Straight-Element Grid Analysis of Horizontally Curved Beam Systems

HERBERT A. WEISSMAN

HORIZONTALLY CURVED beam systems undoubtedly find their greatest applicability today in the framing of curved highway bridges. Although this paper will utilize in its discussions the specific structural components of a steel *I*-shaped beam system with a concrete slab deck, the basic principles are applicable to other horizontally curved beam systems.

Much research[1] has gone into the theoretical and practical aspects of curved beam analysis. A method will be presented here which will enable a designer to secure sufficiently accurate results by using a method of analysis (more practically a computer method of analysis) with which he is familiar, namely the statically indeterminate analysis of plane grid systems with straight elements. STRESS[2] is probably the most popular computer program for the analysis of indeterminate structures, and reference will be made to a computer program which aids in the preparation of input data for STRESS.

A single horizontally curved beam which is not torsionally restrained at the ends is not stable under vertical loads. In curved bridge construction, diaphragms (steel beam or more commonly trussed members, normal to the main beams) provide the transverse load capability necessary for stability by resisting the torsion in the main beams. Because the deck slab frames only into the top flange of the beams, it cannot effectively resist torsional forces in the beam. However, it does provide good lateral load distribution capability by tending to equalize vertical deflections of adjacent beams.

In normal bridge construction (curve or straight) the concrete slab is made to act compositely with the main steel through the use of shear connectors. Because the modular ratio, η, between steel and concrete is assumed to be different[3] for long-duration loads than for short-duration loads, three different analyses must normally be employed:

1. Dead load of steel and concrete $\eta = \infty$
2. Superimposed dead load $3\eta = 30$
3. Live load $\eta = 10$

THE PLANE GRID SYSTEM

The horizontally curved highway bridge is not a true grid system. Such a system can consist only of discretely connected beam elements (straight or curved) lying in a plane and subject only to forces perpendicular to the plane and/or moments acting in the plane.

The points where the beams intersect are referred to as joints. Because the elastic properties of the beam elements can be determined from their cross-sections, length and curvature (if any), the elastic energy of the system and the final state of stress can be found by dealing only with the forces and deflections at the joints. The beam elements do not have to be prismatic as long as the manner of variation of the properties is known. The beam properties involved are the shear area (A_z), the twisting moment of inertia (I_x), and the bending moment of inertia (I_y).

Thus, in accordance with the definition of a grid, there are three loads (and three associated deflections—degrees of freedom) at each joint. Each member has three load components acting on it. Figure 1 shows the

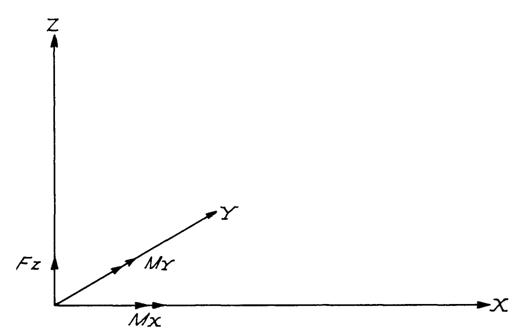

Fig. 1. Load components in a grid system

Herbert A. Weissman is the head of the Structural Department in the New York City Office of Goodkind & O'Dea, Consulting Engineers and Planners.

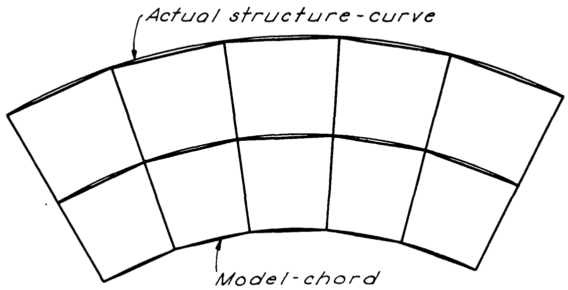

Fig. 2. Curved bridge—Model "A"

three joint load components, where the X-Y plane is the plane of grid. System coordinate axes will be indicated by capital letters. Member coordinate axes will be indicated by lower-case letters with the member x-axis coinciding with the longitudinal axis of the member and the member and system Z-axes being parallel.

Horizontally curved highway bridges deviate from a true grid system in the following ways:

1. Diaphragms most often consist of trussed (axially loaded) members framing into the flanges of the curved beams.
2. The I-beam cross-section warps under torsional loads, introducing additional degrees of freedom.
3. The concrete slab is a continuous member (not discrete) and its elastic line is offset from that of the beams.

