

Design Recommendation for Two-Sided Bolted Rectangular HSS End-Plate Connections in Axial Tension

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ABSTRACT

Recent studies conducted on the design of bolted hollow structural section end-plate connections have demonstrated that the use of the plate ultimate stress in place of the plate yield stress provides an improved correlation to experimental data. This research applies this flexural strength model to two-sided bolted rectangular hollow section end-plate connections to achieve a heightened degree of consistency across North American end-plate connection design. An updated design approach is compared to historical databases, and a suitable resistance factor is computed using the numerical Monte Carlo simulation (MCS) technique. The MCS is compared against the reliability assessment method of AISI S100, as well as a recent reformulation of the separation factor approach, providing an opportunity for the evaluation of these approaches. A design example is then given to illustrate the recommended design procedure, in LRFD.

KEYWORDS: hollow structural section, bolted connection, end-plate splice, prying action, design recommendation.

INTRODUCTION

Bolted end-plate connections are prevalent in structures that incorporate hollow structural sections (HSS), as they allow for larger spans, changes in cross section, and the ease of assembly. In round HSS connections, the bolts are arranged in a ring around the tube, whereas for square and rectangular HSS, the bolts are arranged in rows along either all four sides of the HSS or along two sides only. While four-sided bolted connections are more common in practice, two-sided connections, seen in Figure 1, still enjoy considerable usage.

Due to the bolts being positioned outside of the HSS cross section, end-plate connections will experience prying forces on the bolts when subjected to member tensile loading. Thus, connection nominal strength equations have historically used the prying effect model of the AISC *Steel Construction Manual*, hereafter referred to as the AISC *Manual* (2023), that is derived from the tee-stub model of Struik and de Back (1969). These procedures originally used the plate yield stress (F_{yp}) in their formulation; however, work by

Thornton (1992, 1996) and Swanson (2002) showed that use of the plate ultimate stress (F_{up}), with the Struik-de Back model, provided an improved correlation between experimental and predicted results. Accordingly, the 13th Edition AISC *Manual* (2005) replaced F_{yp} with F_{up} when considering prying effects.

A similar change has been made to nominal strength models for bolted HSS end-plate connections. Works by Fidalgo and Packer (2022) with round HSS and Thornton (2017) with four-sided rectangular HSS connections (which utilized test data from Willibald et al., 2003) have both demonstrated an improved correlation using F_{up} . These changes are reflected in the recent AISC Design Guide 24, *Hollow Structural Section Connections* (DG 24) (Packer and Olson, 2024), which presents methods for the design of round HSS and both four- and two-sided rectangular HSS bolted end-plate connections. However, no study has been conducted to investigate the viability of the use of F_{up} for two-sided bolted rectangular HSS end-plate connections. Thus, DG 24 used F_{yp} for these connections only.

This study aims to address this inconsistency through the development of an updated two-sided bolted rectangular HSS end-plate design model that demonstrates an improved correlation between experimental and predicted capacities. This is achieved through the incorporation of F_{up} into the prying effect model, as well as changes based on observations from previous physical experiments. Following this, a reliability study is conducted, using a Monte Carlo simulation (MCS), to determine a resistance factor that provides a satisfactory degree of structural reliability. The use of this numerical approach produces the “true” resistance factor, which presents the opportunity for an evaluation of the

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relative accuracies of several closed-form reliability techniques. These techniques include the reliability assessment method presented in AISI S100, *North American Specification for the Design of Cold-Formed Steel Structural Members* (2020), recalibrated to apply to structural steel, as well as a recent reformulation of the separation factor approach. The resistance factors computed using these approaches are compared against the MCS resistance factor to assess the viability of the closed-form approaches and provide recommendations on the conduction of a proper reliability study for structural steel.

NOMINAL STRENGTH MODEL

Axial Tension Expression Improvement

When a two-sided bolted HSS end-plate connection is loaded in axial tension, the lateral offset between the tube and the bolts creates flexure within the end plates, which generates a prying force along the end-plate edges. Since the prying force is proportional to end-plate rotation, increasing the connection stiffness by selecting thicker end plates will reduce the prying force. Thus, two potential failure modes govern these connections: When the end plates are thin, plate plastic hinging will occur, whereas bolt rupture will govern when the end plates are sufficiently thick. This forms the basis of the prying effect model advanced by Thornton (1992) and Swanson (2002), upon which the DG 24 strength equations for four- and two-sided bolted HSS end-plate connections are based.

An assumption of the traditional prying effect model is that during plate plastification, two yield lines can form across the end plates. These yield lines are located at the

inside face of the bolts and at the element face (or the weld toe). While this behavior has been observed for tee-stubs and four-sided bolted rectangular HSS end-plate connections, the inner or hogging yield lines of the two-sided connections have been seen to occur within the tube wall dimension, rather than at the HSS weld toes. This observation is based on the work of Packer et al. (1989), whose physical experiments displayed a clear flaking of the white-wash, on the underside of the plate, within the tube wall region. This behavior is attributed to a lack of restraint from the unbolted sides, which allows for an inward progression of yielding as the member load increases, and is therefore uncharacteristic of the four-sided bolted connections.

DG 24 therefore includes the thickness of the HSS wall (t) in Equation 1 to compute b' , which represents the distance between the hogging and sagging yield lines. This parameter is then used within Equation 2 to obtain the plate thickness required to avoid prying (t_c), which, notably, uses F_{yp} . The remainder of the nominal strength model is identical to that of a four-sided connection, with computations performed on a per-bolt basis, making the connection capacity highly dependent on the ratio of the applied load per bolt without prying (T_r) to the available tensile strength per bolt (T_c). This ratio (T_r/T_c) is used to compute the prying ratio (β) and flexibility factor (α^*) in Equations 3 and 4, respectively, which are used in Equation 5 to compute the required plate thickness (t_p).

$$b' = b - \frac{d_b}{2} + t \quad (1)$$

$$t_c = \sqrt{\frac{4T_c b'}{pF_{yp}}} \quad (2)$$



Fig. 1. Examples of two-sided bolted rectangular HSS end-plate connections.

$$\beta = \frac{1}{\rho} \left(\frac{T_c}{T_r} - 1 \right) \quad (3)$$

$$\alpha^* = \min \left[\frac{1}{\delta} \left(\frac{\beta}{1-\beta} \right), 1 \right] \quad (4)$$

$$t_p = t_c \sqrt{\frac{T_r}{T_c} \left(\frac{1}{1+\delta\alpha^*} \right)} \quad (5)$$

where p is the bolt tributary width, $\rho = b'/a'$, $\delta = 1 - d_h/p$, d_h is the bolt diameter, d_h is the bolt hole diameter, and a' is the distance from the plate edge to the sagging yield line. Other geometric variables are illustrated in Figure 2. An adjusted available tensile capacity per bolt ($T_{c,adj}$) can then be computed, including the effects of prying using Equations 6 to 8, which employ the prying adjustment factor (Q). The overall connection capacity (T_a) is then simply the product of the adjusted bolt tensile strengths and the number of bolts (n_b).

