Preliminary Design of Curved Bridges

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IN ORDER to utilize several computer programs[1] for the analysis of radial curved bridge systems of open cross-section, it is necessary to evaluate section properties. This paper presents and explains a simplified method for evaluating the internal forces in radial curved girder systems; the equations and factors presented permit the designer to determine required cross-sectional properties.

METHOD OF DEVELOPMENT

The relationship between a system of curved girders and a single curved girder can be determined by utilizing a Slope Deflection technique[2] for the evaluation of the girder system and by using beam theory[3] to evaluate the behavior of the single girder. In addition, the relationship between a single curved girder and a straight girder can also be formulated.[4] Therefore, a series of factors can be developed, relative to the internal forces and deformations, comparing the single straight:single curved:curved system. This type of formulation has been pursued in developing the equations that follow. The relationships between the single straight and single curved girders were investigated to examine the influence of the length and radii. The relative stiffnesses of all girders were obtained by utilizing those sections recommended in the Bureau of Public Roads *Standard Plans for Highway Bridges*.[5]

Table 1 describes the bridge systems that were studied. For each system, the relationship between a simple span single straight: single curved: curved system was determined. Reduction factors were developed to make the data applicable to continuous spans.

AASHO HS20-44 truck loadings were used throughout. Two trucks, positioned as shown in Fig. 1, were used in evaluating the forces and deformations in the single girder studies (straight and curved). The load to girder 1, the main girder to be analyzed, was evaluated by considering a simple distribution between girders as shown in Fig. 1a. The total load on girder 1, equal to P_{eq}, was then used to evaluate the maximum simple span bending moment (Fig. 1b). The same position and magnitude of this single line of wheel loads was used for both the single curved and straight girder analysis. In analyzing the single span girder *systems* the following number of trucks were used:

1. Four Girder System: Two Lanes—Two Trucks
2. Six Girder System: Three Lanes—Three Trucks
3. Eight Girder System: Four Lanes—Four Trucks

These trucks were located to give maximum bending and torsional moments. Partial span loadings, alternate span loadings etc., were also examined for the continuous spans.

EQUATIONS

Amplification Factor, K_1—All the internal forces and deformations for a single curved and straight girder were evaluated, using various computer programs[3,4],

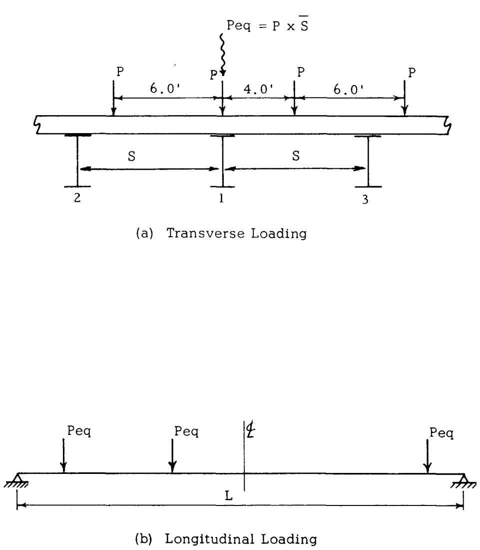

Fig. 1. Loading condition

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Table 1. Bridge System Characteristics

Span Length L (ft)	Girder Spacing S (ft)	Member Size	Cover Plate Size (in.)	Radius R (ft)	Number of Girders	Deck Composite Concrete (in.)
				(100 interval)		
50	7	30WF99	$9 \times \frac{3}{4}$	100–600	4, 6, 8	7
60	7	33WF118	$10 \times \frac{15}{16}$	100–600	4, 6, 8	7
70	7	36WF150	$10 \times 1\frac{1}{8}$	100–600	4, 6, 8	7
80	7	36WF194	$10 \times 1\frac{3}{4}$	100–600	4, 6, 8	7
90	7	36WF245	$15 \times 1\frac{1}{8}$	100–600	4, 6, 8	7
100	7	36WF280	$15 \times 1\frac{3}{8}$	100–600	4, 6, 8	7

and the data described in Table 1. The ratio of the reactions for these two girders is:

$$K_1 = \frac{f_{sc}}{f_{ss}} \qquad (1)$$

This factor describes the immediate effect of curvature relative to a straight member. Analysis of a plot of K_1 vs. L and R, the mid-length and corresponding radius of the system, respectively, gives the following general equations:

$$K_{moment} = \frac{0.15}{n} (L/R) + 1 \qquad (2)$$

$$K_{St.\ V.\ Torsion} = (4000n)(L/R)^2 - 600\,(L/R) \qquad (3)$$

$$K_{warping\ torsion} = (50n)(L/R)^2 - 3\,(L/R) \qquad (4)$$

$$K_{bimoment} = [(35n)(L/R)^2 - 15\,(L/R)] \times 10^3 \qquad (5)$$

$$K_{shear} = 1.0 \qquad (6)$$

$$K_{rotation} = [n^2 (2L/R)^3 - (n - 0.4)(L/R)^2] \times 10^{-2} \qquad (7)$$

$$K_{deflection} = S \cdot e^{P(L/R)} \qquad (8)$$

where:

S and P are evaluated using Fig. 2
$n = R/100$ for $R \geq 100'$
R = Radius (ft.)
L = Span length (ft.)