Each of the above deviations from true grid action will require certain assumptions to be made in order to model

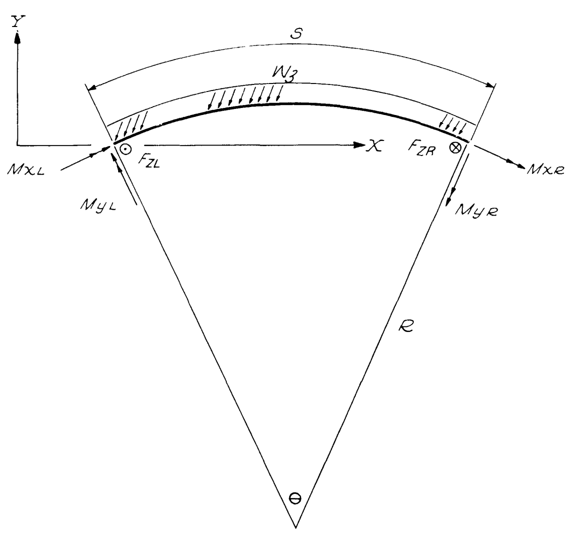

Fig. 3. Curved beam segment

the structure back into a grid. It should be noted that all grid analyses—including those using curved elements—must rationally account for these non-grid characteristics.

EFFECT OF USING STRAIGHT ELEMENTS

In order to use conventional indeterminate analysis (and conventional indeterminate analysis computer programs), it is necessary to replace curved sections of beams by straight lines. The number and arrangement of straight line segments will affect the degree to which the straight line structure simulates the curved structure. The maximum spacing[3] of diaphragms in straight bridges is normally in the order of 25 ft. Since diaphragms in curved bridges must be thought of as main members, necessary for the stability of the structure, they are usually spaced more closely than on straight bridges. A logical positioning of joints would appear to be at the intersection of diaphragms and beams as shown in Fig. 2 and designated as Model "A" (a simply supported single span of three stringers is shown for simplicity). This creates an angular discontinuity in the beam at its intersection with the diaphragm.

Consider first a portion of curved beam between diaphragms as indicated in Fig. 3. Assume that the system X-axis is parallel to the chord. Taking moments about the X-axis:

$$\Sigma w_z \times e + (M_{yL} + M_{yR}) \sin\frac{\theta}{2} - (M_{xL} + M_{xR}) \cos\frac{\theta}{2} = 0 \quad (1)$$

where e is the eccentricity of the load from the chord. For short segments the angle θ is small and $\cos \theta/2 = 1$, $\sin \theta/2 = \theta/2$. Equation (1) can be rewritten as

$$\Sigma w_z \times e + M_{y\text{av.}} \times \theta = \Sigma M_x \quad (2)$$

where

$$M_{y\text{av.}} = \tfrac{1}{2}(M_{yL} + M_{yR})$$

and

$$\Sigma M_x = M_{xL} + M_{xR}$$

Equation (2) indicates that between diaphragms the torsional moment—necessary to resist the components of the bending moment due to curvature (adjacent sections are not parallel) and the eccentricity due to curvature—builds up as the distance S between sections increases. The main beams are weak in torsion and the diaphragms, which can resist these forces by bending, act as elastic supports for the torsional loads acting along the length of the beam.

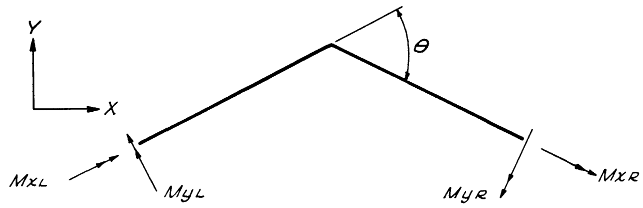

Fig. 4. Chord intersection point—without diaphragm

Consider now a portion of a curved beam which is modelled by a series of chords. Figure 4 shows the forces acting at an intersection point of the chords without any diaphragm (vertical forces are not shown). Without loss of generality, assume that the system X-axis bisects the angle between the chords. Taking moments about the X-axis:

$$(M_{yL} + M_{yR}) \sin \frac{\theta}{2} - (M_{xL} + M_{xR}) \cos \frac{\theta}{2} = 0 \quad (3)$$

For small θ, Eq. (3) can be rewritten as:

$$\tfrac{1}{2} (M_{yL} + M_{yR}) \, \theta = \Sigma M_x \quad (4)$$

where ΣM_x has the same definition as in Eq. (2). In most of the areas of the bridge, the bending moment is much larger than the torsional moment, with the result that M_{yL} and M_{yR} will be approximately equal. Equality of the bending moments gives a good modeling to the structure. The twisting moments will in general not be equal on both sides of the joint since they depend on the rotations of adjacent joints. This produces a discontinuity which does not exist in the curved structure. Values of the twisting moment at such a joint are not reliable and should not be used for design.

Figure 5 shows two possible "building blocks" of straight line segments which can be used to model a curved beam. Each building block corresponds to the arc length S used in Fig. 3 for a curved segment. The difference between Figs. 5a and 5b is that in 5a the angular discontinuity occurs at the boundary of adjacent segments; in 5b it occurs within the segment. Figure 5b would, therefore, seem to be a better "building block". This will become more evident when diaphragms are introduced into the structure.

