Homogeneous and Hybrid Girder Design in the 1969 AISC Specification

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THE RULES for design of plate girders in the 1969 AISC Specification are basically unchanged from the 1963 edition, except that a highly significant extension has been made to provide for the design of hybrid beams. In addition, Section 1.10 has been editorially revised to increase clarity of intent. The principal formulas governing girder design are unchanged, but the designer now has the option of achieving an economical design by either of two concepts: (1) tension field action in the thin web of a homogeneous girder, or (2) hybrid design using a low strength steel web plate welded to high strength steel flanges.

BENDING

In the design of plate girders by either concept, they may be proportioned for bending on the basis of the moment of inertia of the gross cross section. In both cases, however, a reduced allowable bending stress, different for each concept, is necessary.

Homogeneous Girders—The case of a homogeneous thin web girder in which the compression portion of the web is on the verge of buckling as a column due to axial bending stresses is represented in Fig. 1. The upper diagram, Sect. AA, represents the lateral deformations resulting from bending compression stresses about the neutral axis of a relatively thin girder web. Below the neutral axis, tension stresses due to bending tend to hold the web straight. For a short distance down from the top flange—say about 30 times its thickness—the web plate is restrained from buckling by the flange.

The effect of the displacement is to somewhat reduce the bending resistance furnished by the compression portion of the web. This reduction is represented by the area **OC′C** in the stress diagram (Sect. BB). The area to which this reduction applies is only a small part of the total area of the web and is relatively close to the neutral axis; thus its effect is small. A much smaller area, located in the compression flange, can furnish as much bending resistance as that lost by buckling in the web.

It is convenient, in designing the compression flange, to provide for this web deficiency by the use of an adjusted allowable bending stress, F'_b, slightly lower than that which would otherwise be permitted. Formula (1.10-5) gives the reduced allowable stress in terms of F_b, the ratio of the web area to flange area, and the depth-to-thickness ratio of the web:

$$F'_b \leqslant F_b \left[1.0 - 0.0005 \frac{A_w}{A_f}\left(\frac{h}{t} - \frac{760}{\sqrt{F_b}}\right)\right] \quad (1.10\text{-}5)$$

In most cases, the reduction in allowable flange stress is only a few percent. No reduction is required when the web depth-to-thickness ratio is less than 162 for A36 material. When the web depth-to-thickness ratio does exceed 162, Formula (1.10-5) reduces to:

$$F'_b \leq 22 - 0.011 \frac{A_w}{A_f}\left(\frac{h}{t} - 162\right)$$

Hybrid Girders—When high strength flanges are welded to a lower strength web, the member is termed a hybrid girder. During the initial stages of loading (Fig. 2a), strains and stresses will be proportional to the distance from the neutral axis. As loads are increased, eventually stresses in the web adjacent to the flanges will reach the yield strength of the web material (Fig. 2b).

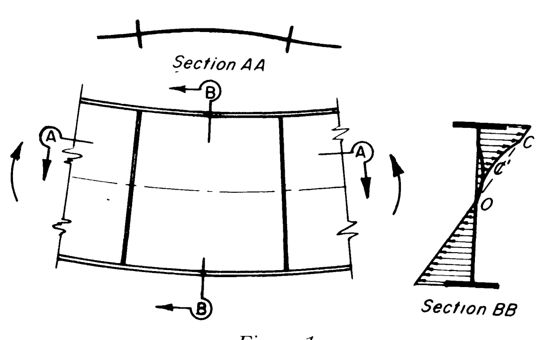

Figure 1

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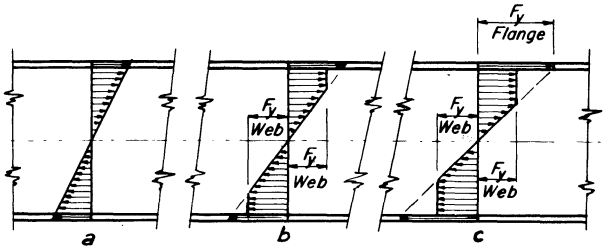

Figure 2

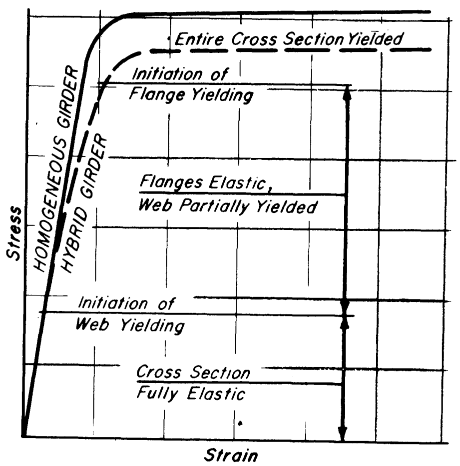

Figure 3

Since the flange stresses will still be well below the yield strength of the flange material, web strains will be effectively controlled. Further increases in load are safely possible up to the point where stresses in the flanges reach the yield strength of the flange material (Fig. 2c).