Distribution Factor, K_2—The evaluation of the true distribution of load to each girder, and thus the realistic values of internal forces, can be considered by analyzing the curved girders as a system. The number of trucks used in the analysis would be dependent on the number of girders as described previously. The ratio of these resulting maximum forces to those in a single curved girder gives:

$$K_2 = \frac{f_{cs_y}}{f_{sc}} \qquad (9)$$

Plotting this ratio vs. L and R leads to general Equations (10) through (30). In all instances the parameters L and R refer to the mid-length and corresponding radius of the system, respectively. These equations have been evaluated for the four, six and eight girder systems.

Four Girder System

$$K_{moment} = (n + 2)\left(\frac{0.4L}{R}\right) + 0.48 \qquad (10)$$

$$K_{St.\ V.\ Torsion} = \frac{0.085}{M} (R/L) \text{ for } R/L \geq 3.3 \qquad (11a)$$

$$= \frac{0.64}{n + 1} \text{ for } R/L \leq 3.3 \qquad (11b)$$

$$K_{warping\ torsion} = \frac{0.35}{M^2} (R/L) + 0.3 \qquad (12)$$

$$K_{bimoment} = \frac{0.11}{M} (R/L) \text{ for } L \leq 70 \text{ ft} \qquad (13a)$$

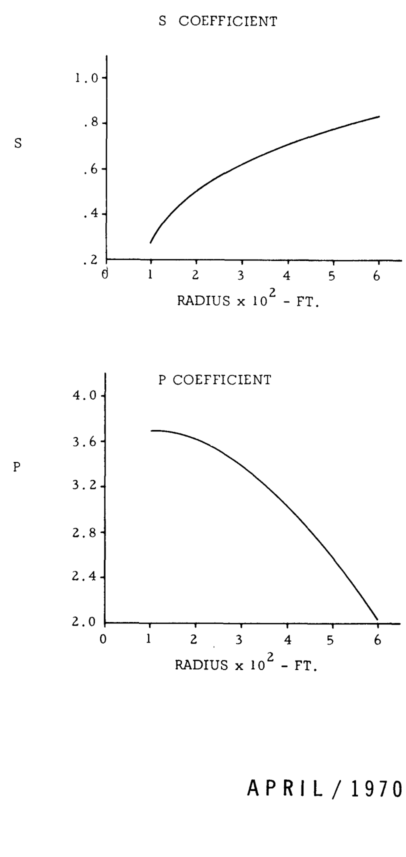

Fig. 2. P and S coefficients for Amplification Factor K_1 deflection

$$= \frac{(M-1)}{6}(R/L) \text{ for } L \geq 70 \text{ ft} \quad (13b)$$

$$K_{shear} = (n+2)\left(\frac{0.4L}{R}\right) + 0.55 \quad (14)$$

$$K_{rotation} = \frac{0.098}{M}(R/L) \text{ for } R/L \geq 3.3 \quad (15a)$$

$$= \frac{0.74}{n+1} \text{ for } R/L \leq 3.3 \quad (15b)$$

$$K_{deflection} = -YLn\,(L/R) + X \quad (16)$$

Six Girder System

$$K_{moment} = (n+3)\left(\frac{0.4L}{R}\right) + 0.6 \quad (17)$$

$$K_{St.\ V.\ Torsion} = \frac{0.105}{M}(R/L) \text{ for } R/L \geq 3.3 \quad (18a)$$

$$= \frac{0.68}{M+1} \text{ for } R/L \leq 3.3 \quad (18b)$$

$$K_{warping\ torsion} = \frac{0.15}{M^2}(R/L) + 0.1 \quad (19)$$

$$K_{bimoment} = \frac{0.11}{M}(R/L) \text{ for } L \leq 70 \text{ ft} \quad (20a)$$

$$= (M-1)/6\,(R/L) \text{ for } L \geq 70 \text{ ft} \quad (20b)$$

$$K_{shear} = (n+2.5)\left(\frac{0.4L}{R}\right) + 0.65 \quad (21)$$

$$K_{rotation} = \frac{0.11}{M}(R/L) \text{ for } R/L \geq 3.4 \quad (22a)$$

$$= \frac{0.84}{n+1} \text{ for } R/L \leq 3.4 \quad (22b)$$

$$K_{deflection} = -YLn\,(L/R) + X \quad (23)$$

Eight Girder System

$$K_{moment} = \left(\frac{n}{2}+3\right)\left(\frac{0.4L}{R}\right) + 0.54 \quad (24)$$

$$K_{St.\ V.\ Torsion} = \frac{0.115}{M}(R/L) \text{ for } R/L \geq 3.3 \quad (25a)$$