Compare Eq. (4) for Fig. 5b with Eq. (2) for the curved segment. There is no term in Eq. (4) for the eccentricity of the load acting directly on the segment. The twisting moment in each arm of Fig. 5b will be constant unless a uniform (more correctly a uniformly varying) twisting moment is introduced along the member. It is interesting to note that if a segment of a curve is broken up into five equal chords, the maximum offset of the curve from the chord is about 1/25 the offset of the curve from a single chord joining the ends. If ten chords are used, the offset ratio becomes 1/100. Placing

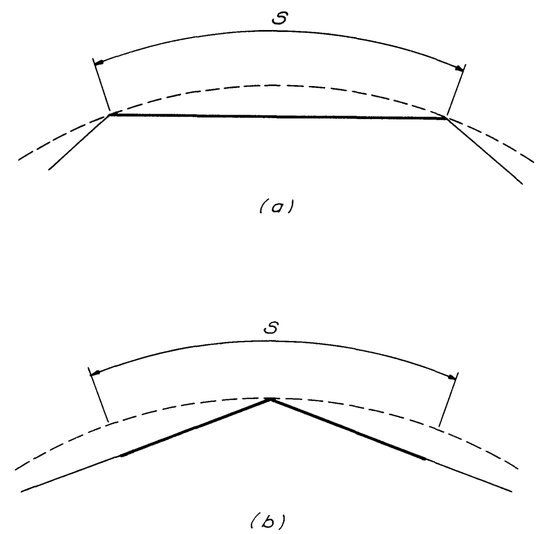

Fig. 5. Straight element "building blocks": (a) Element I, (b) Element II

a uniformly varying twisting moment on the straight members will more closely simulate curved beam action. This sort of accuracy is not normally warranted.

In Fig. 5b the torsional moment in the beam is introduced as a concentrated effect at the angular discontinuity. This torsion is a function of the longitudinal bending moment at the discontinuity as indicated in Eq. (4). In Eq. (2), it is a function of the average of the bending moments at the ends. Still, there is a good correlation between Eqs. (2) and (4).

Figure 6a shows a diaphragm framing into a portion of curved beam. Consider the forces acting at the joint as shown in Fig. 6b (vertical forces are not shown). Here

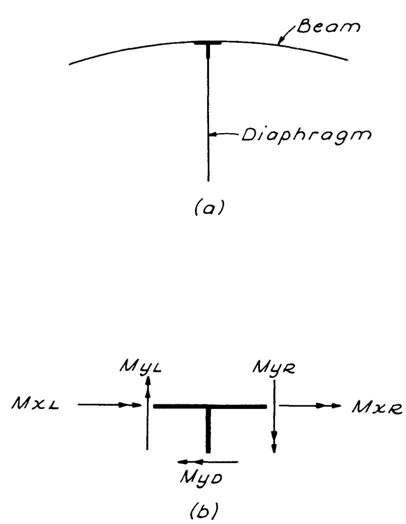

Fig. 6. Curved beam with diaphragm: (a) Plan, (b) forces at joint

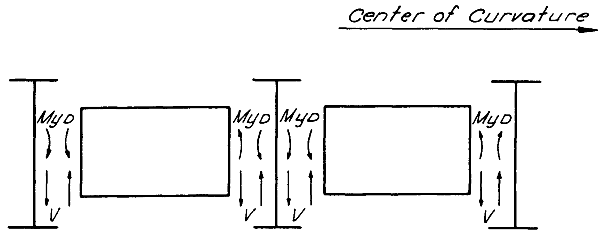

Fig. 7. Cross-section of curved beam system

the bending moment components are parallel and, as was previously noted, the diaphragm acts as an elastic support for the torsional stresses in the beam. Calling the bending moment in the diaphragm M_{yD}, and taking moments about the X-axis:

$$M_{xL} + M_{xR} - M_{yD} = 0 \qquad (5)$$

It is the bending moment in the diaphragms that stabilizes the individual curved beam. To keep the diaphragms in equilibrium the curved beams must supply upward and downward reactions to the diaphragm ends. It is these reactions which cause the vertical redistribution of