Figure 3 is a plot of the same action as parallel stress-strain diagrams for a hybrid beam vs. a homogeneous girder *made of the same grade of material as the flange of the hybrid beam* and with a web of such proportions that tension field action is not counted upon. Up to the point where initial yielding occurs in the lower strength web material of the hybrid beam, the two curves are identical. As the loading is increased the curve for the homogeneous girder will continue to be linear; however, the curve for the hybrid beam will become slightly nonlinear as yielding penetrates further and further into the web. The difference between the two curves will be small, since the web contributes only a small part of the stiffness. Eventually, as the load is increased still further, the stress in the flange will reach the yield strength of the flange material and large deflections will take place. The load at this level is well above the working load, which would be at about ⅔ of the initial flange yielding.

At all levels of load, the stresses and deflections are completely predictable; therefore, hybrid design is a valid engineering concept. As with a thin web girder designed on the basis of tension field action, the allowable stress in the flanges must be adjusted to a slightly lower value than that which would be permitted if the entire girder were made of the high strength material. Formula (1.10-6) gives F'_b in terms of F_b, the ratio of the web area to the area of one flange and the ratio of the web yield strength to the flange yield strength, α.

$$F'_b \leq F_b \left[\frac{12 + \left(\frac{A_w}{A_f}\right)(3\alpha - \alpha^3)}{12 + 2\left(\frac{A_w}{A_f}\right)} \right] \qquad (1.10\text{-}6)$$

In girders of usual proportions using 100 ksi material in the flanges and 36 ksi material in the web, this formula will indicate a reduction in allowable flange stress of about 5 to 10 percent.

SHEAR

In the design of bending members, two types of structural action are recognized as providing resistance to shearing forces. Up to the level of loading where the combined effect of the axial compressive bending stress in the web and the shear stress produce elastic buckling, the shear forces are resisted by conventional beam shear. It was long thought that the onset of buckling determined the limit of strength; however, buckling of a girder web is not significant as far as the strength of a girder is concerned. Subsequent to web buckling the girder will continue to carry increasingly larger loads by virtue of tension field action, provided adequate transverse stiffeners are installed. The action may be likened to the action of a Pratt truss in which the forces in the tension diagonals resist the shear forces. In a girder with transverse stiffeners, tension stresses diagonally across each web panel continue to resist shear forces after web buckling occurs, up to the point where tensile yielding takes place. In girders utilizing tension field action, anchorage is required for the tension field near the ends and in panels adjacent to panels with large web openings. The Specification requires that shear stress in such panels be limited to that provided by beam shear only—Formula (1.10-1) governs.

Homogeneous Girders—The concept of tension field action to provide shear resistance in a bending member has been basic in the design of aircraft since metal replaced fabric covering. In building construction, the concept has been recognized since it was introduced into the 1961 AISC Specification. Extensive analytical research, confirmed by numerous full scale tests, was conducted as the basis for the 1961 rules. No adverse performance in members designed for tension field action has been noted since the provisions were adopted.

It is not simple to visualize the complex actions that take place at all stages of loading in a girder web subject to shear forces, but if the problem is considered in parts it can be clarified somewhat.

First, failure of a girder can occur by general yielding of the web in shear. Von Mises' yield-stress criterion predicts the value of yield stress in shear to be $F_y/\sqrt{3}$; thus, one limit for allowable shear stress would be:

$$F_v \leq \frac{F_y}{\sqrt{3}} \times \frac{1}{\text{Safety Factor}}$$

In the Specification this is reduced to $F_v \leq 0.40\ F_y$.

Next, according to plate buckling theory, the critical stress at which buckling will take place in a plate subject to shearing forces is:

$$f_{cr} = \frac{\pi^2 E \times k}{12(1 - \nu^2)(h/t)^2}$$

It is seen that f_{cr} is inversely proportional to the square of h/t. As the slenderness of the web increases, the critical stress at which elastic buckling will take place is sharply reduced. The critical stress is also directly proportional to k, which reduces as the ratio of the stiffener spacing, a/h, increases. The effects of these two geometrical parameters, h/t and a/h, according to AISC Specification Formula (1.10-1), $F_v \leq F_y C_v/2.89$, is provided for in the term C_v. Graphically their effect is reflected by the dashed curves of Fig. 4. Since the buckling formula is for critical *elastic* buckling, account has been taken of the effect of residual stresses due to welding as reflected by the slight sharp changes in slope at upper stress levels. Notice that as h/t increases, the allowable F_v is sharply reduced. As a/h is reduced, the effect is essentially to shift the curves to the right. The upper cut-off for all curves is at $0.4\ F_y$.