$$\alpha' = \frac{1}{\delta(1+\rho)} \left[\left(\frac{t_c}{t_p} \right)^2 - 1 \right] \quad (6)$$

$$Q = \begin{cases} 1 & \alpha' \leq 0 \\ \left(\frac{t_p}{t_c} \right)^2 (1+\delta\alpha') & 0 < \alpha' \leq 1 \\ \left(\frac{t_p}{t_c} \right)^2 (1+\delta) & 1 < \alpha' < 1.5 \\ 0 & \alpha' \geq 1.5 \end{cases} \quad (7)$$

$$T_{c,adj} = T_c Q \quad (8)$$

$$T_a = T_{c,adj} n_b \quad (9)$$

In the 1980s, 23 physical experiments were performed (Figure 3) on two-sided bolted HSS end-plate connections

under axial tension (Birkemoe and Packer, 1986; Packer et al., 1989). These researchers then used the experimental results to validate a capacity prediction method. Although the method was highly accurate, practitioners did not adopt it because it required computing multiple failure mechanisms to determine the governing case. One conclusion of this work was that under axial tension loading, the bolt force eccentricity—one-half a bolt diameter, as reported by Struik and de Back (1969)—was negligible. Incorporating this into the DG 24 method changes the computation of b' to Equation 10, as the sagging yield line passes through the bolt centerlines. Additionally, the parameter a' can then be taken as a .

$$b' = b + t \quad (10)$$

The bolt tributary width (p) is defined as the plate width (w_p) divided by the number of bolts per side of the connection. This approach reduces the connection to a 2D strip, whereby the tee-stub prying effect model is applied to a pair of bolts. However, Dowswell (2011) recognized that if p is excessively large, independent yield line patterns will develop around the bolts; therefore, Equation 11 imposes a maximum bolt tributary width.

$$p_{max} = 4\sqrt{b'(a+b)} \quad (11)$$

DG 24 states that, for two-sided bolted HSS end-plate connections, the bolt centerlines should not be positioned beyond the HSS width as this may create an uneven load distribution on the bolts. For some connection geometries, equating p to the upper limit given in Equation 11 will result in excessively wide end plates that artificially boost the connection nominal strength, despite there being no actual increase in capacity. It is thus recommended that a more practical upper limit of $w_p \leq H + 4$ in. be imposed, where H is the HSS dimension parallel to the bolt line. This limit is based on the experimental connection geometries

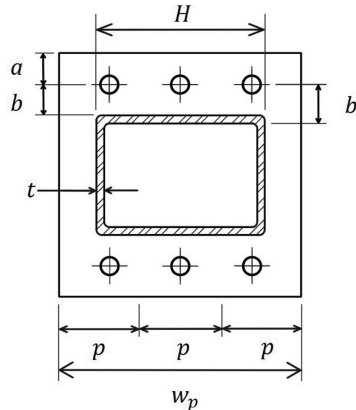


Fig. 2. Two-sided bolted end-plate connection cross section, with b' per Equation 10.

of Packer et al. (1989) and typical design practice, with the 4 in. allowing for the placement of welds. Such connection proportions will prevent unconservative strength prediction and the development of independent yield line patterns. Using this limit, an alternative formulation for p_{max} is shown in Equation 12.

$$p_{max} = \frac{H + 4 \text{ in.}}{n_b/2} \quad (12)$$

Use of F_{up} in lieu of F_{yp} in Equation 2 is also recommended for consistency with other bolted end-plate nominal strength models in DG24.

In order to evaluate the proposed capacity prediction method, 23 laboratory experiments by Birkemoe and Packer (1986) and Packer et al. (1989) are considered, with the measured properties listed in Table 1. The experimental results (T_{nx}) and corresponding failure modes are listed

in Table 2, along with the predicted capacities based on the nominal strength method presented herein, using both F_{yp} and F_{up} (denoted in Table 2 as T_{ny} and T_{nu}). In some cases, the value of α' exceeded 1.5, indicating excessive prying, which, using Equation 7, will give a connection capacity equal to zero in Table 2. The test LB-16 was not considered in the capacity prediction method evaluation due to its governing limit state—tube yielding—not being considered as a connection failure mode.

A comparison between the experimental results and predicted capacities, using the recommendations made herein, is seen in Figure 4. The discrepancy in the number of tests (n) assessed for F_{yp} and F_{up} is due to the exclusion of data when α' exceeded 1.5 (shown by N/A in Table 2). Figure 4 (in which both data sets use Equation 10 for b') shows that the use of F_{up} significantly reduces the mean value relative to F_{yp} [while the two coefficients of variation (CoVs) are very similar], indicating an improved correlation.



(a) Experimental setup



(b) Ultimate failure

Fig. 3. Experiments on two-sided bolted end-plate connections, using bolt load-indicating washers.

Table 1. Measured Geometric and Material Properties of Experimental Specimens

Test Name	h (in.)	b (in.)	t (in.)	n_b	d_b (in.)	F_{ub} (ksi)	w_p (in.)	a (in.)	b (in.)	t_p (in.)	F_{yp} (ksi)	F_{up} (ksi)
LB-1	4	4	0.225	4	0.625	98.1	6	1.63	1.75	0.794	43.5	71.1
LB-2	4	4	0.225	4	0.625	104	6	1.63	1.75	0.500	41.6	65.0
LB-3	4	4	0.225	4	0.75	106	6	1.63	1.75	1.01	39.9	69.3
LB-4	5	7	0.188	6	0.75	106	9	1.18	2.17	1.01	39.9	69.3
LB-5	5	7	0.188	6	0.75	106	9	1.18	1.77	1.01	39.9	69.3
LB-6	5	7	0.188	6	0.75	106	9	1.18	1.38	1.01	39.9	69.3
LB-7	5	7	0.188	6	0.75	106	9	1.57	1.38	1.01	39.9	69.3
LB-8	5	7	0.188	6	0.75	106	9	2.17	1.38	1.01	39.9	69.3
LB-9	5	7	0.188	4	0.75	106	9	1.18	1.38	1.01	39.9	69.3
LB-10	5	7	0.188	6	0.625	103	9	1.18	1.38	1.01	39.9	69.3
LB-11	5	7	0.188	6	0.875	108	9	1.18	1.38	1.01	39.9	69.3
LB-12	5	7	0.188	6	0.875	106	9	1.18	1.38	0.895	48.6	81.1
LB-13	5	7	0.188	6	0.875	104	9	1.18	1.38	0.639	42.5	67.6
LB-14	5	7	0.188	6	0.625	104	9	1.18	1.38	0.500	41.6	65.0
LB-15	5	7	0.188	6	0.625	98.1	9	1.18	1.38	0.794	43.5	71.1
LB-16	5	7	0.188	6	0.875	108	9	1.97	1.38	1.01	39.9	69.3
A	4	4	0.225	4	0.625	111	5.12	1.18	1.46	0.794	43.5	71.1
B	4	4	0.225	4	0.625	111	5.12	1.16	1.85	0.794	43.5	71.1
C	4	4	0.225	4	0.625	111	5.12	1.18	1.65	0.794	43.5	71.1
D	4	4	0.225	4	0.625	111	5.12	1.97	1.61	0.794	43.5	71.1
E	4	4	0.225	4	0.75	112	5.12	1.75	1.63	0.794	43.5	71.1
F	4	4	0.225	4	0.75	112	5.12	1.16	1.85	1.01	39.9	69.3
G	4	4	0.225	4	0.75	112	5.12	1.18	1.85	0.895	48.6	81.1

