Analysis of Curved Girder Bridges

CHARLES CULVER, DARRYL BROGAN, AND DAVID BEDNAR

GEOMETRIC, aesthetic, and economic considerations have led to the increased use of horizontally curved girders for highway bridges and interchange facilities which involve curved alignment. Despite the slightly higher fabrication costs associated with curving the girders, the net costs for such bridges, in certain cases, are lower than those for curved bridges with straight beams placed along the chords of the curved roadway. This overall economy is a result of the elimination of many substructure units (piers) and simplified form work necessary for the roadway slab.

Since curved girders may be classified as linear elastic structures, all the various methods of linear structural analysis may be applied to the design of curved bridges. Although considerable analytical work has been done in this area,[1] no attempt has been made to correlate the various methods of analysis or evaluate the degree of approximation inherent in several of the design procedures.[2]

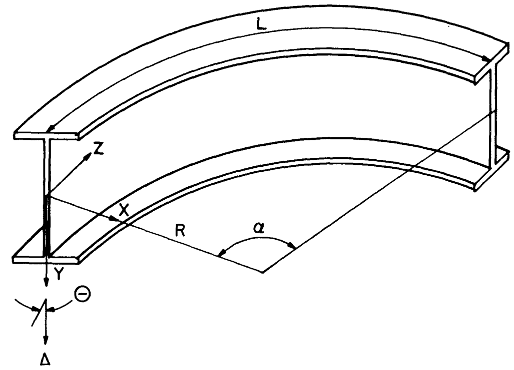
The purpose of this paper is twofold. First, the fundamental differences between the behavior of individual straight girders and curved girders loaded normal to the plane of curvature will be established. Second, numerical results obtained from an approximate method of analysis widely used in the design of curved bridges[3] will be compared with a rigorous analysis of curved grid systems.

BEHAVIOR OF INDIVIDUAL GIRDERS

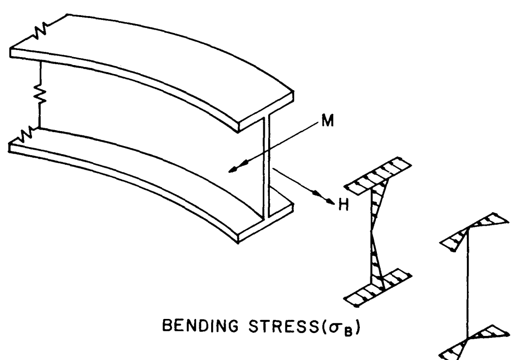
Consider the horizontally curved girder shown in Fig. 1a. The x-y axes are the principal centroidal axes of the cross section and the z axis is tangent to the curved center line of the member. The cross-sectional shape is assumed to be constant along the entire length of the member and doubly symmetric, i.e., the shear center and centroid coincide. The support conditions on both ends are assumed to be the same with the vertical deflection, Δ, and rotation about the z axis, θ, being prevented. Rotations about both the x and y axes are assumed to be unrestrained. These support conditions correspond to "simple supports" used in the analysis of straight girders.

Due to the curvature in the horizontal plane, the girder in Fig. 1a will deflect and twist when loaded in the y-z plane. The normal stresses produced by these displacements are shown in Fig. 1b. In addition to the normal stress due to bending, warping normal stresses due to nonuniform torsion[4] are also developed. The warping normal stresses are sometimes referred to as "flange bending stresses" or stresses produced by "cross bend-

Charles Culver is Assistant Professor of Civil Engineering, Carnegie-Mellon University, Pittsburgh, Pa.
Darryl Brogan is a Senior Civil Engineering student, Carnegie-Mellon University, Pittsburgh, Pa.
David Bednar is an Engineer with Richardson, Gordon and Associates, Consulting Engineers, Pittsburgh, Pa.