Formula (1.10-1) and the dashed curves predict the level of stress at which shear buckling will occur. They ignore the very large reserve of shear capacity due to membrane action that may be counted upon.

If a term is added to take account of the contribution of the tension field, Formula (1.10-2) becomes:

$$F_v \leq \frac{F_y}{2.89}\left[C_v + \frac{1 - C_v}{1.15\sqrt{1 + (a/h)^2}}\right]$$

One can recognize $F_y/2.89$ times C_v, the first term within the brackets, as the value of F_v in Formula (1.10-1). The second term within the brackets times $F_y/2.89$ is the contribution due to tension field action. The solid curves of Fig. 4 indicate the allowable shear stress in a girder web, taking into account the combined effect of beam shear and membrane action, and the geometrical parameters. Notice that tension field action has the effect of flattening the slope of the curves and that the effect is greatest for small values of a/h.

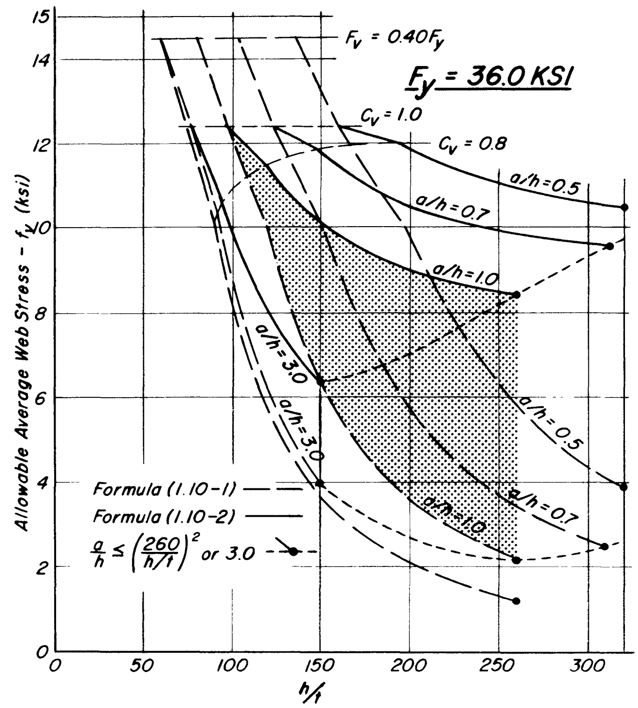

Figure 4

Note in particular the shaded area. This area indicates the increase in allowable shear stress gained by means of tension field action, for the case of $a/h = 1$.

The dotted curves indicate arbitrary limitations on a/h in terms of h/t, not concerned with the strength of the girder, imposed by the Specification to facilitate handling during fabrication and erection.

Hybrid Girders—For homogeneous girders, the allowable shear is limited by general shear yielding in stocky webs and beam shear plus tension field action when the webs are of such proportions that buckling can occur. This is shown by the upper portion of the dashed curves plus the solid curves. Recall now the explanation of the concept of hybrid beam design and consider the resistance to shear provided by the web. Once significant yielding due to axial bending strains has taken place, tension field action to resist shear forces cannot be counted upon. Hybrid girders must rely upon conventional beam shear in the web, Formula (1.10-1), represented by the dashed curves.

Three other limitations on the applicability of hybrid girder designs under the 1969 Specification deserve mention:

1. The concurrent axial load must be less than $0.15F_y$ times the gross area.
2. Flanges must be of equal area.
3. Flanges must be of the same grade of steel.

(Composite hybrid design is not recognized.)