RELIABILITY STUDY

Structural reliability is achieved through load and resistance factor design (LRFD), where a design is considered safe if the product of the nominal capacity (R_n) and a corresponding resistance factor (ϕ) exceeds the product of the nominal load effects ($S_{n,i}$) and constituent load factors (γ_i).

$$\phi R_n \geq \sum \gamma_i S_{n,i} \quad (13)$$

Load and resistance factors are intended to account for the uncertainty within the nominal terms and are based on probabilistic analyses. Whereas load factors are presented as load combinations in so-called loading codes [e.g., ASCE 7 (2022), NBC (NRC, 2020)], resistance factors are specific to nominal strength equations and must be computed on a case-by-case basis. This establishes the notion of target structural reliability as resistance factors must be selected such that their use within LRFD produces an acceptable probability of failure. When probabilistic distributions are used to represent the resistance (R) and the load effect (S),

the failure probability (p_F) can be determined using their difference, termed the safety margin (G):

$$p_F = p(G < 0) \quad (14)$$

Ellingwood et al. (1980) more usefully represent the failure probability using the reliability index (β^+), which is equal to the number of standard deviations (σ_G) from the mean value of the safety margin ($\bar{G}$) to the failure condition.

$$\beta^+ = \frac{\bar{G}}{\sigma_G} \quad (15)$$

In the United States, benchmark values of the reliability index and their associated failure probabilities are defined in ASCE 7, *Minimum Design Loads and Associated Criteria for Buildings and Other Structures* (2022), for a 50 yr design life. These target reliability indices are based on both risk category and failure mode, though it is typically assumed that design equations apply to Risk Category II structures. Thus, ductile failure has a target reliability

Table 2. Connection Experimental and Predicted Capacities						
Test Name	T_{nx} (kips)	Experimental Failure Mode	T_{ny} (kips)	T_{nu} (kips)	T_{nx}/T_{ny}	T_{nx}/T_{nu}
LB-1	99.6	Bolt fracture	63.3	77.6	1.57	1.28
LB-2	78.7	Plate plastification	0	0	N/A	N/A
LB-3	140	Bolt fracture	97.1	122	1.44	1.15
LB-4	178	Bolt fracture	123	161	1.45	1.11
LB-5	193	Bolt fracture	138	181	1.40	1.07
LB-6	215	Bolt fracture	158	207	1.36	1.04
LB-7	218	Bolt fracture	164	208	1.33	1.05
LB-8	219	Bolt fracture	172	208	1.27	1.05
LB-9	179	Bolt fracture	128	141	1.40	1.27
LB-10	179	Bolt fracture	128	142	1.40	1.26
LB-11	252	Bolt fracture	192	242	1.31	1.04
LB-12	243	Bolt fracture	187	230	1.30	1.06
LB-13	209	Plate plastification	0	0	N/A	N/A
LB-14	110	Plate plastification	0	82.6	N/A	1.33
LB-15	153	Bolt fracture	103	132	1.48	1.16
LB-16	262	HSS yielding	199	253	1.32	1.03
A	103	Bolt fracture	66.8	82.4	1.54	1.25
B	87.5	Bolt fracture	58.5	72.2	1.50	1.21
C	92.2	Bolt fracture	62.5	77.1	1.47	1.20
D	98.9	Bolt fracture	66.1	83.1	1.50	1.19
E	122	Bolt fracture	0	104	N/A	1.17
F	120	Bolt fracture	84.9	110	1.42	1.10
G	121	Bolt fracture	80.7	105	1.50	1.15

index of 3.0, and brittle failure has a recommended index of 4.0. Herein, the computation of a resistance factor for the two-sided bolted HSS end-plate connections is performed using a target reliability index of 4.0, due to the potential for sudden bolt fracture to govern the connection design.

Several practical methods exist for the computation of resistance factors, which are said to achieve an acceptable probability of failure by using the target reliability index as an input. Herein, a resistance factor is computed using a Monte Carlo simulation (MCS), a numerical technique that converges to the true solution by performing a large number of iterations. A single MCS iteration simulates a design scenario: Values of all participating material and geometric variables are randomly sampled from probabilistic distributions that reflect actual parametric variation and are used to compute the corresponding nominal capacity. This process is repeated for the load effect and then compared against the capacity to determine the degree of structural reliability under these conditions. When this process is repeated a large number of times across a range of representative

connections, the MCS converges on the overall reliability of the nominal strength model. A suitable resistance factor is selected based on the reliability index distributions produced by trial resistance factor values, relative to the target reliability index.

Rudman et al. (2022) present detailed steps for the completion of a MCS; however, slight adaptations were required herein due to the nature of the nominal strength model. Unlike typical models, where a capacity is computed and compared against the load effect, the method presented herein calculates an initial plate thickness using an applied load per bolt (T_r), which is then rounded upward to a discrete value. This is incorporated in the MCS by randomly generating an applied load per bolt, which informs the selection of the requisite bolt diameter and allows for the initial plate thickness to be computed. Randomly sampled values are only employed after this discretization because designers would use nominal values to arrive at this step. MATLAB (MathWorks, 2023) was used to perform the simulations, using a range of live-to-dead load (L/D) ratios

of $0 \leq L/D \leq 3$ and consisted of 1 million iterations per L/D and representative connection. To ensure a conservative resistance factor, representative reliability index values for each trial resistance factor were taken as the minimum across the range of L/D . The calibration of the resistance factor using the minimum reliability index ensures a standardized approach across all limit states in LRFD, and follows similar studies by Rudman et al. (2022) and Rahbarimanesh and Tousignant (2024).

Representative Connections

The representative connections used in the MCS vary based on randomly generated applied loads, though finite ranges of input parameters are possible. For this study, the connection geometrical parameters were defined based on the following ranges: $\frac{3}{16}$ in. $\leq t \leq \frac{1}{2}$ in., 1.5 in. $\leq a, b \leq 2$ in., $\frac{1}{2}$ in. $\leq d_b \leq 1$ in., and 2 in. $\leq p \leq 4$ in. A value of 65 ksi was selected as the tensile strength of the end plates, based on the minimum specified tensile strength of ASTM A572 Grade 50 (ASTM, 2021). Bolt tensile strength was assigned a value of 120 ksi, based on the minimum specified tensile strength of ASTM F3125 Grade A325 bolts (ASTM, 2025). Due to the nominal strength model performing computations on a per-bolt basis, an input number of bolts was not required for the MCS, though typical bolting arrangements were considered to derive the ranges of geometric

parameters. A total of 30 representative connections were thus generated for each resistance factor.

Statistical Parameters

Computation of the nominal capacity of two-sided bolted end-plate connections requires the following material and geometric parameters: plate ultimate stress, HSS thickness, plate thickness, bolt diameter, distance to bolt holes, bolt tributary width, number of bolts, and bolt tensile strength (which is a function of the bolt ultimate stress and the bolt hole diameter). Excluding the number of bolts, log-normal probabilistic distributions for the variation in all parameters were created using the corresponding mean ratio and coefficient of variation (CoV). Cumulative distribution functions (CDF) required for the MCS were then created from these distributions.