$$= \frac{0.72}{n+1} \text{ for } R/L \leq 3.3 \quad (25b)$$

$$K_{warping\ torsion} = \frac{0.2}{M^2}(R/L) + 0.2 \quad (26)$$

$$K_{bimoment} = \frac{0.11}{M}(R/L) \text{ for } L \leq 70 \text{ ft} \quad (27a)$$

$$= \frac{(M-1)}{6}(R/L) \text{ for } L \geq 70 \text{ ft} \quad (27b)$$

$$K_{shear} = (n+3)(0.3\,L/R) + 0.6 \quad (28)$$

$$K_{rotation} = \frac{0.116}{M}(R/L) \text{ for } R/L \geq 3.2 \quad (29a)$$

$$= \frac{0.86}{n+1} \text{ for } R/L \leq 3.2 \quad (29b)$$

$$K_{deflection} = -YLn\,(L/R) + X \quad (30)$$

where

Y and X are evaluated using Fig. 3
n = $R/100$, where $R \geq 100$ ft
M = $L/50$, where $L \geq 50$ ft
R = Radius, ft
L = Span length, ft

Reduction Factor, K_3—Because many bridge structures are continuous, it is desirable to obtain a factor which can be applied to the simple span data to obtain values of preliminary forces in continuous spans. This factor can be written as:

$$K_3 = \frac{f_{csy}}{f_{sc}} \cdot N \quad (31)$$

where N = Number of spans (2 or 3).

Utilizing a computer program,[1] the maximum forces in a two or three span curved bridge system of four,

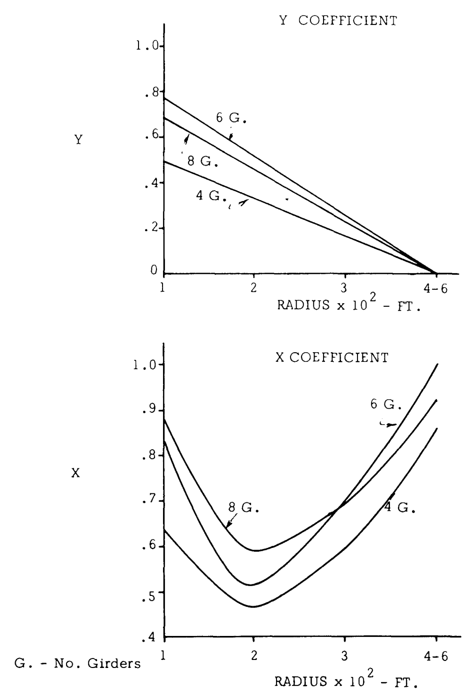

Fig. 3. X and Y coefficients for Distribution Factor K_2 deflection

Table 2. K_3 Reduction Factor for Maximum Function in Two and Three Span Bridges

No. of Spans	$K_{Bending\ Moment}$	$K_{Deflection}$	$K_{Rotation}$	$K_{St.\ V.\ Torsion}$	$K_{Warping\ Torsion}$	$K_{Bimoment}$	K_{Shear}
Two Span	.75	.70	.70	.75	.45	.35	1.00
Three Span	.65	.60	.70	.75	.40	.35	.90

six, and eight girders were evaluated under various critical loadings. Table 2 lists the resulting K_3 values. The factors are independent of the number of girders involved. It should be emphasized that the tabular values apply only to two and three span girder systems which have equal span lengths not exceeding 100.0 ft per span. For example, for a three-span system the total maximum bridge length would then be $3L = 300.0$ ft.

EVALUATION OF GIRDER FORCES AND DEFORMATION

With the various factor equations available, it is now possible to evaluate preliminary forces in a curved girder bridge, relative to the forces in a straight girder. The procedure is as follows:

1. Evaluate maximum function "$F_{straight\ beam}$" for a single straight girder of length L and load P_{eq} (Fig. 1). These functions would be $F_{bending}$, F_{shear}, and $F_{deflection}$. The remaining functions $F_{rotation}$, $F_{St.\ V.\ torsion}$, $F_{warping}$ and $F_{bimoment}$, are all assumed to be equal to one.
2. Evaluate the Amplification Factors, K_1 Equations (2) through (8), for the mid span length L and radius R of the bridge system.
3. Evaluate the Distribution Factors, K_2, Equations (10) through (30), for the given mid span length L, number of girders in system, and radius R.
4. Select the Reduction Factor, K_3, from Table 2, if system is a continuous span.
5. Determine the maximum function, F, of the curved girder system, i.e.,

$$F_{bending\ curved} = F_{bending} \times (K_1 \times K_2 \times K_3)$$

or, in general,

$$F_{curved}\ (\text{max.}) = F_{straight}\ (\text{max.}) \times (K_1 \times K_2 \times K_3) \quad (32)$$

In addition, to evaluate the maximum forces and deflections in the system, it is desirable to determine these maximum forces in all girders so the members may be properly proportioned. Therefore a set of tables have been developed, giving a factor which predicts the percent reduction in the maximum function to yield the respective value in that particular girder. Tables 3 through 8 provide data for the 4, 6, and 8 girder systems, for radii of 100, 200, 300, 400, 500, and 600 ft and span lengths of 50 and 100 ft. Girders are located in numerical order with girder 1 on the shortest radius. Tables for span lengths of 60, 70, 80, and 90 ft are described in Reference 4.

