fig. 2,

For Part I, the Unit Rotation about the $X-X$ axis $= 1/b\beta$.

For Part II, the Unit Rotation about the $Y-Y$ axis $= 1/b$.

D = internal dissipation of energy.

D = (Rotation) $\times$ (length of line) $\times$ (plastic moment).

$$D = \frac{1}{b\beta} \times 4b \times M_p + \frac{1}{b} \times 4b\beta \times M_p + \frac{1}{b} \times 2d \times M_p = 4M_p\left(\frac{1}{\beta} + \beta + \frac{d}{2b}\right)$$

W = the external work.

W = the volume of pyramids of deflections times the unit pressure.

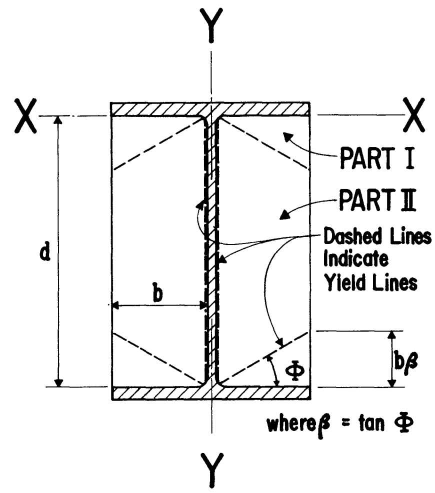

Fig. 2. Plan of column base plate

$$W = \frac{1}{3} F_p \times b \times b\beta \times 4 + \frac{1}{2} F_p \times b(d - 2b\beta)2$$

$$= F_p b^2 \left(\frac{d}{b} - \frac{2}{3}\beta\right)$$

Let $\lambda = d/b$ and set $D = W$,

$$4M_p\left(\frac{1}{\beta} + \beta + \frac{\lambda}{2}\right) = F_p b^2\left(\lambda - \frac{2}{3}\beta\right)$$

$$M_p = \frac{F_p b^2}{4} \times \frac{\lambda - \frac{2}{3}\beta}{1/\beta + \beta + \lambda/2} \qquad (4)$$

There is only one value of β for which F_p is a minimum, or M_p is a maximum. Differentiating with respect to β and setting $M_p' = 0$, leads to:

$$-\frac{2}{3}\left(\frac{1}{\beta} + \beta + \lambda/2\right) - \left(\lambda - \frac{2}{3}\beta\right)\left(-\frac{1}{\beta^2} + 1\right) = 0 \qquad (5)$$

rearranging,

$$\frac{\lambda - \frac{2}{3}\beta}{1/\beta + \beta + \lambda/2} = -\frac{2}{3} \times \frac{1}{1 - 1/\beta^2} = \frac{2\beta^2}{3(1 - \beta^2)}$$

but from Eq. (4),

$$\frac{\lambda - \frac{2}{3}\beta}{1/\beta + \beta + \lambda/2} = \frac{4M_p}{F_p b^2}$$

Therefore,

$$\frac{2\beta^2}{3(1-\beta^2)} = \frac{4M_p}{F_p b^2}$$

or

$$M_p = \frac{F_p b^2}{2} \times \frac{\beta^2}{3(1-\beta^2)} \tag{6}$$

To allow for lack of full plastic moment at the corners, M_p should be increased by 10 percent. Also, $F_y t^2/4$ can be substituted for M_p and a factor of safety of 2 inserted.