Statistical parameters describing the variation of ASTM A572 Grade 50 plate ultimate stress were taken from data collated by Lui et al. (2007), specifically pertaining to plate thicknesses greater than $\frac{1}{2}$ in. The variation in the tensile strength of ASTM F3125 Grade A325 bolts was taken from Moore et al. (2008), who also produced statistical parameters for Grade A325 bolt diameters. The variation in plate width and thickness for ASTM A572 Grade 50 were obtained from a study by Schmidt and Bartlett (2002a). Plate width statistics are assumed to be representative of all

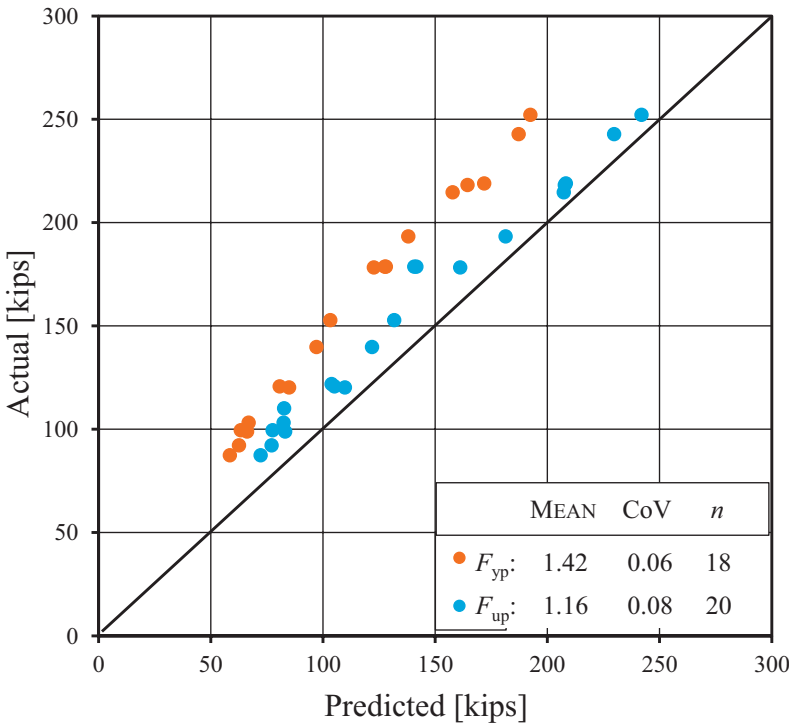


Fig. 4. Comparison of actual to nominal capacities using both the plate yield and ultimate stress.

Table 3. Statistical Parameters Describing Each Participating Variable			
Variable	Source	Mean (Actual/Nominal) Ratio	CoV
F_{up}	Liu et al. (2007)	1.26	0.070
F_{ub}	Moore et al. (2008)	1.22	0.047
d_b		0.994	0.003
t	Schmidt and Bartlett (2002a)	0.975	0.025
t_p		1.04	0.025
b, p		0.998	0.004
D	Schmidt and Bartlett (2002b)	1.05	0.100
L		0.78	0.320

in-plane (fabricator-dependent) plate geometric parameters, including the bolt tributary width (p) and the distance from the bolt hole centers to the tube face (b). This study also surveyed the thickness of ASTM A500 (2023) Grade B/C HSS. Statistical parameters that describe the variation in the load effect terms are also required to perform the MCS. Probabilistic distributions of the dead and live load using normal and Gumbel distributions, respectively, were produced by Ellingwood et al. (1980) and Ellingwood and Culver (1977). These were both converted to equivalent log-normal distributions by Schmidt and Bartlett (2002b), from which statistical parameters were taken. A summary of these values is seen in Table 3.

A probabilistic distribution is required for the professional factor, which measures the variation in accuracy of the nominal strength method. This distribution was created using the actual-to-predicted data from Figure 4 and was fitted with a log-normal curve using a maximum likelihood

estimation (MLE) technique, resulting in Figure 5(a). From this, a CDF was produced, as seen in Figure 5(b).

Results

Three trial resistance factors were assessed using the MCS, with values of $\phi = 0.90, 0.95$, and 1.00 . Figure 6 presents the reliability index distributions associated with each trial resistance factor over the range of $0 \leq L/D \leq 3$. The reliability index distributions exhibit a dip in reliability at L/D of 0.1 due to the change from the principal dead to the principal live load combination. However, Figure 6 shows that the minimum reliability is created at $L/D = 3$. All three curves are observed to exceed the target reliability index of 4.0 over the full range of L/D , and thus all trial resistance factors achieve satisfactory structural reliability. However, a (very conservative) resistance factor value of $\phi = 0.90$ is selected to align with the current recommendations in

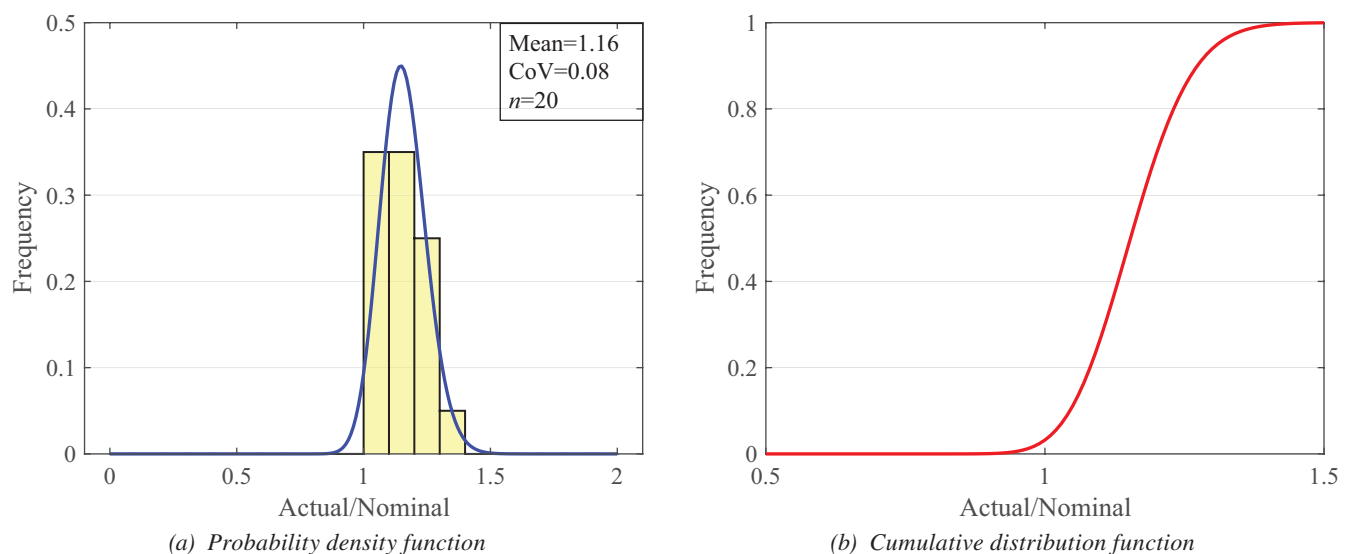


Fig. 5. Professional factor probabilistic distributions for the nominal strength model recommended by the authors.