(a) COORDINATE SYSTEM



BENDING STRESS(σ_B)

WARPING NORMAL STRESS(σ_W)

$\sigma_{TOTAL} = \sigma_B + \sigma_W$

(b) INTERNAL FORCES AND STRESSES

Fig. 1. Horizontally curved girder

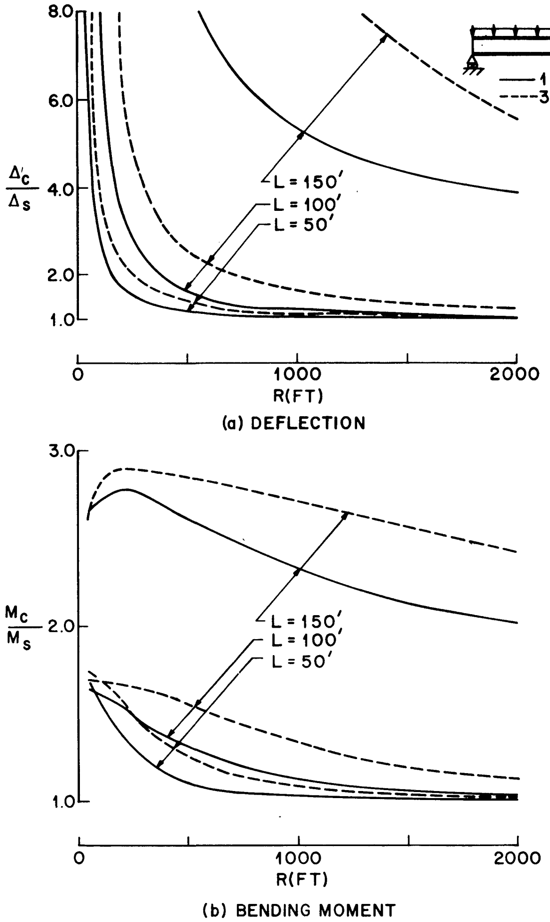


Fig. 2. Comparison of straight and curved girders

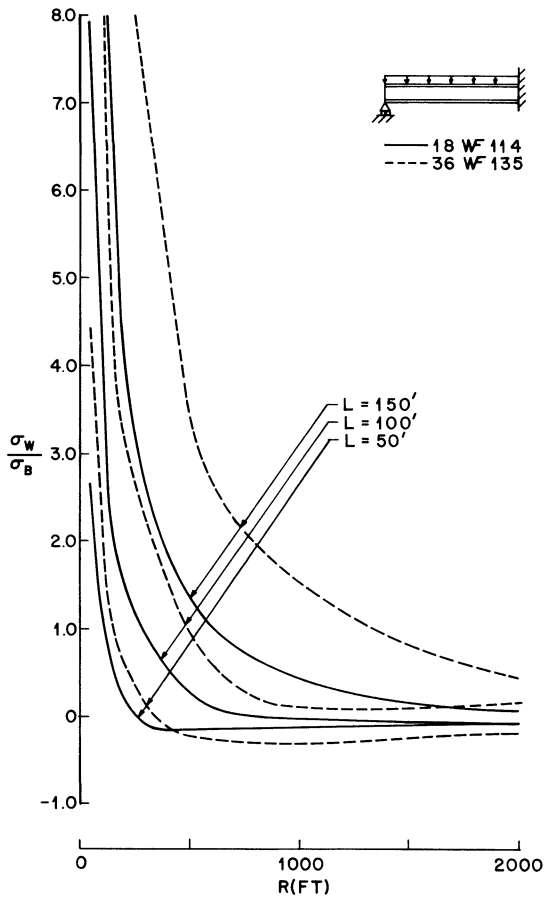


Fig. 3. Magnitude of normal stress due to torsion

ing". The shearing stresses due to St. Venant Torsion and warping torsion are not shown in Fig. 1b.

Expressions for the internal forces, bending moment M and torque H, and the vertical deflection and twist for curved girders were developed by Dabrowski.[5] Influence coefficients relating the end slopes to unit end moments were also developed, thus permitting the solution of continuous curved beam problems. The assumptions inherent in this solution are consistent with elementary straight beam theory and the results may be considered "exact". Similar results have recently been presented by Brookhart.[6]

In order to illustrate the behavior of curved girders, results obtained using Dabrowski's equations are compared with similar results for straight beams in Figs. 2 and 3.