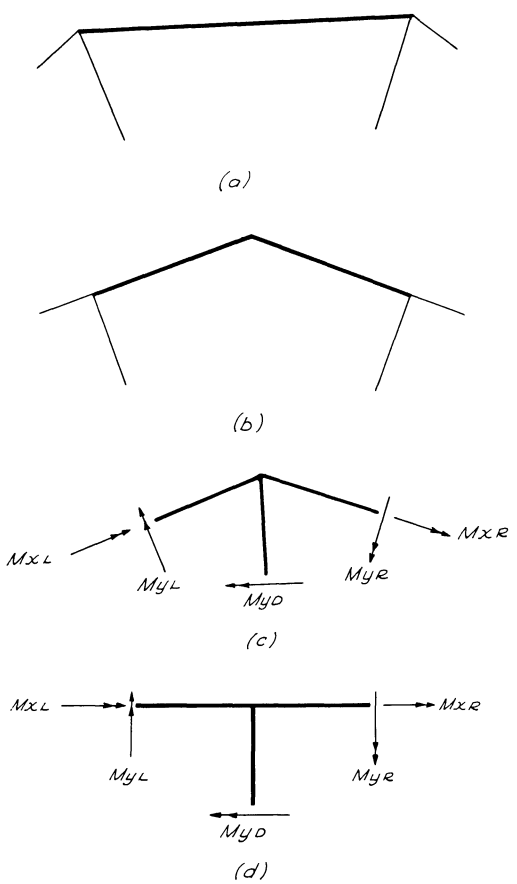

Fig. 8. Straight-element model of curved beam with diaphragm: (a) and (c) Using element I, (b) and (d) using element II

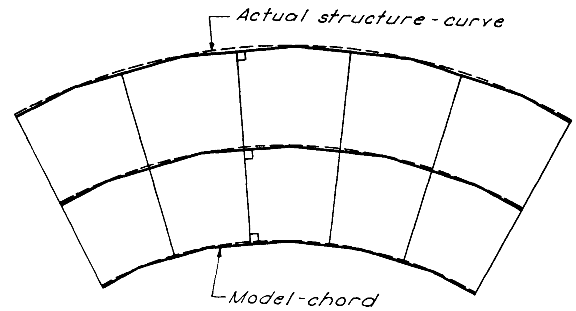

Fig. 9. Curved Bridge—Model "B"

loads which is common to curved beam systems (see Fig. 7).

Figure 8 shows the intersection of diaphragms with a curved beam modeled by the two "building blocks" and free body diaphragms at the diaphragm intersections, Figs. 8c and 8d (vertical forces again are not shown). Taking moments about the X-axis in Fig. 8c and using the same angle relationships as before:

$$\tfrac{1}{2}(M_{yL} + M_{yR})\theta + M_{yD} = \Sigma M_x \qquad (6)$$

In Fig. 8d, moments about the X-axis gives:

$$\Sigma M_x = M_{yD} \qquad (7)$$

Eq. (7) is identical to Eq. (5). Equation (4) remains satisfied for Fig. 8b. For Fig. 8a neither Eq. (4) nor Eq. (5) is satisfied. Clearly the use of Fig. 8b will more accurately model the behavior of curved beams than will the use of Fig. 8a.

Using Fig. 8b the three beam curved bridge structure is shown in Fig. 9 and is designated as Model "B". There are about twice as many joints in Model "B" as there are in Model "A". The chord lengths are about the same. If intermediate points are added on the curve between those of Model "A", a framing with the same number of points as Model "B" will result. This will be called Model "C" and is shown in Fig. 10. The chord lengths here are half those of Model "B"; the diaphragms still are not perpendicular to the beam as in Model "B". A computer program will compare Models "B" and "C" with a more rigorous analysis[4] made for a two span continuous structure with two beams.

THE NON-GRID ELEMENTS

Diaphragms—In order to incorporate a trussed diaphragm (such as the **X** shown in Fig. 11a) into the grid system, equivalent beam properties must be found for it. These can then be used in the computer program in the form of a member stiffness matrix.[2]

Figure 11b shows the position of the members and the forces acting after the left end (assumed to be framing into a non-deforming beam) is given a rotation θ. Let A_c be the area of the top and bottom members, A_d the area of the diagonals, and E the modulus of elasticity. The forces in the members are:

$$P = \frac{A_c E\theta \frac{d}{2}}{h} = \frac{A_c E\theta d}{2h}$$

$$Q = \frac{A_d E \frac{h}{l} \theta \frac{d}{2}}{l} = \frac{A_d E\theta hd}{2l^2}$$

$$H_L = P + \frac{h}{l} Q = \frac{A_c E\theta d}{2h} + \frac{A_d E\theta h^2 d}{2l^3}$$

$$M_L = H_L d$$

$$H_R = P - \frac{h}{l} Q$$

$$M_R = H_R d$$

$$V = 2 \frac{d}{l} Q = \frac{A_d E\theta hd^3}{l^3}$$

The values for a deflection Δ at the left end (Fig. 11c) are:

$$R = \frac{A_d E \frac{d}{l} \Delta}{l} = \frac{A_d E\Delta d}{l^2}$$

$$H_L = H_R = \frac{h}{l} R = \frac{A_d E\Delta dh}{l^3}$$

$$M_L = M_R = H_L d$$

$$V = 2 \frac{d}{l} R = \frac{2A_d E\Delta d^2}{l^3}$$

Note that the shear for a unit rotation is:

$$\frac{V}{G} = \frac{A_d Ehd^3}{l^3}$$

The moment for a unit deflection is the same:

$$\frac{M_L}{\Delta} = \frac{A_d Ehd^3}{l^3}$$

From these values the member stiffness matrix can be formulated. Other types of trussed diaphragms can be solved in a similar manner.