DG 24 for other HSS end-plate connections, as well as the prying effect model given in Part 9 of the AISC *Manual* (2023).

The MCS was conducted using statistical parameters for ASTM A572 Grade 50 plate and ASTM F3125 Grade A325 bolts, thus the computed resistance factor may not be applicable if alternate material grades are used in design. However, the resistance factor of $\phi = 0.90$ is shown in Figure 6 to give reliability well in excess of the target reliability index of 4.0, which expands the validity of the design model. Studies by Xi and Packer (2021) and Rudman et al. (2022) have shown that the MCS results may be sensitive to the mean ratios of key material parameters. A sensitivity analysis was therefore undertaken to determine the effects of variation in the mean values of F_{up} and F_{ub} , with the results shown in Figure 7. This figure shows that the resistance factor of $\phi = 0.90$ still produces adequate structural reliability when the mean ratios of these parameters are as low as 1.10. Applicability for mean ratios of 1.10, for both F_{up} and F_{ub} simultaneously, allow for this model to be applicable to most structural steel and structural bolt grades. This value is less than or equal to the mean reported for 34 of 35 structural steel grades covered in AISC 341, *Seismic Provisions for Structural Steel Buildings* (2022a), and all ASTM F3125 bolt grades reviewed by Moore et al. (2008). Thus, the recommended resistance factor can be confidently extended to cover the vast majority of grades of structural steel and structural bolts. The CoVs of these parameters were not varied within the sensitivity analysis because V_R is dominated by the CoV of the professional factor, V_P .

Because the nominal strength model is most commonly used to compute a plate thickness, the resistance factor must be used within the determination of this parameter instead

of for the determination of connection capacity. However, DG 24 also allows designers to select the plate thickness that avoids prying. Therefore, it is recommended that the plate thickness needed to avoid prying be computed using Equation 16, with $\phi = 0.90$. This allows the resistance factor to be inversely proportional to the design plate thickness and proportional to the connection capacity. A comparison of actual and factored nominal (available) capacities using the recommended design method (where Equation 16 is used in lieu of Equation 2, and Equation 10 is used in lieu of Equation 1), is presented in Figure 8. In this figure, the mean ratio is seen to be adequately conservative, and all data points lie above the 45° line.

$$t_c = \sqrt{\frac{4T_c b'}{\phi p F_{up}}} \quad (16)$$

AISI S100

The current study presents an opportunity for the evaluation of the reliability assessment method referred to as Approach 1 in AISI S100, Chapter K (2020), which is intended for the determination of the design strength by experimentation of replicate laboratory specimens. This approach is evaluated via a comparison to the MCS solution using the experimental results of two-sided bolted HSS end-plate connections.

AISI S100 states that the resistance factor shall be computed using Equation 17:

$$\phi = C_\phi M_m F_m P_m e^{-\beta^+ \sqrt{V_M^2 + V_F^2 + C_p V_P^2 + V_Q^2}} \quad (17)$$

where M_m and V_M , F_m and V_F , and P_m and V_P are the mean values and CoVs of the material, fabrication, and professional factor, respectively. The sample size correction factor, C_p , is computed using Equation 18.

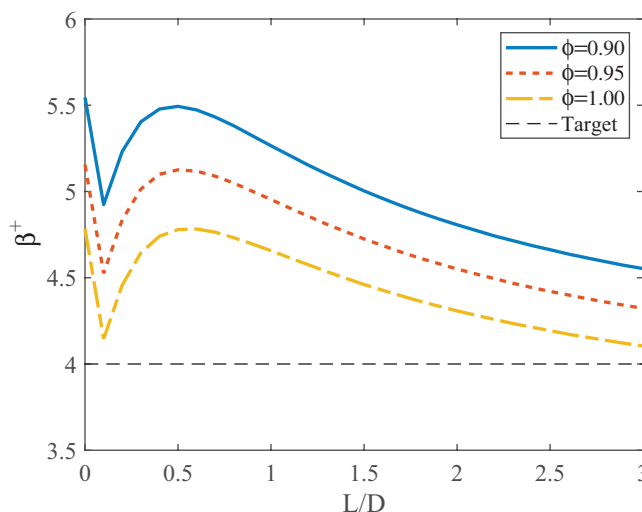


Fig. 6. Reliability index distributions for each trial resistance factor value.

$$C_p = \frac{(1+1/n)m}{(m-2)} \text{ for } n \geq 4 \quad (18)$$

$$= 5.7 \text{ for } n = 3$$

where $m = n - 1$, and n is the number of specimens. Work by Meimand and Schafer (2014) has shown that this reliability

assessment method can be extended to apply for various materials and associated statistical parameters. The AISI S100 method is intended to apply to light-gauge steel structural elements; thus the code is calibrated for $L/D = 5$. The parameter C_ϕ is computed using Equation 19; using load factors from ASCE 7 (2022), the statistical parameters for the dead and live load effect from Ellingwood et al. (1980)

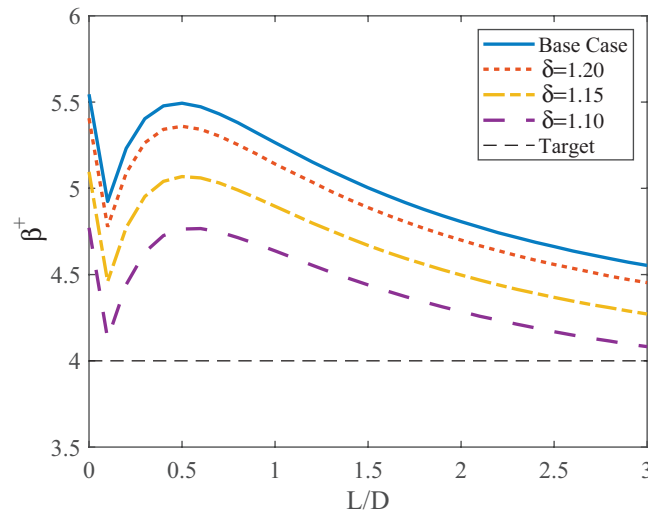


Fig. 7. Sensitivity analysis reliability index distributions for variations in the mean value of the plate and bolt ultimate stresses.