A modification of the basic equations is necessary depending on the transverse spacing of girders. Therefore, the Spacing Factor, S, has been evaluated:

$$\bar{S} = 1.29 \text{ for } S = 7 \text{ ft or } 8 \text{ ft}$$

$$\bar{S} = 1.57 \text{ for } S = 9 \text{ ft or } 10 \text{ ft}$$

The spacing factor is then applied to the wheel load P (4.0 kips or 16.0 kips) to evaluate P_{eq}.

BRIDGE EXAMPLE

The selection of the girder stiffnesses in this study was based on the Bureau of Public Roads *Standard Plans*,[5] issued in 1962. These plans were governed by the 1961 AASHO Code, which permitted a maximum stress of 18.0 ksi for the now obsolete ASTM A7 steel. Under the present (1969) specifications, the allowable stress is 20.0 ksi for ASTM A36. Therefore, the equations will provide design forces which require a stiffer section than is presently required. It is desirable to conduct a computer analysis for further refinement.

To illustrate the application of the design equations, the following bridges are to be designed:

1. Single Span, 4 girders @ 8.0 ft
   $L_{max} = 100.0$ ft, $R_{max} = 224.0$ ft
2. Two Span Continuous, 4 girders @ 8.0 ft
   $L_{max} = 200.0$ ft ($L_{min} = 100.0$ ft)
   $R_{max} = 224.0$ ft

From this preliminary data, the basic ratios L/R, n and M are computed as follows:

$$n = \frac{R_{centerline}}{100} = \frac{212}{100} = 2.12 \qquad \bar{S} = 1.29$$

$$M = \frac{L_{centerline}}{50} = \frac{95}{50} = 1.9$$

$$(L/R)_{centerline} = \frac{95}{212} = 0.448$$

The evaluation of the Amplification Factors K_1, Equations (2) through (8), and the Distribution Factors

K_2, Equations (10) through (16) are computed as:

	Amplification Factors K_1	Distribution Factors K_2
K_{moment}	1.032	1.22
$K_{warping\ torsion}$	20.68	0.34
$K_{St.\ V.\ torsion}$	1434.	0.205
$K_{bimoment}$	8178.	0.353
K_{shear}	1.0	1.288
$K_{rotation}$	2.88×10^{-2}	0.237
$K_{deflection}$	2.574	0.731

The above factors can now be applied to the maximum forces in a single girder of a straight girder bridge, assuming a loading described in Figure 1. These maximum forces and deflections are:

$$M_{b\ straight\ beam} = (717.05 \times 12) \times 1.29 = 11100 \text{ kip-in.}$$

$$V_{straight\ beam} = 24.3 \text{ kips}$$

$$\text{Deflection } \Delta = .850 \text{ in.}$$

Rotation ϕ = 1.0
St. Venant torsion = 1.0
Warping torsion = 1.0
Bimoment = 1.0

Applying the general Equation (32) with $K_3 = 1.0$, the forces and deflections in a simply supported single span four curved girder system are:

Single Span:

$$M_b = 1.032 \times 1.22 \times 11100 = 13975.0 \text{ kip-in.}$$

$$T_{warping} = 20.68 \times 0.34 \times 1.0 = 7.05 \text{ kip-in.}$$

$$T_{St.\ Venant} = 1434 \times 0.205 \times 1.0 = 294.0 \text{ kip-in.}$$

$$T_{bimoment} = 8178 \times 0.353 \times 1.0 = 2880.0 \text{ kip-in.}$$

$$V = 1.0 \times 1.288 \times 24.3 = 31.3 \text{ kips}$$

$$\phi = 0.288 \times 10^{-2} \times 0.237 \times 1.0 = 0.68 \times 10^{-2} \text{ radians}$$

$$\Delta = 2.574 \times 0.731 \times 0.85 = 1.60 \text{ in.}$$

Utilizing the data above, the forces in the two span continuous system can be obtained by applying the reduction factor K_3, listed in Table 2. The resulting forces are:

Table 3. Modification Factors for Various Girders in the System—Four Girder System
$L = 50$

	Girder	Moment	Deflection	Rotation	Bimoment	St. V. Torsion	Warping Torsion	Shear
$R = 100$	1	35	30	85	100	100	100	45
	2	70	55	65	55	65	40	80
	3	85	75	75	45	65	20	90
	4	100	100	100	70	90	55	100
$R = 200$	1	50	50	100	100	100	100	55
	2	80	70	60	50	60	40	85
	3	90	85	60	35	50	10	95
	4	100	100	100	50	70	60	100
$R = 300$	1	65	60	100	100	100	100	65
	2	90	90	55	45	50	35	95
	3	100	100	20	10	15	5	100
	4	100	100	60	40	15	60	95
$R = 400$	1	70	65	100	100	100	100	65
	2	90	90	55	45	50	35	95
	3	100	100	15	10	10	10	100
	4	100	100	45	50	25	65	100
$R = 500$	1	70	70	100	100	100	100	70
	2	95	95	50	40	50	35	95
	3	100	100	10	5	5	5	100
	4	95	100	25	55	35	65	90
$R = 600$	1	75	70	100	100	100	100	75
	2	95	95	45	35	45	35	95
	3	100	100	5	5	5	5	100
	4	100	100	20	65	40	70	100