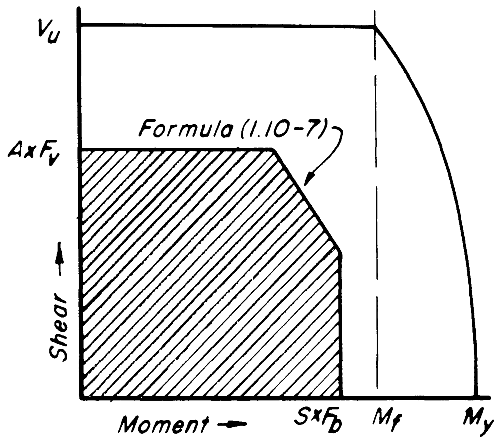

Figure 5

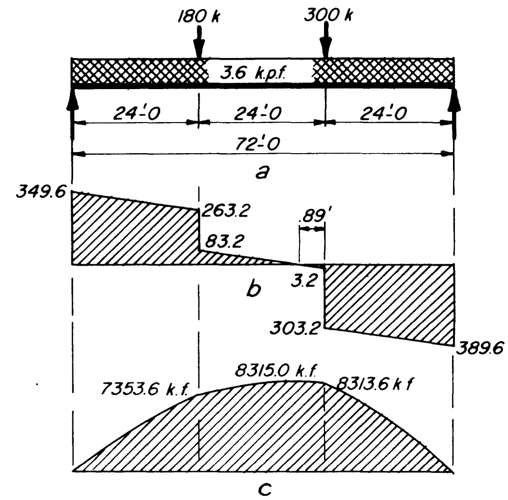

Figure 6

COMBINED BENDING AND SHEAR

At large concentrated loads and at points of interior support for continuous beams, high moment and high shear may occur simultaneously. The possibility of failure of the web due to combined effect of bending tensile stress and tensile stress due to membrane action must be considered.

Shown in Fig. 5 is an interaction diagram for shear and moment applicable to a homogeneous girder. As long as the moment is below the value which can be carried by the flange alone, the ability of the web to resist shear forces is not diminished by the presence of the axial bending stresses in the web. If, on the other hand, the moment is larger than the moment which the flanges can resist unassisted by the web, the web bending tensile stresses reduce the ability of the web to resist shear. The relationship of M_f to M_y will vary, depending upon the geometry of the cross section. At about the economic practical limit of such girders, M_f/M_y can reduce to about 0.75. For girders of usual proportions, M_f/M_y will be larger. It is sufficiently accurate for design purposes to approximate this curve for the limit of practical proportions by straight lines and apply appropriate factors of safety. This has been done in Specification Formula (1.10-7).

In proportioning a girder for real cases in which high moment and high shear occur simultaneously, rework of calculations will be minimized if the combined effect is anticipated and conservative values are assumed for shear and moment stresses taken separately. This will be illustrated in the examples to follow.

A number of other Specification provisions covering the area of stiffeners, the stiffness of stiffeners, web crippling, etc., are similar for both homogeneous and hybrid girders and are unchanged since the 1961 Specification.

EXAMPLES

Two examples will highlight the differences between the tension-field action and hybrid girder design concepts.

Example 1: Assume a girder on a 72-ft span that is required to support a distributed load of 3.6 kips/lin ft and two unequal concentrated loads delivered by 12-in. columns at the one-third points of the span (Fig. 6a). The shear and moment diagrams will be as shown in Figs. 6b and 6c. A homogeneous girder of A36 material will be developed to satisfy these conditions.

The required section modulus is determined in the usual manner as:

$$S = 8315 \times 12/22 = 4{,}585 \text{ in.}^3$$

The AISC Manual, page 2-84, gives the section properties of typical economically proportioned girders; select a girder cross section with 24 × 2½-in. flanges and a 72 × ½-in. web plate. From Manual pages 2-67 and 2-71, calculate the section modulus as 4,730 in.^{3} for the exact dimensions chosen. Since the flanges would probably require splicing, due to the span length, try a 1¼-in. thick flange at each end; this provides a section modulus of 2,577 in.^{3} The maximum moment capacity for the section with the thinner flanges determines the flange splice point; however, the maximum moment capacity is dependent upon the reduced allowable bending stress determined by consideration of combined moment and shear stresses, to be considered later.

Next, check shear stresses for the web plate selected:

At right reaction:	$f_v = 390/72 \times 1/2 = 10.8$ ksi
At left reaction:	$f_v = 350/36 = 9.72$ ksi
In center 1/3:	$f_v = 84/36 = 2.32$ ksi

Table 1

ALLOWABLE SHEAR STRESS (F_v) IN PLATE GIRDER WEBS
TENSION FIELD ACTION NEGLECTED - FORMULA (1.10-1)
Specified yield stress - 36 ksi

