Preliminary Wind Analysis of Multistory Bents

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RECENT PROGRESS in computers has permitted rapid analysis of the multistory bent, including deformations due to horizontal forces. Even so, an approximate method is still needed (and, in certain cases, suffices) for the following purposes:

(a) A rapid answer to the important problem arising in the preliminary stages: the required moment of inertia of the bent girder of a given story, with the horizontal deflection kept below the allowable limit.

(b) Determination of the ratio of moments of inertia between columns and girders — an essential prerequisite in solving the statically-indeterminate problem on a computer.

The proposed method consists of determining:

1. Column areas, on the basis of the gravity loads. In the upper stories, allow for a 10 to 25 percent margin to account for wind moments (which may be critical in relatively light columns).
2. The deformation angle ϕ due to bending of the columns.
3. The deformation angle ψ due to axial deformation of the columns.
4. The required moment of inertia of the girder as a function of the deformation angle θ, due to its own bending. This angle is given by $\theta = \alpha - (\phi + \psi)$, where α is the allowable deformation angle for the given story.

The first part of this paper presents an analysis of the girder and column forces and moments for an arbitrary story. Equilibrium equations are derived in general form, in terms of the external forces and bent geometry—an especially convenient approach for tabular work.

The second part deals with the deformation of the multistory bent. An outline of the basic assumptions is followed by a proposed deformation curve, which yields the allowable deformation angle for any story, based on the allowable deflection index (drift limitation). Subsequently, equations are derived for the horizontal deflections due to each of the following factors: axial deformation of the columns, bending of the columns, and bending of the girders.

The paper concludes with a numerical example.

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PART I. STRESS ANALYSIS

Basic assumptions—The basic assumptions used in the wind analysis of a multistory bent are as follows:

1. Wind forces act as concentrated loads at each floor level.
2. The inflection point of each column is at mid-height.
3. The inflection point of each girder is at mid-span.
4. The column direct stress is proportional to its distance from the centroidal axis of the column group.

The elevation and a plan of a bent are shown in Fig. 1. The general equations derived in this paper represent the moments and shears in an interior bay in terms of those in the end bay. Similarly, the direct stresses in

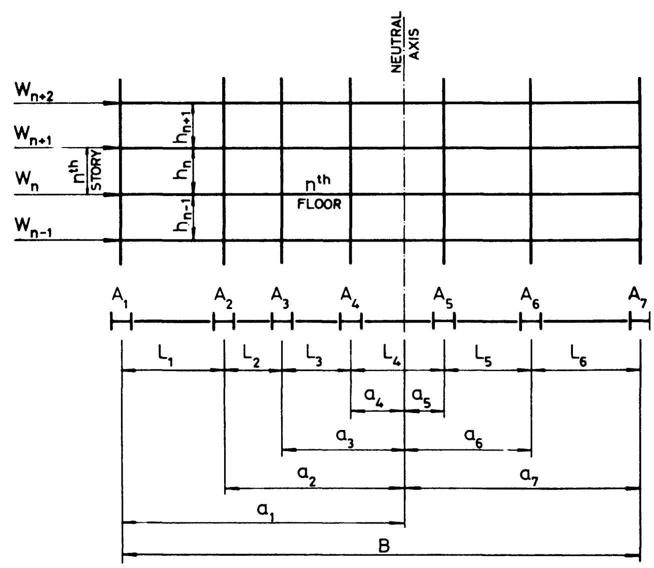

Fig. 1. Elevation and plan of bent

the interior columns are given in terms of those in the end columns. Moments, shears and direct stresses in the first bay are formulated as functions of the wind forces and bent geometry.

This general approach is especially convenient for tabular work, as described in the numerical example.

Definitions used in the analysis

Wind moment (M)—The wind moment M_n is the external bending moment at mid-height of the n-th story due to the total wind forces above this level. For uniformly distributed wind load w (kips/ft), the overturning moment at distance y (measured from the top of the structure) is $M = wy^2/2$.