The maximum vertical deflection of a uniformly loaded fixed-pinned curved beam is compared with the corresponding maximum deflection for a straight beam with the same support conditions in Fig. 2a. Note that for a curved beam both the bending slope, Δ'_C, and the rotational slope, θ', are zero at a fixed support. The length of the straight beam used to obtain these results was equal to the center line length or arc length of the curved beam. Results are presented for several values of the span length and radius of curvature. Since a curved beam will bend and twist due to the applied load, deflections are influenced by the bending rigidity EI_x and the two torsional rigidity parameters GK and EI_w[4]. Results in Fig. 2 are given for two rolled shapes with widely different torsional rigidities. As indicated, the maximum deflection of a curved beam, for a particular span length, is greater than the deflection of an equivalent length straight beam. The ratio of these deflections is a function of span length and increases as the radius of curvature decreases. For small radii of curvature, the curved beam is considerably more flexible than the equivalent straight beam. This drastic increase in flexibility due to curvature was experimentally verified by Moorman.[7] The vertical deflection of a curved fixed-fixed **I** beam in Moorman's tests was 22 times as great as the deflection of an equivalent length straight beam. The influence of increased torsional rigidity of the 18WF114 over the 36WF135 on the vertical deflection is apparent.

The ratio of the maximum negative bending moment at the fixed support for the curved and straight beams is shown in Fig. 2b. Although the increase in bending moment due to decreasing the radius of curvature is less

Table 1. Properties of Curved Grids

Grid	Inner girder span length (ft)	Number of diaphragms in span	Flange	Web	I_x (in.^{4})	I_w (in.^{6})	K (in.^{4})	I_x Diaphragm (in.^{4})	Girder spacing (ft)
1	50	3	8″ x ½″	24″ x ⅜″	1,632	6,403	1.089	170.9	15
2	100	5	20″ x ½″	48″ x ⅜″	40,210	1,225,125	45.84	4,021	15

than the corresponding increase in deflection, it is substantial for practical radii of curvature between 150 ft and 1000 ft. Comparison of Figs. 2a and 2b indicates that the influence of torsional rigidity on the bending moment is less than on deflection. Note that the bending moment ratio for a span length of 150 ft reaches a maximum at a radius of curvature of approximately 200 ft. For increasing radii this ratio decreases toward unity as the curved beam approaches the straight beam (infinite radius). This ratio also decreases for radii below 200 ft, since as the curvature decreases a greater portion of the load is resisted by torsion. A similar trend was noted for the other span lengths.

The ratio of the maximum warping normal stress at the inner edge (toward the center of curvature) of the bottom flange due to torsion to the maximum normal stress due to bending is shown Fig. 3. Maximum values of both the torsional and bending stresses occur at the fixed end. Note that as the radius of curvature decreases, the normal stresses due to torsion are considerably larger than the normal stresses due to bending. Due to reversal in the sign of the warping normal stress (compression to tension) at the inner flange tip, the ratio in Fig. 3 becomes negative for certain radii. The significance of these warping normal stresses has been previously pointed out.[6] Since these warping normal stresses are linearly distributed across the flange width, the average value for the entire flange is zero. Due to their relatively high value at the flange tips, however, they should be taken into account in designing curved girders. Although the results in Fig. 3 are for a bare steel girder, test results from a composite curved girder bridge model[8] also indicate the importance of normal stresses due to torsion.

BEHAVIOR OF CURVED GRID SYSTEMS

Most horizontally curved bridges constructed to date (1969) consist of several individual curved girders interconnected by diaphragms or floor beams and lateral bracing. Thus the girders and floor beams act as a curved grid system in resisting the applied loads. The concrete bridge deck also participates in this grid system action in resisting live loads when designed compositely and connected to the girders with shear connectors. The curved grids considered herein are assumed to consist of only the bare steel girders and the diaphragms. The flexible diaphragms frame into the webs of the girders and offer no warping restraint to the girder flanges. The torsional rigidity of the diaphragms is neglected. The bearing shoes at the supports in combination with the diaphragms at these locations are assumed to prevent rotation of the girders at these points ($\theta = 0$). The composite action of the bridge deck may be included by modifying the bending and torsional rigidities of the grid system members[9] or by treating the entire bridge as an orthotropic plate system.[10]

Two practical curved grid systems with the dimensions and properties given in Table 1 were analyzed using three different computer programs. The first program, referred to herein as the "Approximate Method", utilizes equivalent length straight beams in the analysis and computes the torsional stresses from the primary moment diagram.[3] The second program, referred to as the "Flexibility Method", is based on the flexibility method of analysis[11] and utilizes the flexibility influence coefficients derived by Dabrowski for curved girders.[5] The third program, "STRESS", is well known[12] and involves subdividing the individual girders into small straight segments.