Slenderness Ratio - h/t	Aspect Ratio - a/h													
	0.5	0.6	0.7	0.8	0.9	1.0	1.2	1.4	1.6	1.8	2.0	2.5	3.0	∞
60											14.5	14.5	14.5	14.5
70									14.5	14.5	14.2	13.8	13.6	13.0
80						14.5	14.1	13.4	13.0	12.6	12.4	12.0	11.9	11.4
90				14.5	14.3	13.4	12.5	11.9	11.5	11.2	11.0	10.7	10.5	10.1
100			14.5	13.9	12.8	12.0	11.2	10.7	10.4	10.1	9.9	9.3	9.0	8.3
110		14.5	13.8	12.6	11.7	10.9	11.2	9.5	8.9	8.4	8.2	7.7	7.5	6.9
120		14.3	12.7	11.6	10.7	10.0	8.8	8.0	7.5	7.1	6.9	6.5	6.3	5.8
130	14.5	13.2	11.7	10.7	9.8	8.6	7.5	6.8	6.4	6.0	5.8	5.5	5.3	4.9
140	14.2	12.2	10.9	9.8	8.4	7.4	6.5	5.9	5.5	5.2	5.0	4.7	4.6	4.2
150	13.2	11.4	10.1	8.5	7.3	6.5	5.6	5.1	4.8	4.5	4.4	4.1	4.0	3.7
160	12.4	10.7	9.1	7.5	6.4	5.7	4.9	4.5	4.2	4.0	3.9	3.6		3.2
170	11.7	10.1	8.0	6.7	5.7	5.0	4.4	4.0	3.7	3.5	3.4			2.9
180	11.0	9.0	7.2	5.9	5.1	4.5	3.9	3.5	3.3	3.1	3.0			2.6
200	9.9	7.3	5.8	4.8	4.1	3.6	3.2	2.9	2.7					2.1
220	8.2	6.1	4.8	4.0	3.4	3.0	2.6	2.4						1.7
240	6.8	5.1	4.0	3.3	2.9	2.5								1.4
260	5.8	4.3	3.4	2.9	2.4	2.1								1.2
280	5.0	3.7	3.0	2.5										
300	4.4	3.3	2.6											
320	3.9	2.9												

For the center $\frac{1}{3}$, referring to Table 3-36 for $h/t = 144$ and $a/h > 3$ ($24/6 = 4$), the allowable shear stress is found to be 4.2 ksi. Since this is larger than the 2.32 ksi required, the web is adequate without stiffeners. The allowable shear stress indicated in Table 3-36 as 4.2 ksi results from Formula (1.10-1) with a/h taken as infinity.

At the right reaction, where the required shear capacity is 10.8 ksi, the first panel must provide an anchor for the adjacent panels which will rely upon tension field action. Formula (1.10-1) must be used as the criteria for allowable stress. The Specification suggests that, to avoid tension field action in end panels or panels adjacent to panels containing large holes, the smaller dimension a or h must not exceed $348t\sqrt{f_v}$; however, the author prefers to use Formula (1.10-1) directly. This may be readily done by simple interpolation in Table 1, which could be added to the Specification Appendix as Table 3-36A. Whereas Table 3-36 in the Specification gives the maximum allowable web shear permitted by Formula (1.10-1) or (1.10-2), as applicable, when tension field action may be counted upon, Table 1 gives values of allowable shear by Formula (1.10-1) only at various values of a/h and h/t. For h/t equal to 144 and a required allowable shear of 10.8 ksi, interpolation in the table indicates the maximum value of a/h to be 0.68. Multiplying 0.68 by the girder depth, 72 in., indicates that the first stiffener should be located not more than 49 in. from the end bearing. Use 48 in.

Table 2

h/t \ a/h	0.7	0.8	0.9	1.0	1.2
140	12.0	11.6	11.0	10.5	9.8
144	11.9	11.4	10.8	10.3	9.6
150	11.8	11.2	10.6	10.1	9.4

With anchorage provided by the end panel, the remainder of the panels between the right reaction and the concentrated load may take advantage of tension field action. Table 3-36 provides a convenient means for determining the stiffener spacing by interpolation, although in this case the interpolation turns out to be trivial. Table 2 is provided only for the purpose of illustration.

By plotting the interpolated values of allowable F_v vs. the panel length a on a parallel plot of required f_v vs. x, the distance from the reaction, using the same ordinate scale for both curves, i.e., a curve of allowable F_v vs. the shear diagram, a very convenient tool for determining stiffener spacing may be constructed (see Fig. 7).

Without such an aid it is necessary to assume an a/h, calculate the maximum shear in the panel for different positions along the span, and then compare the calculated shear with the allowable shear from Table 3-36. A number of trials are usually required.

To use the graphical aid, begin at 48 in. on the lower abscissa scale (the length of the end panel previously determined) and move vertically to the calculated shear curve for the specific problem. Move horizontally to the curve for allowable F_v; then move vertically downward to the upper abscissa scale and read the maximum size panel for the required shear capacity.