Warping of Cross-section—The torsional stiffness of an *I*-shaped section is greatly influenced by the amount (if any) of resistance to warping caused by either the supports or the nature of the loading. Any theoretical solution[5] to this problem is quite complex. It would certainly not be warranted in a straight-element grid analysis, from which only an accuracy necessary for design is required. However, if lower and upper bounds for the torsional stiffness could be obtained, two grid analyses could be made and design values could be obtained by comparing and evaluating the results. While

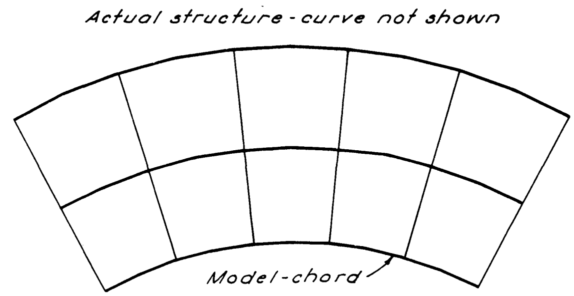

Fig. 10. Curved Bridge—Model "C"

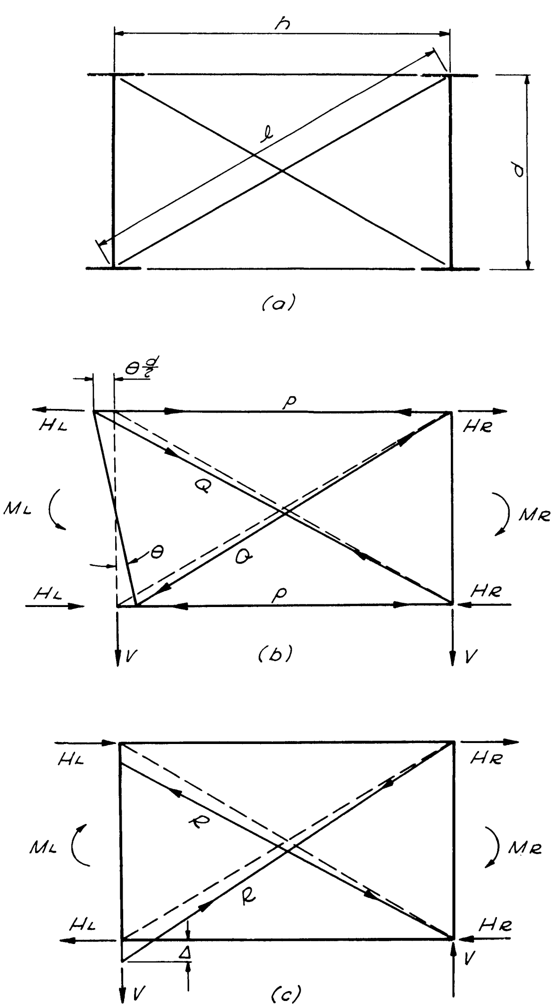

Fig. 11. X-diaphragm: (a) Dimensions, (b) end rotation, (c) end displacement

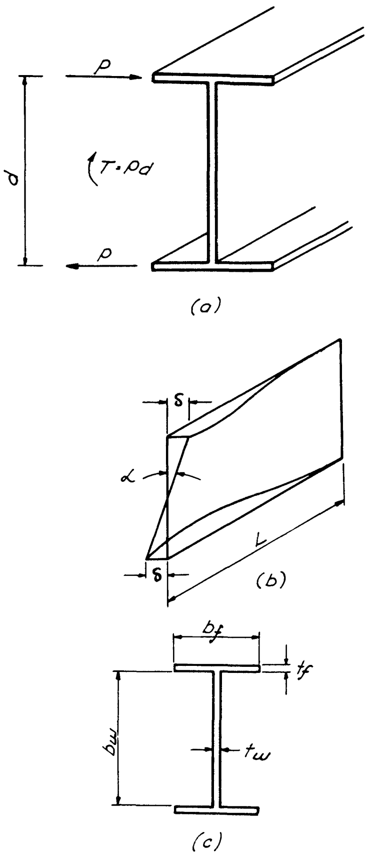

Fig. 12. Torsion in a restrained I-shaped beam: (a) loads, (b) deflections, (c) cross-section

not as desirable as an exact approach (if one could be obtained) such procedures—utilizing upper and lower bounds—are not uncommon in engineering design. Because even the upper bound of torsional stiffness is so much smaller than the bending stiffness of the diaphragms and the main beams, the two analyses will give similar results.[4]