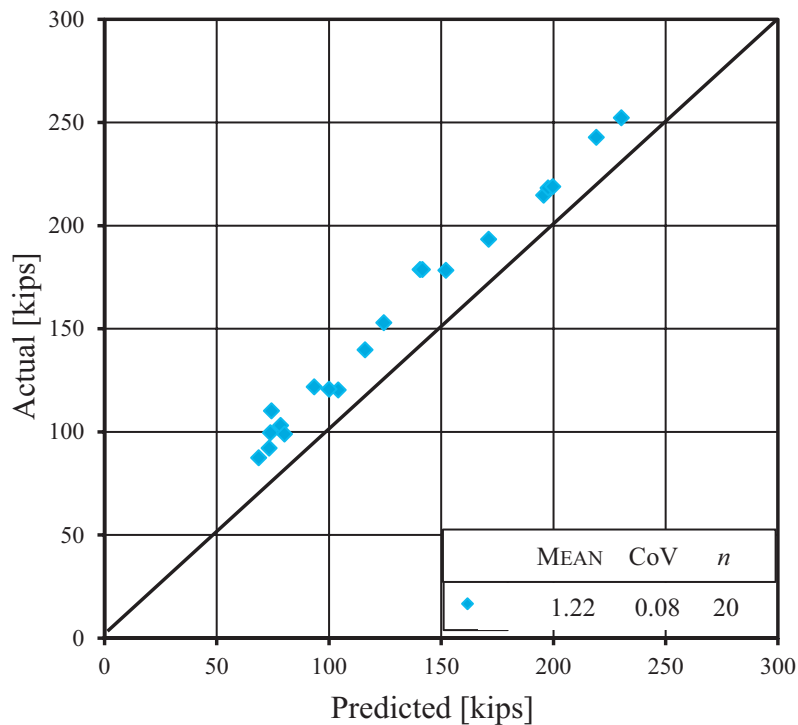


Fig. 8. Comparison of actual-to-factored nominal capacities using the recommended design model.

and $L/D = 5$ produces a value of 1.52, as given in AISI S100 (2020). Recalculating this parameter with $L/D = 3$, the ratio used to calibrate AISC 360, *Specification for Structural Steel Buildings* (2022b), yields a magnitude of 1.48.

$$C_\phi = \frac{\gamma_D + \gamma_L L/D}{B_D + B_L L/D} \quad (19)$$

$$C_\phi = \frac{1.2 + (1.6)(3)}{1.05 + (1)(3)} = 1.48$$

where γ_D and γ_L are the dead and live load factors and B_D and B_L are the dead and live load bias coefficients, respectively. Similarly, the CoV of the load effect can be recomputed, using Equation 20, yielding a value of 0.19 instead of the value of 0.21 given in AISI S100:

$$V_Q = \frac{\sqrt{(B_D V_D)^2 + (B_L V_L L/D)^2}}{B_D + B_L L/D} \quad (20)$$

$$V_Q = \frac{\sqrt{[(1.05)(1)]^2 + [(1)(0.25)(3)]^2}}{1.05 + (1)(3)} = 0.19$$

The mean values and CoVs for the material and fabrication factors are given in AISI S100, Table K2.1.1-1 (2020), but are intended for light-gauge steel elements and connections.

Thus, these parameters can be supplanted by credible documented statistical data. In the case of two-sided bolted HSS end-plate connections, several material and fabrication variables contribute unequally to the nominal strength. However, the partial derivative approach detailed in Kennedy and Gad Aly (1980), which is typically used in such cases, cannot provide a reasonable approximation of material and fabrication factors due to the discretization of the

end-plate thickness. This approach is therefore assessed twice: once using values from Table K2.1.1-1 for bolted connections in AISI S100 (2020) and again using the minimum mean values and maximum CoVs seen in Table 3 for both the material and fabrication factors ($M_m = 1.22$, $V_M = 0.070$, $F_m = 0.975$ and $V_F = 0.025$), which are taken from the literature. AISI recommends the use of target reliability indices of 2.5 and 3.5 for structural members and connections in LRFD, respectively, which is not consistent with ASCE 7 (2022) nor Section B3.1 of the AISC 360 Commentary (2022), where the latter states that the resistance factors presented therein achieve an implied reliability index of 4.0 for connections. Thus, target reliability indices of 3.5, per AISI S100 (2020), and 4.0, per AISC 360 Commentary Section B3.1 (AISC, 2022b) and ASCE 7 (2022), are also used to assess this approach. The results of these analyses are shown in Figure 9.

Figure 9 shows that, as expected, use of a reduced target reliability index ($\beta^+ = 3.5$ vs. $\beta^+ = 4.0$) produces less conservative results for ϕ . Using $\beta^+ = 4.0$, both predictions for ϕ are conservative, though the use of statistical parameters from the literature provides a better approximation to the true (MCS) solution. This is due to the mean values and CoVs given in AISI S100 being very conservative, particularly for the material mean overstrength. The use of generally larger mean values and smaller CoVs, as presented in Table 3, produces higher resistance factors.

Separation Factor Approach

The disassociation of the load and resistance terms of Equation 13 using a separation factor (α_R) forms the basis of separation factor-based approaches (SFA), a group of reliability assessment methods widely used to compute resistance

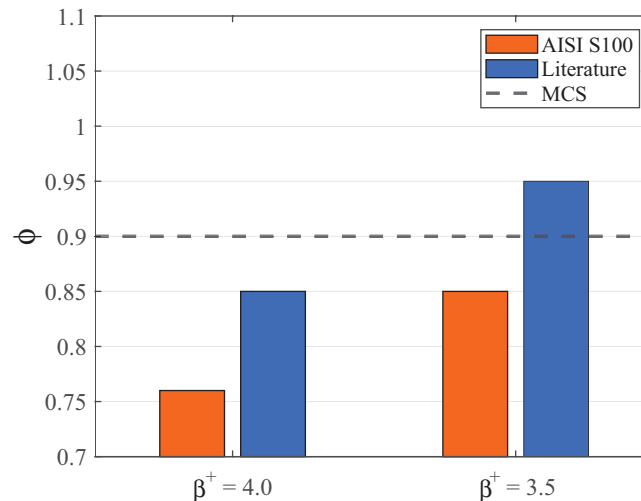


Fig. 9. Comparison of AISI S100 (2020) reliability assessment method against MCS.

factors in North America over the past half century (Lind, 1971; Galambos and Ravindra, 1973). The general form of the method is shown as Equation 21, where the load ratio correction factor (C_R), used to correct the solution when $\beta^+ \neq 3.0$, is computed using Equation 22 (Franchuk et al., 2002).

$$\phi = C_R \rho_R \exp(-\beta^+ \alpha_R V_R) \quad (21)$$

$$C_R = 0.0062\beta^{+2} - 0.131\beta^+ + 1.338 \quad (22)$$

where ρ_R and V_R refer to the bias coefficient and the CoV of the resistance, respectively, originally taken as the mean ratio and CoV of the professional factor. However, recognition that material and fabrication factors can influence the reliability of a nominal expression led to researchers taking the bias coefficient as the product of the mean values of the material, fabrication, and professional factors. Similarly, the CoV of the resistance is typically taken as the square root of the sum of the squares of the three individual CoVs. This principle is reflected in many North American reliability assessment methods (e.g., Equation 17) and is termed the expanded separation factor approach (ESF).

A value of 0.55 for α_R , suggested by Galambos and Ravindra (1973), is most common, though other definitions exist [e.g., ASCE 7 (2022) suggests 0.7, Lind (1971) suggests 0.75]. Jollymore et al. (2025) recently performed a review of the SFA, finding that so long as V_R was reasonably low, different values of α_R best approximate the true solution for unique target reliability indices. Recommendations by Jollymore et al. are summarized in Equation 23, where C_R is not required.

$$\phi = \begin{cases} \rho_R \exp(-0.55\beta^+ V_R; 0.05) \leq V_R \leq 0.30, \beta^+ \approx 3.0 \\ \rho_R \exp(-0.70\beta^+ V_R; 0.10) \leq V_R \leq 0.40, \beta^+ \approx 3.5 \\ \rho_R \exp(-0.80\beta^+ V_R; 0.10) \leq V_R \leq 0.40, \beta^+ \approx 4.0 \end{cases} \quad (23)$$

The experimental results of two-sided bolted HSS end-plate connections are also used to evaluate the Jollymore-corrected SFA via comparison to the true (MCS) resistance factor. Use of a target reliability index of 4.0 corresponds to $\alpha_R = 0.8$, and material and fabrication factors are taken as the minimum mean values and maximum CoVs found in Table 3.