Floor moment (M_{FL})—The floor moment is the moment produced at floor level by the shear forces acting at points of contraflexure above and below the floor in question (Fig. 2).

$$M_{FL} = H_n{}^B \frac{h}{2} + H_n{}^T \frac{h}{2} \tag{1}$$

where

M_{FL} = floor moment
$H_n{}^T$ = total shear acting above given floor
$H_n{}^B$ = total shear acting below given floor
h = story height

The shear forces are obtainable from the wind load w:

$$H_n{}^T = wy_n \quad ; \quad H_n{}^B = w(y_n + h)$$

$$M_{FL} = w(y_n + h)\frac{h}{2} + wy_n \frac{h}{2} = \frac{w}{2}(2y_nh + h^2)$$

It can be shown that the floor moment is also obtainable as the difference of the overturning moments acting below and above the given floor level:

$$M_n{}^B = \frac{w(y_n + h)^2}{2} \quad ; \quad M_n{}^T = \frac{wy_n{}^2}{2}$$

$$M_n{}^B - M_n{}^T = \frac{w(y_n + h)^2}{2} - \frac{wy_n{}^2}{2} = \frac{w}{2}(2y_nh + h^2)$$

$$M_{FL} = M_n{}^B - M_n{}^T \tag{2}$$

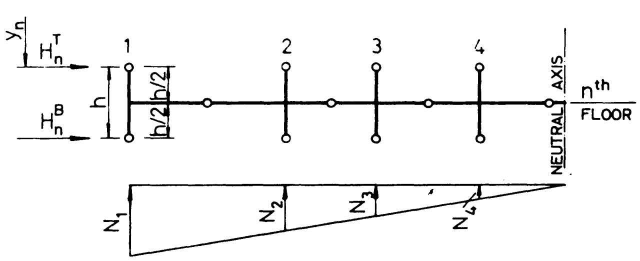

Fig. 2. Column direct forces distribution

Relative column area (R)—The relative column area is the ratio of the area of each of the interior columns and the end column:

$$R_i = A_i/A_e \tag{3}$$

where

R_i = relative column area of i-th column
A_i = cross-sectional area of i-th column
A_e = cross-sectional area of end column

Direct stress intensity (D)—The direct stress intensity is the ratio of the distances, from the neutral axis, of each interior column and of the end column:

$$D_i = a_i/a_e \tag{4}$$

where

D_i = direct stress intensity of i-th column
a_i = distance of i-th column from neutral axis
a_e = distance of end column from neutral axis

Relative column direct stress (K)—Assuming linear distribution, the direct stresses N are proportional to the column distance from the centroidal axis and to the relative column areas. The relative column direct stress is the product of the relative column area R and the direct stress intensity D

$$K_i = R_iD_i \tag{5}$$

Equivalent base (B_E)—The equivalent base is the section modulus of the column group in the bent, obtained as the quotient of the moment of inertia of the column group divided by the distance of the end column from the neutral axis:

$$B_E = \frac{I}{a_e} \quad ; \quad I = \sum R_ia_i^2$$

$$B_E = \sum \frac{R_ia_i^2}{a_e} \tag{6}$$

where

B_E = equivalent base
I = moment of inertia of column group

Substitution of

$$a_e = a_i/D_i \tag{4}$$

and

$$R_i = K_i/D_i \tag{5}$$

yields the following simple formula for the equivalent base:

$$B_E = \sum K_ia_i \tag{7}$$

Column direct forces—The column direct forces in the end column (Fig. 3) are readily obtainable from the familiar formula $f = M/S$, with the section modulus S replaced by the equivalent base:

$$N_1^B = M^B/B_E \qquad N_1^T = M^T/B_E \tag{8}$$

Assuming linear distribution (Fig. 2), the column direct forces in the interior columns (Fig. 4) are obtained with the aid of the relative column direct stress coefficients (K_i):

$$N_i^T = K_i N_e^T \qquad N_i^B = K_i N_e^B \tag{9}$$

Girder shears—The girder shears are obtained from the column direct forces (Fig. 3):

$$V_1 = N_1^B - N_1^T$$

Substitution of Eq. (8) yields a simple expression for the shear in the end bay:

$$V_e = \frac{M^B}{B_E} - \frac{M^T}{B_E} = \frac{M_{FL}}{B_E} \tag{10}$$

The girder shears in the interior bays are now obtained by equating the vertical forces (Fig. 4):

$$V_1 + N_2^B = V_2 + N_2^T$$

$$V_2 = V_1 + N_2^B - N_2^T$$

Substitution of $N_2^B = k_2 N_e^B$ and $N_2^T = k_2 V_e^T$ yields the following expression:

$$V_2 = V_1 + k_2(N_e^B - V_e^T) = V_1 + k_2 V_1 = V_1(1 + k_2)$$

or, in general form,

$$V_i = V_{i-1}(1 + k_i) = g_s V_e$$

where g_s = girder shear coefficient.