Two Span:

M_b = 13957 × 0.75 = 10450.0 kip-in.
$T_{warping}$ = 7.05 × 0.45 = 3.18 kip-in.
$T_{St.\ Venant}$ = 294 × 0.75 = 221.0 kip-in.
$T_{bimoment}$ = 2880 × 0.35 = 1010.0 kip-in.
V = 31.3 × 1.0 = 31.3 kip-in.
ϕ = $0.68 \times 10^{-2} \times 0.70 = 0.476 \times 10^{-2}$ radians
Δ = 1.60 × 0.70 = 1.12 in.

All of the forces described above refer to a single isolated girder. The forces can now be used to estimate a given cross section.

The basic equations to be applied in evaluating normal stresses only, as described in References (6) and (9), are:

$$\sigma_b = \frac{M_b C}{I} \tag{33}$$

$$\sigma_w = \frac{T_{bimoment} \times W_{ns}}{I_w} \tag{34}$$

The resulting shearing stresses can be found by applying expression listed in the Notations (Appendix A). Note that Equations (33) and (34) are dependent on section properties and forces. The forces have been evaluated, but the percentage of stress induced by these forces cannot be directly evaluated. Therefore, some value of stress must be assumed for each equation, and the properties then evaluated. Computation of torsional section properties, for composite beams, can be found in Reference (7).

Try the sections shown in Figs. 4a and 4b, respectively, for the single-span and two-span structures. Their dead load moments and section properties, relative to the bottom flange, are listed in Tables 9 and 10.

From Equations (33) and (34) and the data tabulated in Tables 9 and 10, the maximum normal stresses listed in Table 11 can be developed. The effect of impact has not been included. If the distribution of live load stresses is required in other girders throughout the system, the Modification Factors shown in Tables 3 through 8 would be applied as shown. Note that only live load stresses are reduced. Distribution of stresses describe effects of parallel truck loadings near outside girder.

To examine the validity of the results, computer program,[1,2] were used to predict the forces obtained by

Table 4. Modification Factors for Various Girders in the System—Four Girder System
$L = 100$

	Girder	Moment	Deflection	Rotation	Bimoment	St. V. Torsion	Warping Torsion	Shear
$R = 100$	1	25	20	50	80	60	100	20
	2	40	40	55	80	65	75	55
	3	65	60	80	95	85	80	70
	4	100	100	100	100	100	100	100
$R = 200$	1	20	25	75	100	80	100	20
	2	55	50	70	75	70	35	60
	3	80	70	75	65	70	30	80
	4	100	100	100	70	100	60	100
$R = 300$	1	35	35	90	100	90	100	35
	2	65	55	80	65	80	35	60
	3	85	80	85	60	85	40	75
	4	100	100	100	65	100	55	100
$R = 400$	1	45	40	100	100	100	100	40
	2	70	65	70	60	85	35	65
	3	90	85	70	55	75	35	85
	4	100	100	90	55	80	50	100
$R = 500$	1	50	50	100	100	100	100	50
	2	75	70	75	55	80	30	75
	3	90	85	65	40	65	20	90
	4	100	100	65	45	75	45	100
$R = 600$	1	55	55	100	100	100	100	55
	2	80	75	75	50	80	30	80
	3	95	90	65	40	60	20	95
	4	100	100	65	50	70	50	100

Table 5. Modification Factors for Various Girders in the System—Six Girder System

$L = 50$

	Girder	Moment	Deflection	Rotation	Bimoment	St. V. Torsion	Warping Torsion	Shear
$R = 100$	1	10	10	40	90	65	85	10
	2	40	25	45	90	65	100	55
	3	60	40	40	65	50	95	75
	4	70	55	50	65	55	70	80
	5	85	70	70	75	70	50	90
	6	100	100	100	100	100	70	100
$R = 200$	1	15	15	100	100	100	90	15
	2	50	40	100	90	90	100	65
	3	75	65	60	60	55	95	85
	4	85	75	50	40	40	60	95
	5	95	90	50	30	35	25	100
	6	100	100	70	45	50	40	100
$R = 300$	1	20	20	100	100	100	90	20
	2	60	50	95	90	90	100	65
	3	85	75	55	55	50	95	90
	4	95	85	40	35	30	60	95
	5	100	95	25	15	15	20	100
	6	100	100	30	20	20	30	100
$R = 400$	1	25	25	100	100	100	100	25
	2	70	60	90	90	90	100	70
	3	90	80	50	50	45	90	90
	4	100	90	40	35	25	55	95
	5	100	100	25	10	10	15	100
	6	100	100	30	15	15	25	100
$R = 500$	1	25	20	100	100	100	100	25
	2	65	60	90	90	90	100	70
	3	90	80	50	50	45	95	90
	4	100	95	35	30	25	55	95
	5	100	100	20	20	15	15	100
	6	95	100	25	15	10	30	100
$R = 600$	1	20	20	100	100	100	90	20
	2	65	60	90	90	90	100	65
	3	90	85	45	50	45	100	90
	4	100	95	25	25	20	55	95
	5	100	100	5	25	5	10	100
	6	95	100	5	10	5	30	95