Torsion in an *I*-section with fixed ends (fully restrained against torsion) can be resisted by bending of the flanges.[6] Figure 12a shows an *I*-section resisting a torsional load $T = Pd$. Figure 12b shows the horizontal deflection of the flanges assuming that there is no horizontal rotation at the ends. The deflection is

$$\delta = \frac{PL^3}{12EI_H}$$

where I_H is the horizontal moment of inertia of the flanges (assumed equal in this example). The angle of rotation is $\alpha = \delta/(d/2) = 2\delta/d$. The twisting moment of inertia is

$$I_x = \frac{TL}{G\alpha} = \frac{Pd^2L}{2G\delta} = \frac{6EI_Hd^2}{GL^2} \tag{8}$$

where G is the shear modulus of elasticity. If the flanges have unequal I_H, the center of rotation will be at the "center of gravity" of the two values of I_H. The twisting moment of inertia can then be figured in a similar manner.

If there is no restraint against warping, the resulting torsion is referred to as Saint Venant torsion[6] and the twisting moment of inertia (for an *I*-section) is approximately

$$I_x = \sum \frac{1}{3} bt^3 \tag{9}$$

where t is the thickness of plate and b is the width as shown in Fig. 12c.

The I_x based on Saint Venant torsion is a good lower bound and the sum of this value and (the much larger) value given by Eq. (8) is a good upper bound for use in the grid analyses.

Concrete Slab—The action of the concrete slab poses two problems in trying to fit it into a grid: its continuity and the fact that it is attached only to the top flange of the main beams. If no temporary supports for the beams are used, the slab is effective only for the super-imposed dead load and the live load. Thus its action is only to be incorporated for the analyses at $3\eta = 30$ and $\eta = 10$.

Since it frames only into the top flange of the beam the slab will not offer the transverse resistance to twisting of the beam which the diaphragms do, but it will increase the torsional moment of inertia, I_x, of individual beams. An effective slab width, similar to that used in figuring composite bending properties, could be used in figuring composite torsional properties. The slab transverse bending strength must be represented by a beam element in order to fit into the grid analysis. The stiffness to resist differential deflections of adjacent beams must involve a stiffness to resist the bending rotation of the transverse members. The rotation of transverse members is the same degree of freedom as the twisting of the main beams. Thus a transverse member with bending stiffness must resist torsion in the main beams. Therefore, if the beam element is assumed to frame into the main beams between diaphragms, it will offer a resistance to the twisting of the main beams which actually does not exist. By placing the beam representing the slab at the diaphragms results in there being no transverse resistance to torsion between diaphragms. It will, however, increase the resistance over that which actually exists. Since the bending stiffness of the diaphragms alone is so much larger than the twisting stiffness of the main beams, the resulting force distribution will not be greatly in error.

The question arises as to what value of I_y and I_x should be assigned to a concrete slab about 8 in. thick and about 20 ft wide, where the transverse spacing of beams is about 8 ft. Experimental evidence* exists concerning the action of a concrete slab on straight stringers. If such a structure were analyzed as a grid using a diaphragm spacing of about 20 ft, an effective transverse slab width in bending could be approximated. Using one-half the effective width of the slab will give good results. The torsion moment of inertia should be taken equal[7] to the bending moment of inertia. The properties of the steel diaphragm and the concrete slab would be added together (taking into consideration the different moduli of elasticity) even though their elastic lines are offset.

Slab continuity also poses a problem regarding the distribution of the applied loads. Most of the loads are not applied directly over the modeled grid structure. Factors used in the design[3] of the slab could be used in determining the distribution of loads from the slab to the grid elements. These factors involve an effective width for concentrated loads and continuity in the slab under certain conditions.

COMPUTER PROGRAM FOR STRESS INPUT

Because there are a relatively large number of joints and members in the straight-element grid analysis of curved beams, a large part of the input deck for STRESS will consist of data describing the joint coordinates and the member incidences.[2] Calculating the joint coordinates is a big task; coding them for key punching and punching the cards makes it even bigger. However, a computer program can compute all the joint coordinates and the member incidences and can punch the results out on cards for use directly in the STRESS program. The author has written such a program for radial supports.

Input for the program consists of the number of supports and their stations (along a line of given radius), the number of stringers and their offsets from the station line, the number of diaphragm spaces in each span, and the number of intermediate points between diaphragms. Joints are numbered consecutively along the stringers and then along the diaphragms (on radial lines). The program places an *S* after support joints as required by STRESS. Joint releases must be prescribed at supports to account for actual conditions of articulation.