Results obtained from these three programs are compared in Figs. 4, 5, 6, and 7 for various values of the radii of curvature of the outer girder. In each case, the two-span, two-girder grid systems were loaded with concentrated loads at the center line of the outer and inner girders in both spans simultaneously. Values for the maximum negative bending stress over the center support and the maximum positive bending stress within the span for both the inner and outer girder are given in Figs. 4 and 6. Values for the total normal stress, bending plus warping, at the same locations are given in Figs. 5 and 7. In each case the stresses are nondimensionalized with respect to the corresponding stresses in an equivalent straight grid in which the length of the members are equal to the center line length of the outer curved girder.

Referring to Figs. 4 and 6, note that both the positive and negative bending stress in the outer girder of the two curved grids is greater than the stress in the equivalent straight grid and the stress in the inner girder is less than that in the straight grid. Note also that the three methods of analysis give essentially the same results for the outer

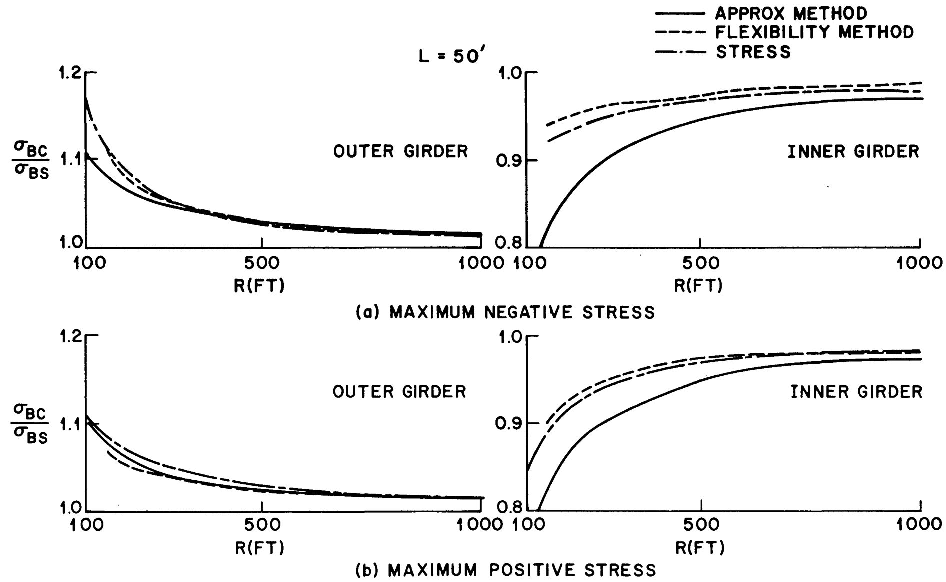


Fig. 4. Comparison of methods of analysis — bending stresses (Grid 1)

girder. For the inner girder, however, the approximate method may underestimate the maximum negative bending stress by as much as 15 percent (see Fig. 6a).

Referring to Figs. 5 and 7, note that the approximate method overestimates the maximum positive total stress and underestimates the maximum negative total stress for both the inner and outer girders in all cases. Since the STRESS program does not consider warping torsion, results for the total stress (bending plus warping) are not given in Figs. 5 and 7 for this method of analysis. It would be possible, however, to obtain approximate values for these warping stresses by means of an appropriate modification to the original STRESS program.