Column shears—The column shears S_1^T, S_1^B, S_2^T, S_2^B (Figs. 3 and 4) are obtained by equating the shear moments:

$$(S_1^B + S_1^T)\frac{h}{2} = V_1 \frac{L_1}{2}$$

The column shears are proportional to the total shear acting above and below the floor level:

$$S_1^T/S_1^B = H^T/H^B$$

Substitution of

$$S_1^T = S_1^B(H^T/H^B)$$

$$V_1 = M_{FL}/B_E \tag{10}$$

$$M_{FL} = (H^B + H^T)\frac{h}{2} \tag{11}$$

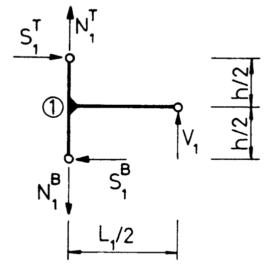

Fig. 3. Forces acting on end joint

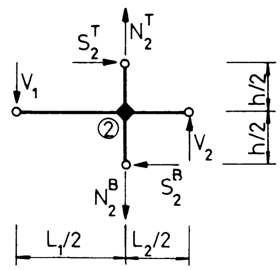

Fig. 4. Forces acting on interior joint

yields

$$\left(S_1^B + S_1^B \frac{H^T}{H^B}\right)\frac{h}{2} = \frac{(H^B + H^T)h}{2B_E}\,\frac{L_1}{2}$$

$$S_1^B = H^B \frac{L_1}{2B_E} = c_1 H^B \tag{12}$$

where

$$c_1 = L_1/2B_E$$

With the column shear in the end column expressed as a function of the total shear, we now express the interior column shears as functions of their end counterpart.

Again equating the moments:

$$(S_2^B + S_2^T)\frac{h}{2} = V_1 \frac{L_1}{2} + V_2 \frac{L_2}{2}$$

Substitution of

$$S_2^T = S_2^B \frac{H^T}{H^B}$$

$$V_1 = \frac{M_{FL}}{B_E} = \frac{(H^B + H^T)\frac{h}{2}}{B_E}$$

$$V_2 = V_1(1 + k_2) = \frac{(H^B + H^T)\frac{h}{2}}{B_E}(1 + k_2)$$

yields

$$\left(S_2{}^B + S_2{}^B \frac{H^T}{H^B}\right)\frac{h}{2} = \frac{(H^B + H^T \frac{h}{2}}{B_E}\frac{L_1}{2} + \frac{(H^B + H^T)\frac{h}{2}}{B_E}(1 + k_2)\frac{L_2}{2}$$

$$S_2{}^B\left(1 + \frac{H^T}{H^B}\right) = (H^B + H^T)\left[\frac{L_1}{2B_E} + \frac{L_2}{2B_E}(1 + k_2)\right]$$

$$S_2{}^B = H^B\left[\frac{L_1}{2B_E} + \frac{L_2}{2B_E}(1 + k_2)\right]$$

and the substitution of Eq. (12)

$$S_1{}^B = H^B \frac{L_1}{2B_E}$$

yields the column shear in the interior column 2:

$$S_2{}^B = S_1{}^B\left[1 + \frac{L_2}{L_1}(1 + k_2)\right]$$

The general formula is:

$$S_i{}^B = S_e{}^B\left[1 + \frac{L_i}{L_1}\sum k_i\right] \tag{13}$$

or

$$S_i{}^B = c_i H^B$$

Girder and column wind moments—The interior wind moments are the product of the girder and column shears and the corresponding lever arms (half-bay and story height distances), Figs. 5 and 6:

$$\left.\begin{array}{ll} M_1{}^{GR} = V_1\dfrac{L_1}{2} & M_1{}^{CT} = S_1{}^T\dfrac{h}{2} \\[2ex] M_2{}^{GL} = V_1\dfrac{L_1}{2} & M_1{}^{CB} = S_1{}^B\dfrac{h}{2} \\[2ex] M_2{}^{GR} = V_2\dfrac{L_2}{2} & M_2{}^{CT} = S_2{}^T\dfrac{h}{2} \\[2ex] & M_2{}^{CB} = S_2{}^B\dfrac{h}{2} \end{array}\right\} \tag{14}$$

where

$M_i{}^{GL}$, $M_i{}^{GR}$ = girder moments at left- and right-hand face of column i