COMPARISON OF RESULTS AND CONCLUSIONS

Figure 13 shows the two beam, two span continuous structure which will be analyzed using Models "B" and

* *Reference 3 lists some studies used to determine lateral distribution of wheel loads.*

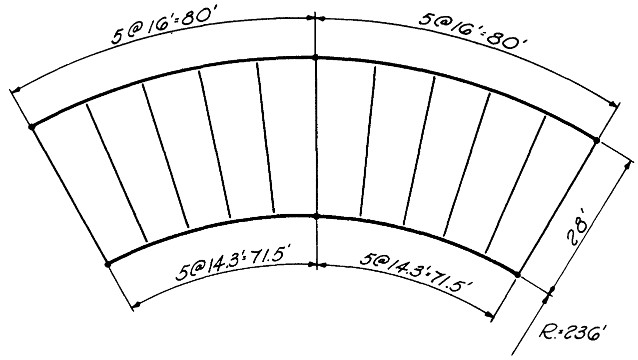

Fig. 13. Two beam, two span continuous curved structure

Table 1. Loading 1
(1 kip/foot uniform load on each girder)
Outside Girder Forces

Point	Force	Analysis		
		Model B	Model C	Rigorous
0.0L	V_c	—	—	—
	V_i	31.2	31.2	31.5
	M	.0	.0	.0
	T_o	—	—	—
	T_i	9.4	6.4	7.7
0.1L	V_o	15.3	15.4	15.4
	V_i	14.2	14.2	14.3
	M	369.	371.	375.
	T_o	−3.6	−0.1	−5.0
	T_i	14.8	8.1	14.5
0.2L	V_o	−1.7	−1.7	−1.7
	V_i	−3.1	−3.1	−3.2
	M	468.	470.	476.
	T_o	−12.3	−5.5	−12.7
	T_i	11.2	5.0	11.9
0.3L	V_o	−19.0	−19.0	−19.2
	V_i	−19.8	−19.9	−20.1
	M	292.	294.	297.
	T_o	−13.7	−7.5	−12.8
	T_i	0.1	−1.5	1.8
0.4L	V_o	−35.7	−35.8	−36.1
	V_i	−35.1	−35.3	−35.5
	M	−149.	−149.	−152.
	T_o	−6.1	−4.6	−3.9
	T_i	−15.7	−9.4	−13.4
0.5L	V_o	−51.0	−51.1	−51.5
	V_i	−51.0	−51.1	51.5
	M	−834.	−836.	−848.
	T_o	12.0	4.5	15.7
	T_i	−12.0	−4.5	−15.7

Table 2. Loading 1
(1 kip/foot uniform load on each girder)
Inside Girder Forces

Point	Force	Analysis		
		Model B	Model C	Rigorous
0.0L	V_o	—	—	—
	V_i	24.9	24.9	25.0
	M	.0	.0	.0
	T_o	—	—	—
	T_i	7.5	5.3	6.3
0.1L	V_a	10.7	10.7	10.7
	V_i	11.8	11.8	11.9
	M	254.	252.	256.
	T_o	−1.6	0.7	−2.5
	T_i	10.8	6.0	10.4
0.2L	V_o	−2.4	−2.4	−2.4
	V_i	−1.0	−1.0	−1.0
	M	321.	319.	324.
	T_o	−8.0	−3.4	−8.2
	T_i	7.7	3.4	8.1
0.3L	V_o	−15.2	−15.1	−15.3
	V_i	−14.4	−14.2	−14.4
	M	206.	205.	208.
	T_o	−9.7	−5.3	−9.0
	T_i	−0.1	−1.3	1.0
0.4L	V_o	−28.5	−28.4	−28.7
	V_i	−29.1	−29.0	−29.2
	M	−99.	−98.	−100.
	T_o	−4.9	−3.7	−3.3
	T_i	−11.5	−6.8	−9.6
0.5L	V_o	−43.3	−43.2	−43.6
	V_i	43.3	43.2	43.6
	M	−614.	−611.	−621.
	T_o	8.4	3.1	11.2
	T_i	−8.4	−3.1	−11.2

Table 3. Loading 2
(1 kip/foot uniform load on outside girder only)
Outside Girder Forces

Point	Force	Analysis		
		Model B	Model C	Rigorous
0.0L	V_o	—	—	—
	V_i	30.0	30.1	30.2
	M	.0	.0	.0
	T_o	—	—	—
	T_i	13.0	10.1	11.7
0.1L	V_o	14.1	14.2	14.2
	V_i	13.7	13.7	13.8
	M	351.	351.	356.
	T_o	0.6	3.9	−0.4
	T_i	16.0	9.5	15.7
0.2L	V_o	−2.2	−2.2	−2.2
	V_i	−2.7	−2.7	−2.7
	M	443.	444.	449.
	T_o	−9.9	−3.4	−10.0
	T_i	9.3	3.4	9.9
0.3L	V_o	−18.6	−18.6	−18.7
	V_i	−18.8	−18.8	−19.0
	M	274.	275.	278.
	T_o	−14.3	−8.4	−13.5
	T_i	−3.5	−4.9	−2.1
0.4L	V_o	−34.7	−34.7	−35.0
	V_i	−34.4	−34.5	−34.7
	M	−151.	−151.	−154.
	T_o	−9.1	−7.7	−7.1
	T_i	−17.6	−11.6	−15.7
0.5L	V_o	−50.3	−50.3	−50.7
	V_i	50.3	50.3	50.7
	M	−825.	−825.	−837.
	T_o	9.8	2.1	13.1
	T_i	−9.8	−2.1	−13.1