For the succeeding panel add the first and second panel lengths and repeat the cycle. The minimum number of web panels can be determined in minutes. In this example, the chart (Fig. 7) plus some arbitrary adjustment to cause the total of all panels to equal the length of the shear span indicates that panels of 48, 66, 78 and 90 in. would provide conservative stresses with the minimum number of stiffeners required.

This type of diagram also provides a convenient tool for investigating the maximum allowable bending tensile stress in the presence of high shear by Formula (1.10-7).

In the panel adjacent to the right concentrated load, which is most likely to be the critical panel, f_v varies from 8.4 to 9.2 ksi and $F_v = 9.5$ ksi. This would appear to be conservative; however, if these values are substituted in Formula (1.10-7), it will be discovered that the allowable bending stress in combination with the shear is reduced from 22 ksi to 16.6 ksi. Calculated bending stress at this point is 21 ksi. Therefore a revision of the initial assumed design is required.

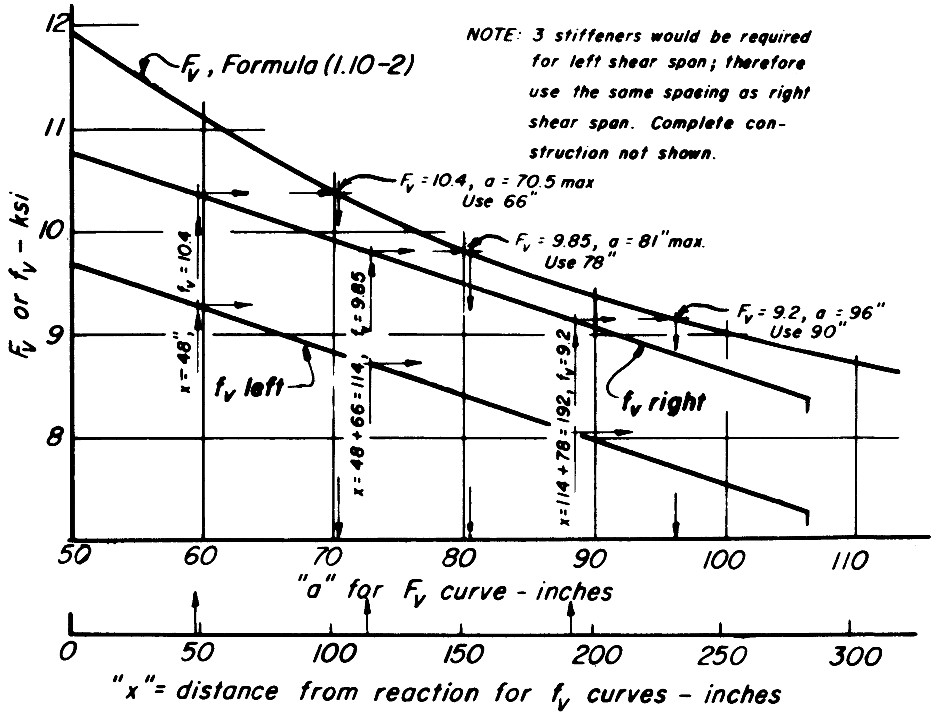

Figure 7

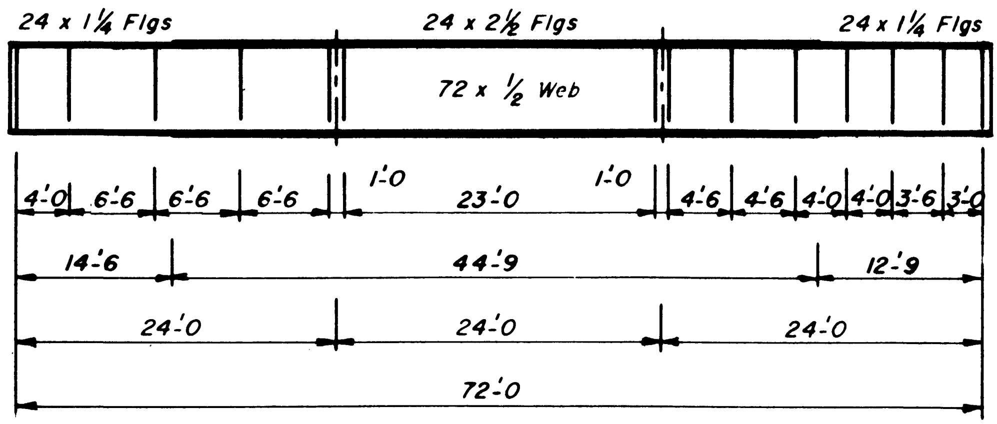

Figure 8

In the panel adjacent to left concentrated load:

$$f_v = 8.0 \quad ; \quad F_v = 9.5$$

$$F_b \leq (0.825 - 0.375 \times 8.0/9.5)36 = 18.2 \text{ ksi}$$

$$f_b = 7{,}354 \times 12/4{,}730 = 18.6 \text{ ksi} > 18.2 \text{ ksi}$$

A check of Formula (1.10-7), using the maximum shear at the left end of the panel and the maximum moment at the right end of the same panel, indicates that there would be a 2 percent bending overstress. This overstress may be ignored for several reasons: (1) Formula (1.10-7) was developed on the assumption of uniform moment and uniform shear throughout the panel; (2) Formula (1.10-7) is conservative and based upon the lowest limits for economically proportioned cross sections; and (3) a 2 percent overstress is so small that it can frequently be ignored. Even the small overstress could be completely eliminated by establishing a 6′-6″ length for all tension field panels in the left shear span.

The moment capacity or the shear capacity must be increased in the right shear span. A little study of Formula (1.10-7) and the diagram for allowable shear (Fig. 4) will indicate that an adequate increase in shear capacity would move the web stress into the region where tension field action is not relied upon; hence, Formula (1.10-7) would cease to be applicable.

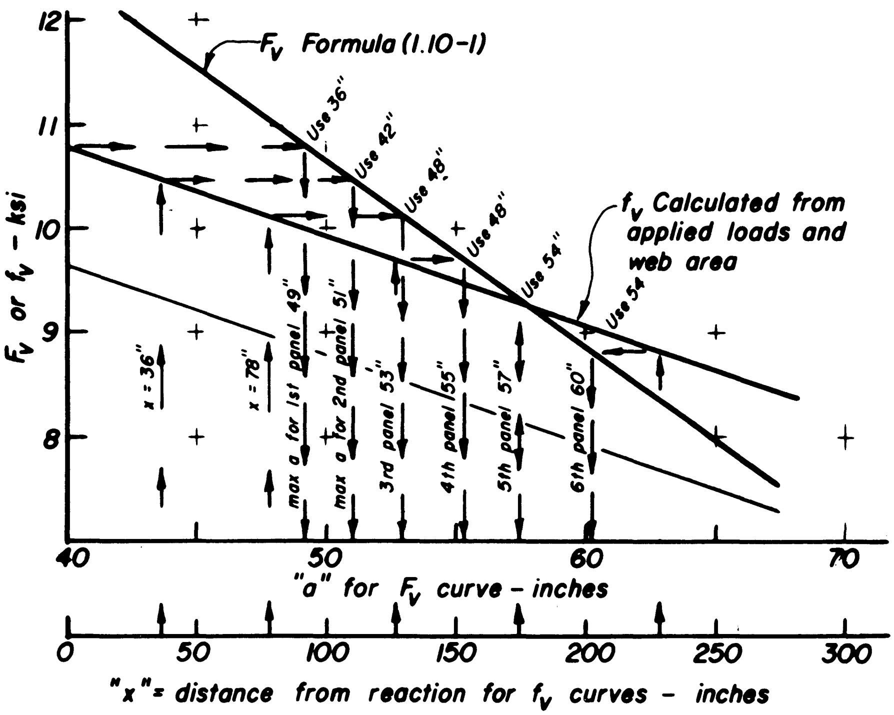

Figure 9

Note that a substantial change in the ratio f_v/F_v would be required, since it is multiplied by 0.375 to make a significant improvement in allowable bending stress.

Note also that the intersection of h/t of 144 and a calculated shear of between 9 and 10 falls in the narrow upper part of the curves for a/h slightly larger than 1.0 (90/72). A shift to a thicker web by moving horizontally to the left, or to a smaller a/h by moving vertically upward, will accomplish the required change to values based upon Formula (1.10-1).

The required design modification may be accomplished by use of a $^{13}\!/_{16}$-in. web plate with stiffeners spaced at about 6′-0″ centers (1,840 lbs of additional web material required) or by reducing the aspect ratio to 0.75 without increasing web thickness, requiring two additional stiffeners. Alternatively, the moment capacity could be increased by increasing the flange thickness to 3 in. (requiring about 4,080 lbs of additional material).

The alternative of reducing the aspect ratio of the web panels to 0.75 in the right shear span is selected.

The stiffener area required to carry the vertical component of the tension fields would be determined from the values shown in italics in Table 3-36.

Clean up checks on web crippling, flange stress reduction, heavy flange to light flange splice points, etc. would indicate that a final girder design such as shown in Fig. 8 is satisfactory.

Example 2: A girder to satisfy the same load conditions (Fig. 6) may be designed by use of hybrid design concept with A514 flanges and A36 web material.