$M_i{}^{CT}$, $M_i{}^{CB}$ = column moments at column i above and below floor level

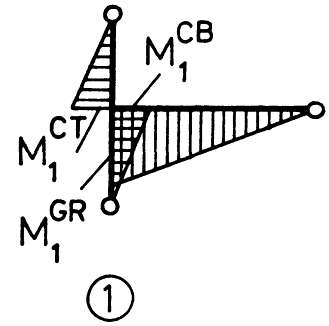

Fig. 5. Moments acting on end joint

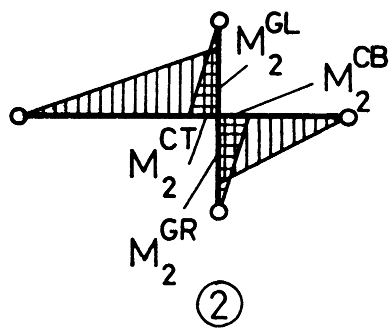

Fig. 6. Moments acting on interior joint

PART II. DEFLECTIONS

Basic assumptions—The lateral deflection (or drift) due to wind forces comprises three components, due respectively to:

a) bending of girders
b) bending of columns
c) axial deformations of columns

$$\Delta = \Delta_g + \Delta_c + \Delta_c' \tag{15}$$

where

Δ = total deflection
Δ_g = deflection due to bending of girders
Δ_c = deflection due to bending of columns
Δ_c' = deflection due to axial deformation of columns

The drift can also be expressed in terms of angles of rotation:

$$\alpha = \theta + \phi + \psi \tag{16}$$

where

α = total angle of rotation of n-th floor
θ = angle of rotation due to bending of girders
ϕ = angle of rotation due to bending of columns
ψ = angle of rotation due to axial deformation of columns

Figure 7 shows the angles and deflections due to column and girder bending.

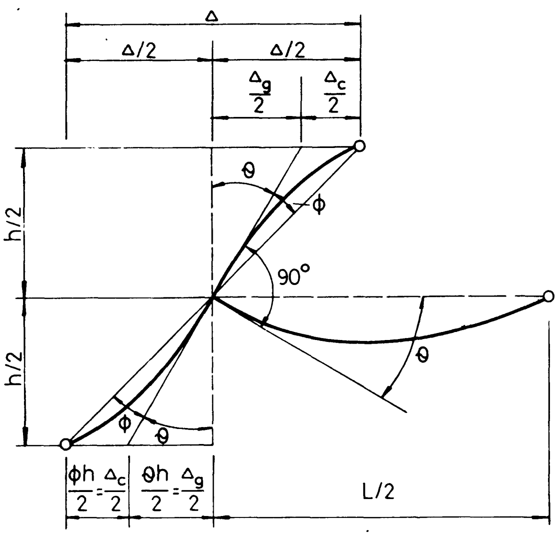

Fig. 7. Angles and deflections due to column and girder bending

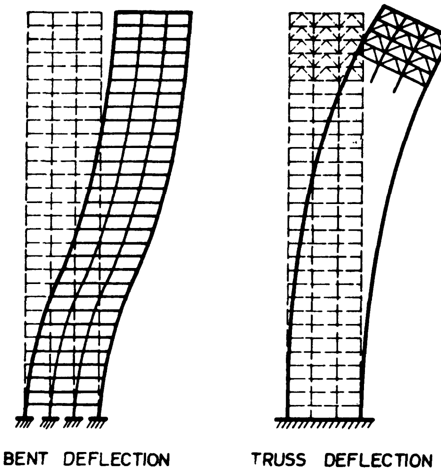

Fig. 8. Typical deflected shape lines

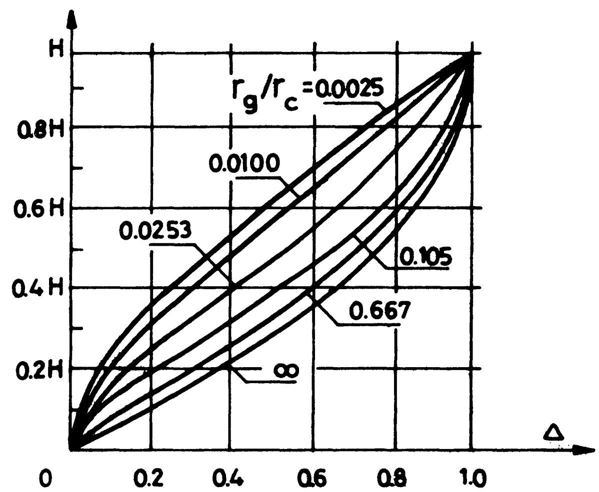

Fig. 9. Deflection lines for a multistory bent with different rigidity ratios (Ref. 4)