Thus: $$F_y \frac{t^2}{4} = \frac{1.10 F_p b^2}{2} \times \frac{\beta^2}{3(1-\beta^2)} \times 2$$

and, $$t = 1.21 b\beta \sqrt{\frac{F_p}{F_y(1-\beta^2)}} \tag{7}$$

To find the value of β, solve Eq. (5) by expanding and collecting,

$$-4/3\,\beta - 4\,\lambda/3 + \lambda/\beta^2 = 0$$

or

$$4\beta + 4\lambda\beta^2 - 3\lambda = 0$$

$$\beta^2 + \beta/\lambda = 3/4$$

from which

$$\beta = \sqrt{\frac{3}{4} + \frac{1}{4\lambda^2}} - \frac{1}{2\lambda} \tag{8}$$

For example:

Compute the minimum bearing plate thickness for a 14 × 8WF column, using $F_p = 0.750$ ksi and $F_y = 36$ ksi. $\lambda = 12.62/3.85 = 3.28$

Using Eq. (8),

$$\beta = \sqrt{\frac{3}{4} + \frac{1}{4 \times (3.28)^2}} - \frac{1}{2 \times 3.28} = 0.880 - .152 = 0.728$$

Using Eq. (7),

$$t = 1.21 \times 3.85 \times 0.728 \sqrt{\frac{0.75}{36[1 - (0.728)^2]}} = 0.711 \text{ in.}$$

The deflection of a small base plate should also be limited to a reasonable value. Roark's[3] equation for the maximum deflection at the middle of the free edge of a plate which is fixed on the opposite edge and supported on the other two edges can be restated as:

$$t = \sqrt[3]{\frac{1.37 F_p b^4}{Ea\,(1 + 10/\lambda^3)}} \tag{9}$$

One of the assumptions made in the derivation of this equation is that the plate is nowhere stressed beyond the elastic limit. The thickness required to satisfy this assumption can be computed from another of Roark's equations by setting the maximum stress, which occurs at the middle of the fixed edge, equal to the yield stress of the steel. Thus:

$$t = \sqrt{\frac{3F_p b^2}{F_y(1 + 3.2/\lambda^3)}} \tag{10}$$

Note that Eq. (10) merely gives the minimum thickness for which Eq. (9) is applicable. Since the yield stress is used with no factor of safety, Eq. (10) cannot be relied upon to give a plate thickness which will be sufficiently strong. The yield line analysis must be used to check the ultimate strength.

Eqs. (7), (9) and (10) have been solved for all rolled steel structural shapes normally used for columns, using the clear inside dimensions of the shapes, ASTM A36 steel ($F_y = 36$ ksi), two common bearing pressures and an allowable deflection of 0.01 in. The computer output is shown in Table 1 along with the minimum recommended thickness for each structural shape. Since they are not direct determinants of plate thickness, the values from Eq. (10) have been reduced as much as 5 percent in some cases in establishing the recommended thicknesses, provided that the computed deflection and ultimate yield line strength are satisfactory.

The computed thicknesses are somewhat conservative because the edges of the plate under the column flanges are undoubtedly partially restrained rather than freely supported, as assumed. This would be especially true if the column is welded to the base plate on all sides. In addition, the recommended thicknesses in Table 1 are still more conservative because no attempt was made to compute the deflection after first yielding. It seems likely that substantial yielding could occur before the deflection would deviate significantly from the elastically computed deflection.

A uniform bearing pressure has been assumed for all computations. Although an analysis of the interaction between the bearing plate and the supporting material is beyond the scope of this paper, it should be noted that the bearing pressure is probably somewhat higher under portions of the plate which deflect the least. This would have the effect of reducing the computed bending stresses and deflection of the plate. Thus the plate thicknesses shown in Table 1 are conservatively stated from this point of view also.

Table 1. Minimum Column Base Plate Thickness
F_y = 36.0 ksi., F_p = 0.750 and 1.125 ksi.