The required section modulus is determined in the usual manner on the basis of the high strength flange material and the gross moment of inertia

$$S = 8{,}315 \times 12/60 = 1{,}663 \text{ in.}^3$$

A reduction in flange stress will be required by Formula (1.10-6); therefore, the section modulus will be arbitrarily increased 10 percent to 1,830 in.³ to minimize recalculation effort.

A trial section using 20 × 1-in. A514 flanges with a 72 × ½-in. web, providing a section modulus of 1,860 in.³, is selected.

The reduced allowable flange stress by Formula (1.10-6) is calculated as follows:

$$\alpha = 36/100 = 0.36$$

$$F'_b = F_b \left[\frac{12 + 36/20(1.08 - 0.36^2)}{12 + 2(36/20)}\right]$$

$$F'_b = 0.89 \times F_b = 53.4 \text{ ksi}$$

$$f_b = 8{,}315 \times 12/1{,}860 = 53.6 \text{ ksi} \approx 53.4 \text{ ksi}$$

In the design of hybrid girder webs, tension field action may not be used; therefore, only Formula (1.10-1) applies. Table 1 provides values of allowable shearing stress for this case.

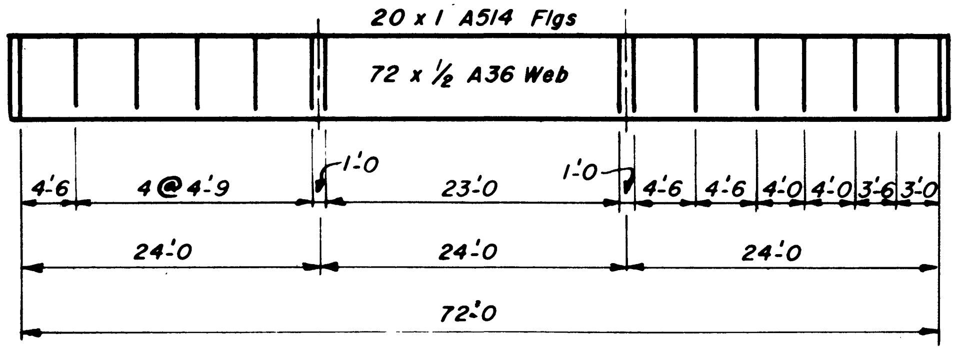

Figure 10

As in Example 1, the spacing of stiffeners may be readily determined by plotting parallel curves of allowable F_v vs. the shear diagram to the same ordinate scale (Fig. 9). Arbitrary adjustments of the web panel sizes, based upon judgment, are required, since the maximum panel lengths for equality between allowable shear and calculated shear result in a total of all panels that exceed the length of the shear spans. However, the minimum number of stiffeners is provided if panel lengths of 36, 42, 48, 48, 54 and 54 in. are selected for the right shear span and 54, 57, 57, 57, and 57 in. are selected for the left shear span.

The final arrangement shown in Fig. 10 will satisfy the load conditions.

SUMMARY

The rules for design of homogeneous girders in the AISC Specification, Section 1.10, are unchanged from the 1963 Specification, except for editorial revisions and rearrangements to improve clarity of intent. The significant difference in Section 1.10 is that rules for the design of symmetrical hybrid members have been added.

On the basis of hypothetical design examples, the following observations may be made:

Both homogeneous and hybrid girders may be proportioned for bending on the basis of the gross section properties.

The allowable bending stress in the compression flange of a homogeneous girder which relies upon tension field action must be investigated and perhaps reduced, because of the reduction of the effectiveness of the compression portion of the web to resist bending compression stress. The required reduction amounts to only a few percent.

The allowable bending stress for both high strength flanges of a hybrid girder must be reduced because of the controlled yielding of the lower strength web. The reduction is about 7 percent for girders of the usual proportions using 100 ksi material in the flanges and 36 ksi material in the web.

Tension field action may not be counted upon in hybrid beam design.

Under the 1969 Specification, hybrid girders must have flanges of equal area and made of the same yield strength material.

In the case of homogeneous girders which rely upon tension field action, the effect of high combined bending and shear may require significant modification of a design proportioned on the basis of bending and shear considered separately.

There are significant opportunities for economy in the hybrid beam concept. For the examples compared in this paper, the material in the web plate was the same for both designs. The homogeneous girder required eight intermediate stiffeners, whereas the hybrid girder required nine intermediate stiffeners. The homogeneous girder required 24,900 lbs of A36 material for the flanges plus four flange splices; the hybrid girder required 11,700 lbs of A514 material for the flanges plus two flange splices.

Only on rare occasions would it seem logical to design a girder using high strength material throughout.