Deflection line—The deflection line of a multistory building under horizontal forces depends on the stiffening systems used and their rigidities. Figure 8 shows typical deflected shape lines for two types of stiffening systems—bents and trusses. The deflection line of a bent is governed by the rigidity ratio of its elements, girder and columns, and as the column stiffness increases the deflection line approaches that of a truss. Figure 9 shows deflection lines for a multistory bent with different rigidity ratios.[4] In a multistory structure comprising bents and trusses, the distribution of lateral forces and their effect on the deflection line form a highly complex problem. The floors act as horizontal diaphragms and distribute the wind forces to the bents and trusses according to their own rigidity distribution. The bent restrains truss deflection in the upper stories, while the influence of the truss is especially pronounced in the lower stories — in which the truss is "pulling back." The interaction of a bent and a truss is a problem of special interest, but outside the scope of the present study.

The deflection line proposed here (Fig. 10) is based on computer analysis of five different multistory bents. Denoting by α the allowable deflection angle (deflection index) and replacing the curved deflection line with straight segments, the deformation angles are obtained in terms of the maximum deflection angle α as follows:

$$\left.\begin{array}{ll} \alpha_{9-10} = \alpha & \alpha_{9-5} = 1.1\alpha \\ \alpha_{8-9} = 1.2\alpha & \alpha_{3-4} = \alpha \\ \alpha_{9-8} = 1.5\alpha & \alpha_{2-3} = 0.8\alpha \\ \alpha_{6-9} = 1.2\alpha & \alpha_{1-2} = 0.6\alpha \\ \alpha_{5-6} = 1.2\alpha & \alpha_{0-1} = 0.4\alpha \end{array}\right\} \qquad (17)$$

Deflection due to axial deformation of columns (chord drift)—The column areas are usually proportional to the height of the structure, and the column axial forces—to the same height squared. Hence, the axial stresses in the columns due to wind forces increase linearly from zero at the top of the bent to maximum at the base* (Fig. 11).

The chord drift at any point of the bent is obtained by the virtual-work method (Fig. 12):

$$\Delta_x = \int_0^H \frac{mM_x}{EI}\,dx$$

where

Δ_x = horizontal deflection at distance x
m = virtual bending moment due to unit virtual force applied at x
M_x = real bending moment due to wind forces at x
E = modulus of elasticity = 29,000,000 psi
I = moment of inertia of column group