In view of the considerable number of variables involved, such as span length, girder spacing, number of diaphragms, etc., it is difficult to draw general conclusions concerning comparisons of the three methods of analysis. Based on the results in Figs. 4 through 7, however, the following trends may be noted:

1. Maximum positive and negative bending stresses for the outer girder obtained from the three methods of analysis are essentially the same.
2. The approximate method underestimates maximum positive and negative bending stresses in the inner girder as the radius of curvature decreases.
3. Total stresses obtained from the Approximate Method and the Flexibility Method agree to within 15 percent or less for radii of curvature greater than 300 ft. For smaller radii, the difference between these two methods of analysis increases.

The comparisons given above were concerned with specific grid systems and illustrate the influence of the method of analysis on the calculated stresses. It is also of interest to consider the influence of various parameters such girder spacing, span length, etc. on the behavior of curved grids. One important variable is the spacing and bending stiffness of the diaphragms or floorbeams. These diaphragms influence the load transfer between the various girders and affect the overall stiffness of the grid system. Although the influence of these diaphragms is less significant after the bridge deck is in place, they do affect the stresses and deflections of the steel grid produced by dead loads.

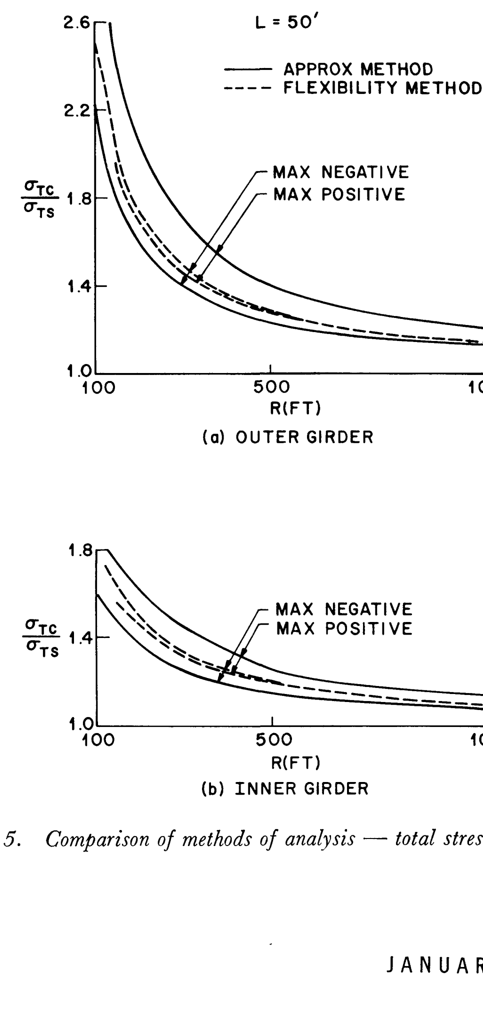


Fig. 5. Comparison of methods of analysis — total stress (Grid 1)

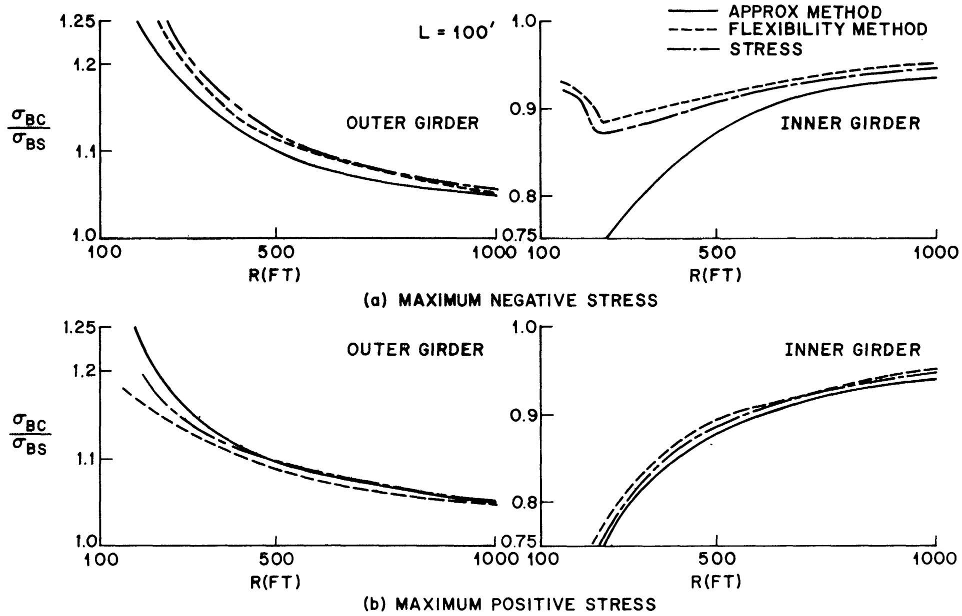


Fig. 6. Comparison of methods of analysis — Bending stress (Grid 2)