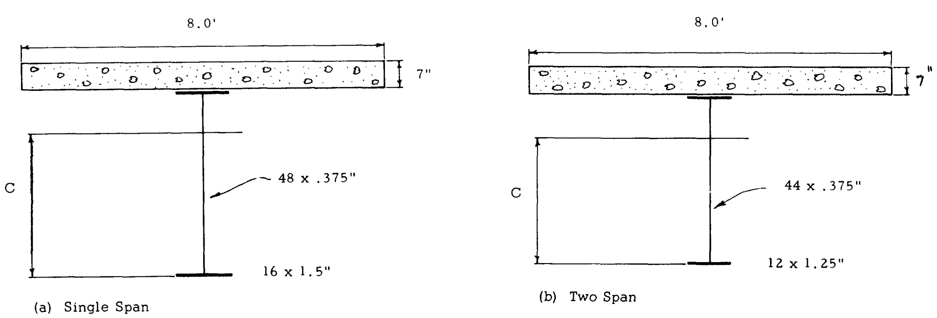

Fig. 4. Example problem — section properties

Table 6. Modification Factors for Various Girders in the System—Six Girder System
$L = 100$

	Girder	Moment	Deflection	Rotation	Bimoment	St. V. Torsion	Warping Torsion	Shear
$R = 100$	1	5	5	20	30	25	50	5
	2	20	15	25	40	30	50	20
	3	30	25	35	50	40	60	35
	4	40	35	50	65	65	80	55
	5	60	60	75	85	80	100	65
	6	100	100	100	100	100	100	100
$R = 200$	1	5	10	40	60	45	65	5
	2	30	25	40	60	45	75	25
	3	45	40	40	50	40	50	45
	4	60	50	50	60	50	60	60
	5	75	70	75	75	75	85	70
	6	100	100	100	100	100	100	100
$R = 300$	1	10	15	100	100	100	100	15
	2	35	30	85	90	80	85	35
	3	55	50	60	40	55	50	50
	4	70	65	65	50	55	55	70
	5	80	75	65	65	75	80	80
	6	100	100	95	85	95	90	100
$R = 400$	1	10	15	100	100	100	100	20
	2	40	35	90	90	85	85	45
	3	65	55	55	55	60	50	60
	4	75	70	60	45	55	40	80
	5	85	80	65	50	60	45	90
	6	100	100	85	65	85	70	100
$R = 500$	1	15	15	100	100	100	100	20
	2	45	45	90	90	90	80	45
	3	70	70.	60	50	65	45	65
	4	80	75	60	40	60	35	80
	5	90	85	70	40	65	40	95
	6	100	100	80	50	80	60	100
$R = 600$	1	15	15	100	100	100	100	20
	2	50	45	90	90	90	75	50
	3	75	70	60	50	65	40	65
	4	85	80	40	40	60	35	85
	5	95	90	45	45	55	40	95
	6	100	100	65	50	70	55	100

the equations. In addition, the sections that were assumed (Figs. 4 (a) and 4 (b)) were used for computer analysis. Tables 12 and 13, show the results from both the equations and the computer programs, for the live load forces. As can be seen excellent agreement occurs, indicating the validity of the equations.

RESULTS AND CONCLUSIONS

A series of simplified equations are presented, which will permit evaluation of internal forces and deformation in a single, two- and three-span curved girder system. These forces can then be utilized to estimate initial section properties, which are necessary in utilizing various computer programs listed in Refs. (1) and (8).

The studies which were conducted, resulting in the design equations, have the following limitations:

1. Girder spacing may be 7, 8, 9, or 10 ft.
2. Individual girder span lengths varied from 50 to 100 ft.
3. The girders of the system must have a constant curvature and are limited to radii of 100 to 600 ft.
4. The number of girders in the system may be 4, 6 or 8.
5. Radii and span lengths of the system are the mid-radii and length.
6. Only two- and three-span continuous bridges were examined, with all interior and end spans of equal length.