Column Size	B In.	D In.	Minimum Theoretical Thickness						Design Values	
			F_p = 0.750 ksi			F_p = 1.125 ksi				
			Eq 10	Eq 9	Eq 7	Eq 10	Eq 9	Eq 7	0.750	1.12
4 WF	1.89	3.47	0.383	0.255	0.271	0.469	0.292	0.332	3/8	1/2
5 WF	2.37	4.36	0.481	0.346	0.341	0.589	0.396	0.417	1/2	5/8
6 WF	2.88	5.46	0.593	0.457	0.420	0.727	0.523	0.515	5/8	3/4
8 × 6.5 WF	3.13	7.13	0.694	0.562	0.498	0.850	0.644	0.610	3/4	7/8
8 × 8 WF	3.86	7.13	0.785	0.665	0.556	0.962	0.761	0.681	3/4	1
10 × 8 WF	3.84	8.88	0.855	0.744	0.615	1.047	0.851	0.754	7/8	1
10 × 10 WF	4.83	8.88	0.981	0.894	0.694	1.201	1.023	0.850	1	1 1/4
12 × 8 WF	3.85	10.91	0.901	0.805	0.672	1.103	0.922	0.824	7/8	1 1/4
12 × 10 WF	4.83	10.91	1.068	0.999	0.766	1.308	1.143	0.938	1 1/4	1 1/4
12 × 12 WF	5.81	10.91	1.192	1.158	0.844	1.460	1.325	1.034	1 1/4	1 1/2
14 × 8 WF	3.85	12.62	0.921	0.837	0.711	1.128	0.958	0.871	7/8	1 1/4
14 × 10 WF	4.81	12.62	1.108	1.056	0.814	1.357	1.209	0.997	1 1/4	1 1/2
14 × 12 WF	5.79	12.62	1.265	1.250	0.904	1.549	1.431	1.107	1 1/4	1 1/2
14 × 14.5 WF	7.04	12.62	1.411	1.453	0.999	1.728	1.663	1.224	1 1/2	1 3/4
14 × 16 WF	7.41	12.62	1.443	1.505	1.024	1.767	1.722	1.255	1 1/2	1 3/4

Eq. 10—Thickness determined by max. elastic bending stress = 36000 psi.
Eq. 9—Thickness determined by max. deflection = 0.01 in.
Eq. 7—Thickness determined by yield line theory with a factor of safety of 2.

NOTATION

a = Deflection of the bearing plate, inches
b = Clear distance from face of column web to edge of flange, inches
d = Clear distance between column flanges, inches
D = The internal dissipation of energy, in the yield line theory
E = Young's modulus of elasticity, ksi
F_b = Allowable bending stress in bearing plate, ksi
F_p = Allowable bearing pressure on the supporting material, ksi
F_y = Specified minimum yield point of the steel being used, ksi
M_p = Ultimate plastic bending moment on unit length of plate = $F_y Z_p$
n = The maximum overhanging cantilever span of th bearing plate, inches
t = Thickness of bearing plate, inches
W = The external work, in the yield line theory
Z_p = Plastic Section Modulus, in.^{3} per inch
λ = d/b
β = Tan ϕ
ϕ = Angle between yield line and the X-axis

REFERENCES

1. *American Institute of Steel Construction* Manual of Steel Co struction *Sixth Edition, 1963.*
2. *R. H. Wood* Plastic and Elastic Design of Slabs and Pla *The Ronald Press, 1961.*
3. *Raymond J. Roark* Formulas for Stress and Strain *McGra Hill Book Company, Inc., Third Edition, 1954, page 205.*

Errata

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Page 11

Due to a drafting error, the curves in Figs. 2 and 3 for a span length L = 150 ft are incorrect. The following data may be used to revise these curves:

R (ft)	Fig. 2 Δ_c/Δ_s		Fig. 2 M_c/M_s		Fig. 3 σ_W/σ_l	
	18WF114	36WF135	18WF114	36WF135	18WF114	36WF1
50	—	—	1.63	1.63	15.4	33.4
200	6.71	20.40	1.56	1.65	2.81	6.10
500	2.22	4.57	1.43	1.60	0.86	2.24
1000	1.42	2.24	1.23	1.48	0.19	0.84
1500	1.22	1.71	1.13	1.35	0.03	0.34
2000	1.12	1.45	1.08	1.25	−0.02	0.10