Use of the traditional $\alpha_R = 0.55$ and Equations 21 and 22 leads to a resistance factor of $\phi = 1.00$, whereas the use of $\alpha_R = 0.80$ per Equation 23 with Equation 21 gives $\phi = 0.90$. The use of the larger Jollymore-corrected α_R values in Equation 23 will always lead to more conservative results when $\beta^+ > 3.0$. Figure 6 shows that both of these resistance factor values exceeded the target reliability index across the full range of L/D .

CONCLUSIONS

This paper presents a reformulation of the AISC DG 24 (Packer and Olson, 2024) design method for two-sided bolted rectangular HSS end-plate connections under axial tension, including the use of F_{up} in lieu of F_{yp} . These changes create uniformity across the rectangular HSS bolted end-plate connection design models in DG 24 and also with the prying effect model of the AISC *Manual* (2023). The revised design method is verified through comparison between actual and predicted connection capacities. A reliability study was conducted using a Monte Carlo simulation (MCS) to determine a suitable resistance factor for the design method that met the ASCE 7 (2022) recommended target reliability index of 4.0. It is thus recommended that the computation of the tensile resistance of two-sided bolted HSS end-plate connections follow the current DG 24 design method for four-sided bolted RHS end-plate connections under axial tension loading, with the following changes:

- Parameter b' be computed using Equation 10.
- Parameter t_c be computed using Equation 16 (which employs F_{up}), with the resistance factor of $\phi = 0.90$.
- Parameter p be taken as equal to, or less than, the limit given by Equation 12.

Reliability analyses were also performed using the reliability assessment method given in AISI S100 (2020), as well as a recent reformulation of the separation factor approach. This paper illustrates how the MCS method can be used to evaluate simple closed-form methods for calculating resistance factors. For the particular connection database studied herein, it was found that:

- The AISI S100 reliability assessment method (Equations 17 to 20) can be used to compute resistance factors for non-lightweight materials when adjusted to consider typical structural steel loading ratios. The approach was able to compute an accurate yet conservative resistance factor when statistical parameters are taken from the literature for the material and fabrication factors and the target reliability indices given by Section B of the AISC 360 Commentary and ASCE 7 are used.
- Use of the default statistical parameters and target reliability indices in AISI S100 are poorly suited for the computation of resistance factors for structural steel members and connections.
- The traditional separation factor approach (SFA) for calculating the resistance factor (Equations 21 and 22 with $\alpha_R = 0.55$) produced a result close to the true (MCS) value. On the other hand, the modified SFA approach by Jollymore et al. (2025), using Equation 23 with $\alpha_R = 0.80$ in conjunction with Equation 21, was conservative.

EXAMPLE PROBLEM—DESIGN

Given:

Determine the design plate thickness required for a two-sided bolted HSS end-plate connection to resist a dead load $P_D = 20$ kips and a live load $P_L = 65$ kips in axial tension, using ASTM A500 Grade C HSS6×4× $\frac{5}{16}$ and six $\frac{3}{4}$ -in.-diameter ASTM F3125 Grade A325 bolts. The end plate is ASTM A572 Grade 50 with $w_p = 10$ in., $a = 1.5$ in., and $b = 1.5$ in., as shown in Figure 10.

LRFD Solution:

From AISC *Manual* Table 1-11, the geometric properties for HSS6×4× $\frac{5}{16}$ are:

$$H = 6 \text{ in.}$$

$$t = 0.291 \text{ in.}$$

From AISC *Manual* Table 2-5, the plate ultimate strength for ASTM A572 Grade 50 is:

$$F_{up} = 65 \text{ ksi}$$

Using load factors from ASCE 7 (2022), the required tensile strength is:

$$\begin{aligned} P_u &= 1.2P_D + 1.6P_L \\ &= 1.2(20 \text{ kips}) + 1.6(65 \text{ kips}) \\ &= 128 \text{ kips} \end{aligned}$$

The applied load per bolt, with no prying, can then be computed:

$$\begin{aligned} T_r &= \frac{128 \text{ kips}}{6 \text{ bolts}} \\ &= 21.3 \text{ kips/bolt} \end{aligned}$$

From AISC *Manual* Table 7-2, the available tensile strength per $\frac{3}{4}$ in. ASTM F3125 Grade A325 bolt is:

$$T_c = 29.8 \text{ kips/bolt}$$

Because $T_c \geq T_r$, the design can proceed. Calculate b' using Equation 10:

$$\begin{aligned} b' &= b + t \\ &= 1.5 \text{ in.} + 0.291 \text{ in.} \\ &= 1.79 \text{ in.} \end{aligned} \tag{10}$$

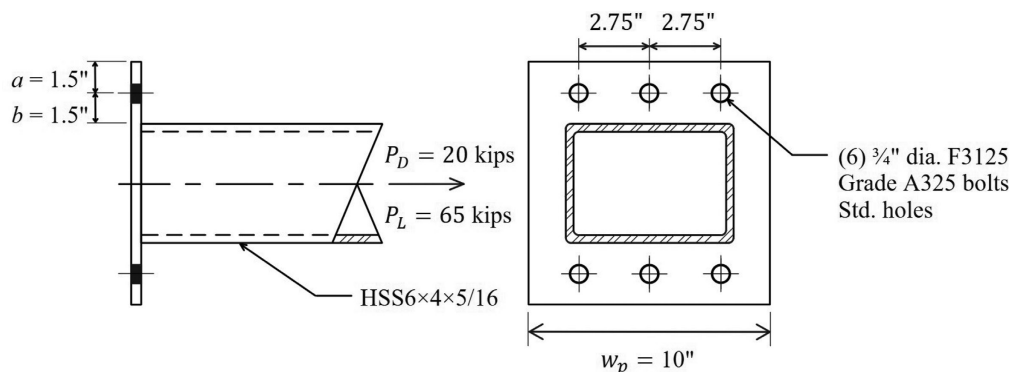


Fig. 10. Design example for a two-sided rectangular HSS end-plate connection.