The influence of diaphragm spacing and stiffness on the behavior of the outer girder of a two-span, two-girder grid system is shown in Figs. 8 and 9. The deflections and stresses for the outer girder resulting from concentrated loads applied simultaneously in each span on both the inner and outer girder at the diaphragms are compared with the corresponding values for a single curved girder with the same dimensions and loading.

The stiffening influence of the diaphragms is apparent in Fig. 8. For a diaphragm stiffness equal to 5 percent of the girder stiffness, the maximum deflection of the girder in the grid system is only 55 percent of that of the individual girder.

The influence of diaphragm spacing and stiffness on the maximum negative bending stress over the center support is shown in Fig. 9a. The effect of the diaphragms of these stresses is similar to that for deflections. The bending stresses and deflections are not significantly influenced by the number of diaphragms and the stiffening effect remains constant as the diaphragm stiffness increases beyond 5 percent of the girder stiffness.

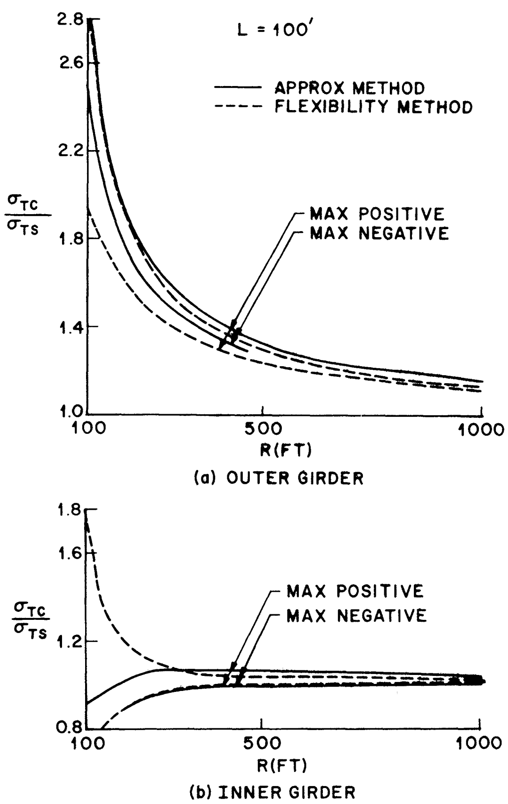


Fig. 7. Comparison of methods of analysis — total stress (Grid 2)