Table 7. Modification Factors for Various Girders in the System—Eight Girder System

$L = 50$

	Girder	Moment	Deflection	Rotation	Bimoment	St. V. Torsion	Warping Torsion	Shear
$R = 100$	1	5	5	50	100	90	100	20
	2	50	20	45	75	70	85	80
	3	65	30	25	25	30	15	100
	4	65	35	15	10	10	10	85
	5	60	40	40	60	50	95	75
	6	70	55	55	50	60	45	85
	7	85	75	75	50	75	30	90
	8	100	100	100	80	100	70	100
$R = 200$	1	25	20	100	100	100	90	30
	2	70	50	75	70	70	75	80
	3	90	65	20	15	15	10	100
	4	85	65	5	10	10	15	85
	5	80	65	40	55	35	100	75
	6	90	80	45	35	35	45	85
	7	95	90	40	20	25	10	90
	8	100	100	40	45	30	80	90
$R = 300$	1	30	30	100	100	100	90	30
	2	80	65	70	70	70	65	80
	3	100	80	15	10	10	10	100
	4	90	75	10	15	15	20	85
	5	85	75	30	55	30	100	75
	6	95	90	35	30	25	45	85
	7	100	95	20	10	10	10	85
	8	100	100	5	40	10	80	85
$R = 400$	1	35	35	100	100	100	85	35
	2	80	15	70	70	65	65	80
	3	100	90	10	10	5	10	100
	4	90	85	15	20	20	20	85
	5	85	85	25	55	25	100	80
	6	95	95	30	30	25	45	85
	7	100	100	10	5	5	5	90
	8	95	100	85	45	15	85	80
$R = 500$	1	35	35	100	100	100	85	35
	2	80	75	70	70	70	65	80
	3	100	95	10	5	5	10	100
	4	90	90	20	20	20	20	85
	5	85	90	20	55	25	100	80
	6	90	95	25	30	20	45	85
	7	95	100	5	5	5	5	90
	8	90	95	15	50	20	90	80
$R = 600$	1	35	35	100	100	100	80	35
	2	80	80	65	70	70	65	80
	3	100	95	10	5	5	10	100
	4	90	90	20	20	20	20	85
	5	85	85	20	55	25	100	75
	6	90	95	25	30	20	45	85
	7	95	100	25	5	5	5	90
	8	90	95	20	55	25	95	80

Table 8. Modification Factors for Various Girders in the System—Eight Girder System

$L = 100$

	Girder	Moment	Deflection	Rotation	Bimoment	St. V. Torsion	Warping Torsion	Shear
$R = 100$	1	10	10	10	40	25	70	10
	2	30	15	15	35	25	45	35
	3	40	20	15	30	20	40	55
	4	45	25	25	30	25	40	50
	5	45	30	40	60	45	65	50
	6	60	50	60	80	65	75	55
	7	70	70	85	95	90	95	70
	8	100	100	100	100	100	100	100
$R = 200$	1	20	20	55	100	70	100	20
	2	50	35	45	80	50	60	55
	3	65	40	25	25	25	30	75
	4	70	45	20	25	15	25	75
	5	65	50	35	55	25	70	70
	6	75	60	60	65	60	45	75
	7	85	80	80	85	80	50	85
	8	100	100	100	90	100	75	100
$R = 300$	1	30	30	100	100	100	100	30
	2	60	50	75	60	80	65	65
	3	80	65	35	25	25	25	85
	4	80	65	20	25	10	25	80
	5	75	60	35	45	25	65	75
	6	85	75	55	50	40	40	80
	7	95	85	75	60	50	35	90
	8	100	100	85	70	75	70	100
$R = 400$	1	35	35	100	100	100	100	35
	2	70	60	75	70	80	60	80
	3	90	80	30	25	25	20	95
	4	90	80	10	10	10	25	95
	5	85	75	25	15	25	70	90
	6	90	80	40	30	25	30	95
	7	100	90	45	30	40	20	95
	8	100	100	50	40	60	50	100
$R = 500$	1	35	35	100	100	100	100	35
	2	70	60	75	70	80	60	80
	3	90	80	30	25	25	20	95
	4	90	80	10	10	10	25	95
	5	85	75	25	15	25	70	90
	6	90	80	40	30	25	30	95
	7	100	90	45	30	40	20	95
	8	100	100	50	40	60	50	100
$R = 600$	1	35	35	100	100	100	100	35
	2	70	60	75	70	80	60	80
	3	90	80	30	25	25	20	95
	4	90	80	10	10	10	25	95
	5	85	75	25	15	25	70	90
	6	90	80	40	30	25	30	95
	7	100	90	45	30	40	20	95
	8	100	100	50	40	60	50	100

Table 9. Single Span Moments and Properties

Moments	L.L. (kip-in.)	D.L. (kip-in.)	C (in.)	I_{xx} (in.^{4})	W_{ns} (in.^{3})	I_w (in.^{6})
M_b	13975.0	14651.0	39.38	61150.0	—	—
Bimoment	2880.0	2043.0	—	—	488.0	19630×10^2

Table 10. Two Span Moments and Properties

Moments	L.L. (kip-in.)	D.L. (kip-in.)	C (in.)	I_{xx} (in.^{4})	W_{ns} (in.^{3})	I_w (in.^{6})
M_b	10450.0	7648.0	38.44	38000.0	—	—
Bimoment	1010.0	564.0	—	—	325.0	542000.0