"C" and then compared with a more rigorous curved girder analysis.* For the comparison in Model "B," points on and outside the curve were used (they could also have been on and inside the curve or both outside and inside). For the beams $I_x = 594$ in.^{4}, $I_y = 12{,}290$ in.^{4}; for the diaphragms $I_x = 2.3$ in.^{4}, $I_y = 1500$ in.^{4} Shear deformations were not included. Loading 1 consists of a uniform load of 1 kip/ft on both girders; loading 2 consists of a load of 1 kip/ft on the outside girder, no load on the inside girder. Tables 1 through 4 show the results for loadings 1 and 2, outside girder and inside girder. V_o and V_i, T_o and T_i, are the shear and twisting moment at a section at the outside (away from middle support) and inside (toward middle support). M is the bending moment at the section. Since the results are symmetrical with respect to the middle support, only one-half the structure is shown.

The results for shear and bending moment for Models "B" and "C" are very close to those of the rigorous analysis. The torsional values for both models are not in as close an agreement with the rigorous analysis as are the values for bending moment and shear. However, the values for Model "B," using Fig. 9, are in substantially better agreement than those of Model "C," using Fig. 10. Although it is risky to generalize from one example (one that uses a symmetrical structure with uniform, symmetrical load-

* *The values (and the sign convention) for the vigorous analysis are taken from the computations made by Richardson, Gordon and Associates; these computations are the basis for the results shown in Reference 4.*

Table 4. Loading 2 (1 kip/foot uniform load on outside girder only) Inside Girder Forces

Point	Force	Analysis		
		Model B	Model C	Rigorous
0.0L	V_o	—	—	—
	V_i	−0.9	−1.0	−0.9
	M	.0	.0	.0
	T_o	—	—	—
	T_i	6.3	6.1	6.7
0.1L	V_o	−0.9	−1.0	−0.9
	V_i	−0.5	−0.5	−0.5
	M	−12.	−13.	−13.
	T_o	6.7	6.3	7.1
	T_i	2.6	2.7	2.8
0.2L	V_o	−0.5	−0.5	−0.5
	V_i	.0	.0	.0
	M	−19.	−20.	−20.
	T_o	3.5	3.2	3.8
	T_i	−2.1	−1.7	−2.2
0.3L	V_o	.0	.0	.0
	V_i	0.2	0.3	0.3
	M	−19.	−20.	−20.
	T_o	−0.9	−1.1	−1.0
	T_i	−4.6	−4.2	−4.9
0.4L	V_o	0.2	0.3	0.3
	V_i	.0	.0	.0
	M	−16.	−16.	−17.
	T_o	−3.6	−3.6	−3.7
	T_i	−3.1	−2.4	−3.0
0.5L	V_o	.0	.0	.0
	V_i	.0	.0	.0
	M	−17.	−16.	−17.
	T_o	2.1	−1.9	−1.9
	T_i	2.1	1.9	1.9

ing), it is felt that Model "B" can be used to adequately simulate a horizontally curved beam system.

REFERENCES

1. *McManus, P. F., Nasir, G. A., and Culver, C. G.* Horizontally Curved Girders—State of the Art, *Proc. Am. Soc. Civil Engs., May, 1969.*
2. *Fenves, S. J., Logcher, R. D., Mauch, S. P., and Reinschmidt, K. F.* STRESS: A User's Manual, *M.I.T. Press, Cambridge, Mass. 1964.*
3. Standard Specifications For Highway Bridges, American Association of State Highway Officials, 1969.
4. Highway Structures Design Handbook, United States Steel Corporation, 1965.
5. *McManus, P. F., and Culver, C. G.* Nonuniform Torsion of Composite Beams, *Proc. Am. Soc. Civil Engs., June, 1969.*
6. *Timoshenko, S.* Strength of Materials, *D. Van Nostrand Co. Inc., 3rd Edition.*
7. *Yettram, A. L., and Husain, H. M.* The Representation of a Plate in Flexure by a Grid of Orthogonally Connected Beams, *Int. J. Mech. Sci., Vol. 7, 1965.*

MANUAL OF STEEL CONSTRUCTION—7th Edition

Preparation of the 7th Edition of the AISC Manual of Steel Construction is nearly complete, and copies are expected to be available in July, 1970.

The new edition is being extensively revised and expanded to keep pace with the many new developments in steel construction since the 6th Edition was published in 1963.

A brochure describing the new Manual, including an order form, will be mailed to readers of the Engineering Journal within the next few weeks. We suggest you complete the order form promptly to assure delivery of your copy as early as possible after publication.