Calculate the geometric parameters b' , p , ρ , d_h :

$$\begin{aligned} a' &= a \\ &= 1.5 \text{ in.} \end{aligned}$$

$$\begin{aligned} p &= \frac{w_p}{n_b/2} \\ &= \frac{10 \text{ in.}}{6/2} \\ &= 3.33 \text{ in.} \end{aligned}$$

$$\begin{aligned} \rho &= b'/a' \\ &= 1.79 \text{ in.}/1.5 \text{ in.} \\ &= 1.19 \end{aligned}$$

$$\begin{aligned} d_h &= d_b + 1/16 \text{ in.} \\ &= 3/4 \text{ in.} + 1/16 \text{ in.} \\ &= 13/16 \text{ in.} \end{aligned}$$

Check that $p \leq p_{max}$ using Equation 12:

$$\begin{aligned} p &\leq \frac{H + 4 \text{ in.}}{n_b/2} \\ 3.33 \text{ in.} &\leq \frac{6 \text{ in.} + 4 \text{ in.}}{6/2} \\ 3.33 \text{ in.} &\leq 3.33 \text{ in.} \end{aligned} \tag{12}$$

From AISC 360, Section J3.4, a User Note provides the preferred minimum bolt spacing as:

$$\begin{aligned} 3d_b &\leq 2.75 \text{ in.} \\ 2.25 \text{ in.} &\leq 2.75 \text{ in.} \end{aligned}$$

From AISC 360, Table J3.4, the minimum edge distance from hole center for a $3/4$ in. bolt is:

$$1 \text{ in.} \leq 1.5 \text{ in.}$$

Calculate δ :

$$\begin{aligned} \delta &= 1 - \frac{d_h}{p} \\ &= 1 - \frac{13/16 \text{ in.}}{3.33 \text{ in.}} \\ &= 0.756 \end{aligned}$$

Calculate t_c , the plate thickness to avoid prying, using Equation 16:

$$\begin{aligned} t_c &= \sqrt{\frac{4T_c b'}{\phi p F_{up}}} \\ &= \sqrt{\frac{(4)(29.8 \text{ kips})(1.79 \text{ in.})}{(0.90)(3.33 \text{ in.})(65 \text{ ksi})}} \\ &= 1.05 \text{ in.} \end{aligned} \tag{16}$$

Calculate β using Equation 3:

$$\begin{aligned}\beta &= \frac{1}{\rho} \left(\frac{T_c}{T_r} - 1 \right) \\ &= \frac{1}{1.19} \left(\frac{29.8 \text{ kips}}{21.3 \text{ kips}} - 1 \right) \\ &= 0.332\end{aligned}\tag{3}$$

Calculate α^* using Equation 4:

$$\begin{aligned}\alpha^* &= \min \left[\frac{1}{\delta} \left(\frac{\beta}{1-\beta} \right), 1 \right] \\ &= \min \left[\frac{1}{0.756} \left(\frac{0.332}{1-0.332} \right), 1 \right] \\ &= \min[0.659, 1] \\ &= 0.659\end{aligned}\tag{4}$$

Calculate t_p , the required plate thickness, using Equation 5:

$$\begin{aligned}t_p &= t_c \sqrt{\frac{T_r}{T_c} \left(\frac{1}{1 + \delta \alpha^*} \right)} \\ &= (1.05 \text{ in.}) \sqrt{\frac{21.3 \text{ kips}}{29.8 \text{ kips}} \left[\frac{1}{1 + (0.756)(0.659)} \right]} \\ &= 0.724 \text{ in.}\end{aligned}\tag{5}$$

Use a $\frac{3}{4}$ in. ASTM A572 Grade 50 end plate.

EXAMPLE PROBLEM—ANALYSIS

Given:

Find the available tensile strength of the connection presented in the previous example, with an end-plate thickness of $\frac{3}{4}$ in.

Solution:

Calculate α' using Equation 6:

$$\begin{aligned}\alpha' &= \frac{1}{\delta(1+\rho)} \left[\left(\frac{t_c}{t_p} \right)^2 - 1 \right] \\ &= \frac{1}{0.756(1+1.19)} \left[\left(\frac{1.05 \text{ in.}}{0.75 \text{ in.}} \right)^2 - 1 \right] \\ &= 0.572\end{aligned}\tag{6}$$

Calculate Q using Equation 7, for $0 < \alpha' \leq 1$:

$$\begin{aligned}Q &= \left(\frac{t_p}{t_c} \right)^2 (1 + \delta \alpha') \\ &= \left(\frac{0.75 \text{ in.}}{1.05 \text{ in.}} \right)^2 [1 + (0.756)(0.572)] \\ &= 0.735\end{aligned}\tag{7}$$

Calculate $T_{c,adj}$ using Equation 8:

$$\begin{aligned} T_{c,adj} &= T_c Q \\ &= (29.8 \text{ kips})(0.735) \\ &= 21.9 \text{ kips} \end{aligned} \quad (8)$$

Calculate T_a , the connection available tensile strength, using Equation 9:

$$\begin{aligned} T_a &= T_{c,adj} n_b \\ &= (21.9 \text{ kips})(6) \\ &= 132 \text{ kips} \end{aligned} \quad (9)$$

Because $T_a = 132 \text{ kips} > P_u = 128 \text{ kips}$, the design is satisfactory.

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SYMBOLS

B_D	Bias factor of dead load
B_L	Bias factor of live load
C_R	Load ratio correction factor
C_ϕ	Calibration coefficient
D	Dead load effect
F_m	Mean value of fabrication factor
F_{ub}	Bolt ultimate stress (ksi)
F_{up}	Plate ultimate stress (ksi)
F_{yp}	Plate yield stress (ksi)
G	Safety margin
$\overline{G}$	Mean value of the safety margin
H	HSS dimension parallel to bolt lines (in.)
L	Live load effect
M_m	Mean value of material factor
P_D	Tensile dead load (kips)
P_L	Tensile live load (kips)
P_m	Mean value of professional factor
P_u	Required tensile strength (kips)
Q	Prying adjustment factor

R	Resistance
R_n	Nominal resistance
S	Load effect
$S_{n,i}$	Nominal load effect
T_a	Available tensile strength of connection (kips)
T_c	Available bolt tensile strength (kips)
$T_{c,adj}$	Available bolt tensile strength, adjusted for prying effects (kips)
T_{nu}	Predicted tensile strength of connection using the plate ultimate stress (kips)
T_{nx}	Measured tensile strength of connection from experimentation (kips)
T_{ny}	Predicted tensile strength of connection using the plate yield stress (kips)
T_r	Applied load per bolt, with no prying (kips)
V_D	Coefficient of variation of dead load
V_F	Coefficient of variation of fabrication factor
V_L	Coefficient of variation of live load
V_M	Coefficient of variation of material factor
V_P	Coefficient of variation of professional factor
V_R	Coefficient of variation of resistance
V_Q	Coefficient of variation of load effect
a	Plate edge to bolt centerline distance (in.)
a'	Plate edge to sagging yield line distance (in.)
b	Bolt centerline to HSS face distance (in.)
b'	Bolt centerline to hogging yield line distance (in.)
d_b	Bolt shank diameter (in.)

d_h	Bolt hole diameter (in.)
n	Number of specimens
n_b	Number of bolts
p	Bolt tributary width (in.), probability
p_F	Probability of failure
p_{max}	Maximum allowable bolt tributary width (in.)
t	HSS thickness (in.)
t_c	End-plate thickness to avoid prying (in.)
t_p	End-plate thickness (in.)
w_p	End-plate width (in.)
α^*	Flexibility factor
α'	Design control factor
α_R	Separation factor
β	Prying ratio
β^+	Reliability index
γ_D	Dead load factor
γ_i	Constituent load factor
γ_L	Live load factor
δ	Prying effect factor
ρ	Yield line ratio
ρ_R	Bias coefficient of resistance
σ_G	Standard deviation of the safety margin
ϕ	Resistance factor

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