Table 11. Resulting Stresses

Structure	σ_b-L.L. (ksi)	σ_b-D.L. (ksi)	σ_w-L.L. (ksi)	σ_w-D.L. (ksi)	σ_T (ksi)
Max. F					
Single Span	8.98	6.29	0.72	0.50	16.49
Two Span	10.60	7.71	0.63	0.34	19.28
Distribution Using Table 4 for L = 100 and R = 200					
Single Span					
G1	1.80	6.29	0.72	0.50	9.31
G2	4.94	6.29	0.54	0.50	12.27
G3	7.18	6.29	0.47	0.50	14.44
G4	8.98	6.29	0.50	0.50	16.27
Two Span					
G1	2.12	7.71	0.63	0.34	10.80
G2	5.83	7.71	0.47	0.34	14.35
G3	8.48	7.71	0.41	0.34	16.82
G4	10.60	7.71	0.44	0.34	19.09

7. The support conditions are assumed as follows:

   Exterior $\Delta = \phi = 0$
   Interior $\Delta = 0$

8. The girders are rolled beam members or plate girders in composite action with a 7-in. concrete slab. The stiffness for each girder is constant along its length.

In summary, the general form for computing the maximum function of a curved girder system is:

$$F_{max} = F \cdot K_1 \times K_2 \times K_3$$

where:

F = Maximum value of function, due to a single line of truck loading on straight girder, $P_{eq} = P \times \bar{S}$, Fig. 1

K_1 = Amplification Factors, Equations (2) through (8)

K_2 = Distribution Factors, Equations (10) through (30)

K_3 = Reduction Factors, Table 2

$\bar{S}$ = Spacing Factor = 1.29 for S = 7 ft or 8 ft
= 1.57 for S = 9 ft or 10 ft

Table 12. Single Span Data—Computer and Equations

Method	M_b (kip-in.)	Δ (in.)	$\phi \times 10^{-2}$	St. V. Torsion (kip-in.)	Warp. Torsion (kip-in.)	Bimoment (kip-in.)	Shear (kips)
Computer	13848.	1.37	0.529	250.7	9.0	2674.0	31.9
Equations	13937.	1.60	0.680	285.0	8.0	2680.0	31.3

Table 13. Two Span Data—Computer and Equations

Method	M_b (kip-in.)	Δ (in.)	$\phi \times 10^{-2}$	St. V. Torsion (kip-in.)	Warping Torsion (kip-in.)	Bimoment (kip-in.)	Shear (kips)
Computer	9160.	1.23	0.400	202.	2.55	693.0	29.6
Equations	10450.0	1.12	0.442	214.	3.6	938.0	31.3

For girders in the system other than the basic maximum function girder:

$F_i = F_{max} \times$ Modification Factor, Tables 3 through 8.

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APPENDIX A

NOTATIONS

C	= distance from bottom flange to centroid of section
E	= Modulus of elasticity
G	= Shear modulus of elasticity
I	= Moment inertia of cross section, in.^{4}
I_w	= Warping constant, in.^{6}
K_1	= Amplification Factor $= f_{SC}/f_{SS}$
K_2	= Distribution Factor $= f_{CSy}/f_{SC}$
K_3	= Reduction Factor for continuous spans
K_T	= Torsional constant of cross section, in.^{4}
L	= Length of girder, ft
M	= $L/50$ (coefficient)
M_b	= Normal bending moment kip-in.
P	= Wheel load, 4.0 kips or 16.0 kips
P_{eq}	= Equivalent concentrated load to one girder, due to adjacent truck loads, kips $= P \times \bar{S}$
R	= Radius of girder, ft
S	= Spacing of girder, ft
$\bar{S}$	= Spacing factor
S_{ws}	= Warping statical moment, in.^{4}
$T_{warping}$	= Warping torque $= -EI_w\left(\phi''' + \frac{\eta}{R}\right)$
$T_{St.\,Venant}$	= St. Venant Torque $= GK_T\left(\phi' + \frac{\eta'}{R}\right)$
$T_{bimoment}$	= Bimoment $= -EI_w\left(\phi'' + \frac{\eta''}{R}\right)$
V	= Shear, kips
W_{ns}	= Normalized warping functions, in.^{2}
f_{SC}	= Reactions on a single curved girder
f_{SS}	= Reactions on a single straight girder
f_{CSy}	= Reactions on a system of curved girders
n	= $R/100$ (coefficient)
t	= flange thickness
Δ	= η = vertical displacement
η	= Vertical displacement
η'	= First derivative of η with respect to arc length
η''	= Second derivative of η with respect to arc length
σT	= Total normal stress
σ_b	= Normal bending stress (ksi) $= M_bCI$
σ_w	= Normal warping stress (ksi) $= -T_{bimoment} \times \frac{W_{ns}}{I_w}$
τ_{st}	= St. Venant shearing stress (ksi) $= T_{St.\,Venant} \times t/G$
τ_{ws}	= Warping shearing stress (ksi) $= -T_{warping} \times \frac{S_{ws}}{tI_w}$
ϕ	= Rotation
ϕ'	= First derivative of ϕ with respect to arc length
ϕ''	= Second derivative of ϕ with respect to arc length
ϕ'''	= Third derivative of ϕ with respect to arc length

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