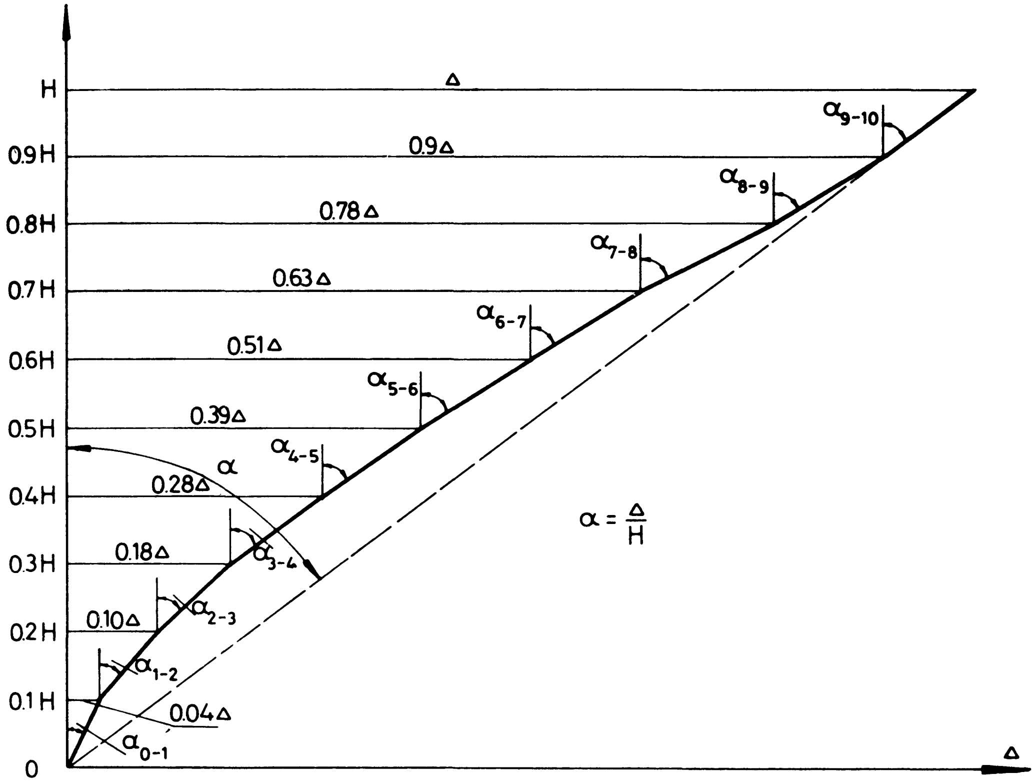

Fig. 10. Proposed deflection line

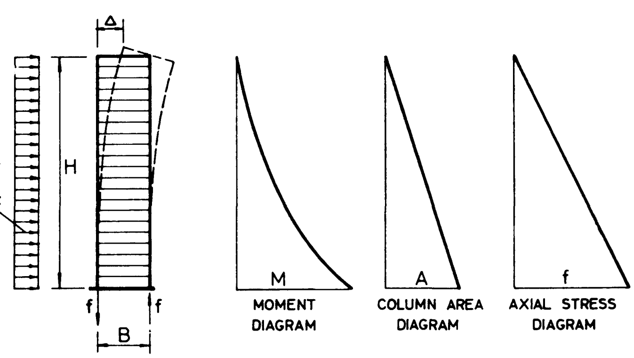

Fig. 11. Axial stress diagram

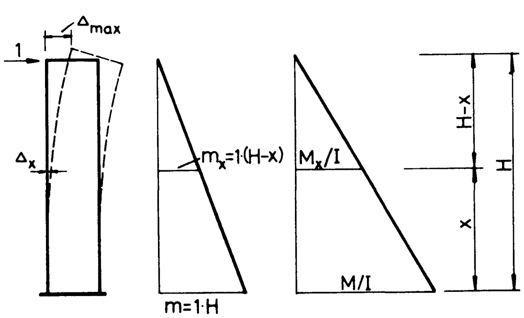

Fig. 12. Moment diagrams due to unit virtual forces and real bending moments

Substitution of

$$m = 1\ (H - x)$$

$$M_{\max} = f_{\max}\frac{I}{a_{\max}} = f_{\max}\frac{I}{B/2}$$

$$M_x = M_{\max}\frac{H - x}{H} = f_{\max}\frac{2I}{B}\frac{H - x}{H}$$

yields the maximum chord drift as follows:

$$\Delta'_{c_{\max}} = \int_0^H \frac{(H - h)f_{\max}2I(H - x)}{EIBH}$$

$$\Delta'_{c_{\max}} = \frac{2f_{\max}}{BHE}\int_0^H (H - x)^2 dx =$$

$$\frac{2f_{\max}}{BHE}\left| H^2x - 2H\frac{x^2}{2} + \frac{x^3}{3}\right|_0^H$$

$$\Delta'_{c_{\max}} = \frac{2f_{\max}H^2}{3BE} \qquad (18)$$

where

$f_{\max}$ = maximum unit stress in end column at base of bent due to wind forces
B = width of bent
H = total height of bent
E = modulus of elasticity