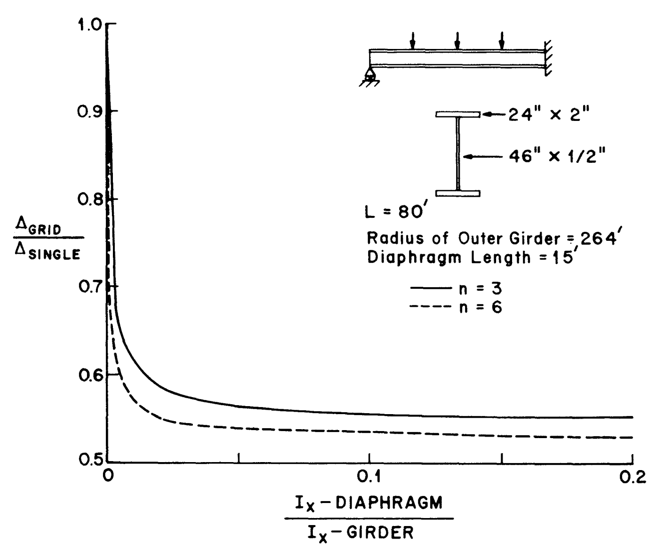


Fig. 8. Influence of diaphragm stiffness on deflection

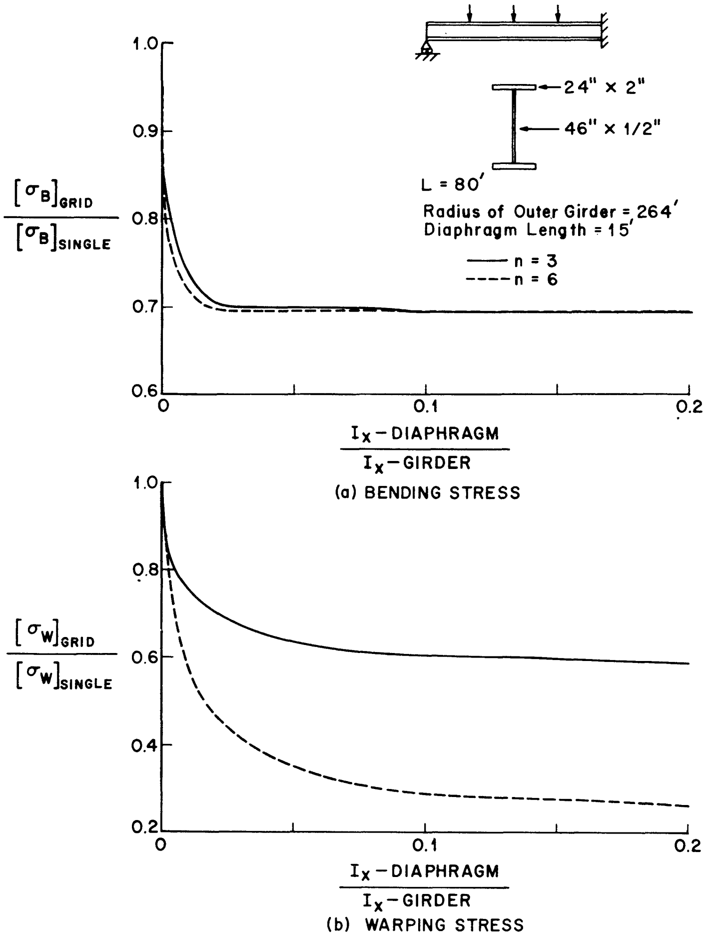


Fig. 9. Influence of diaphragm stiffness on stress

As shown in Fig. 9b, however, the number of diaphragms significantly influences the maximum warping normal stress over the center support due to torsion. For diaphragm stiffnesses greater than 5 percent of the girder stiffness, doubling the number of diaphragms decreases the warping normal stress by 50 percent. This result is to be expected, since torsional deformations of the girders are directly related to bending of the diaphragms.

The above results are based only on load transfer or structural behavior considerations. Economic and stability or lateral bracing requirements must be taken into account in determining the spacing and stiffness of the diaphragms.

SUMMARY AND CONCLUSIONS

Results presented herein indicate that the behavior of individual curved girders is substantially different from that of straight girders. For curved grid systems, differences obtained from the three methods of analysis used herein increase as the radius of curvature decreases. For radii of curvature less than 300 ft, careful attention should be given to the assumptions inherent in any approximate method of analysis.

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APPENDIX A - NOMENCLATURE

The following symbols are used in this paper:

E	Modulus of elasticity
G	Shear modulus
H	Internal torque
Ix	Moment of inertia about x axis
Iw	Warping constant
K	St. Venant Torsion constant
L	Span length measured along center line of girder
Mc	Internal bending moment in curved girder
Ms	Internal bending moment in straight girder
n	Number of diaphragms
R	Radius of curvature of girder
x, y, z	Coordinate axes
α	Central angle of curved girder
Δ_C	Vertical deflection of curved girder
Δ_S	Vertical deflection of straight girder
σ_B	Bending stress
σ_{BC}	Bending stress in curved grid
σ_{BS}	Bending stress in straight grid
σ_{TC}	Total stress in curved grid
σ_{TS}	Total stress in straight grid
σ_w	Warping normal stress
θ	Angle of rotation