* *Confirmed by computer analysis.*

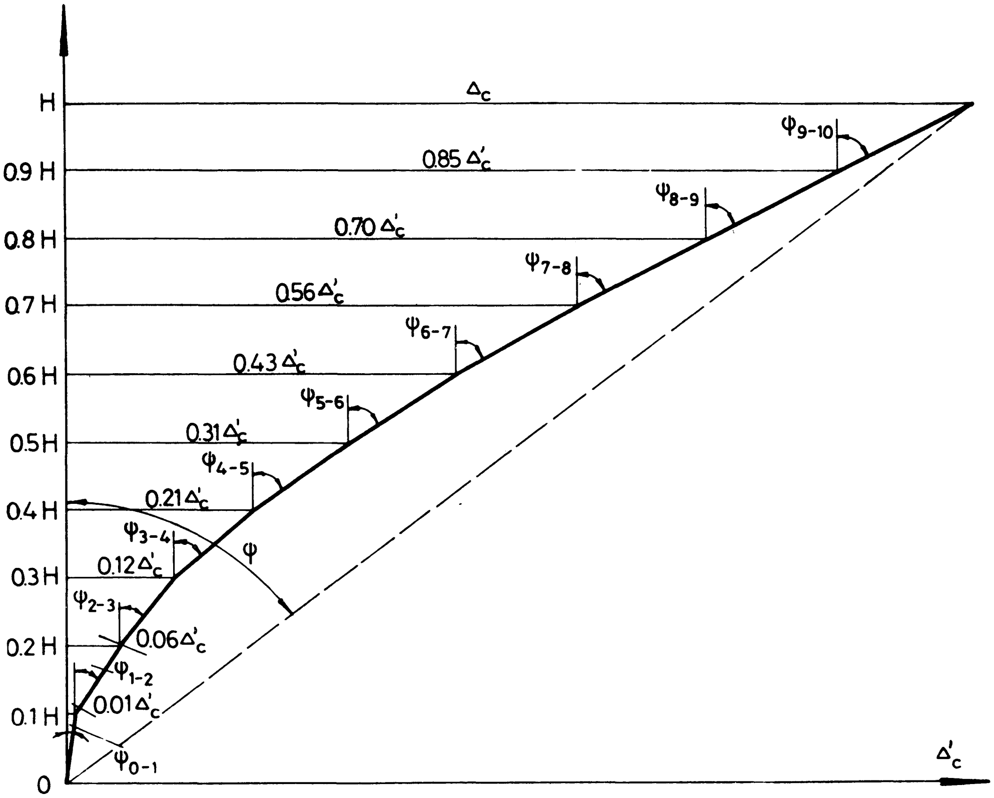

Fig. 13. Deflection line due to chord drift

The chord drift at any point is:

$$\Delta'_c = \frac{f_{\max}x^2}{BE}\left(1 - \frac{x}{3H}\right) \tag{19}$$

Figure 13 shows the deflection line due to chord drift.

We now consider the rotation angle ψ, of the bent. Differentiating Eq. (19) yields the slope of the line tangent to the curve at any point:

$$\frac{d\Delta'_c}{dx} = \frac{2f_{\max}}{BE}\,x - \frac{f_{\max}}{BE}\frac{x^2}{H} = \frac{2f_{\max}}{BE}\,x\left(1 - \frac{x}{2H}\right)$$

and approximating the angle by its tangent,

$$\psi = \frac{2f_{\max}}{BE}\,x\left(1 - \frac{x}{2H}\right) \tag{20}$$

The deformation angle at the top yields:

$$\psi_{top} = \frac{2f_{\max}}{BE}\,H\left(1 - \frac{H}{2H}\right) = \frac{f_{\max}\,H}{BE} \tag{21}$$

The theoretical angle ψ (Fig. 13) obtained as the quotient of the maximum chord drift by the total height H yields:

$$\psi = \frac{2f_{\max}H^2}{3BE}\Big/H = \frac{2f_{\max}H}{3BE} = \frac{2}{3}\,\psi_{top}$$

Deflection due to bending of columns—The deflection component due to column bending is obtained on the assumption that the girders are infinitely rigid. Again by the virtual-work method, the deflection Δ_c between the inflection points above and below the n-th story (Fig. 7) is given by:

$$\Delta_c = \int \frac{mM}{EI}\,dx$$

As all moment diagram segments are triangles, the integral may be replaced by summation (see Fig. 14):

$$\Delta_c = \frac{h/2}{3E}\sum_{i=1}^{n}\frac{m_iM_i}{I_i}$$

Substitution of $\quad m_i = c_i(h/2)$

$$M_i = c_iH\frac{h}{2}$$

assuming $\quad H_n{}^T = H_n{}^B = H_n$

yields

$$\Delta_c = \frac{h}{6E}\sum_{i=1}^{n} 2\,\frac{c_ih}{2}\,\frac{c_iH_nh}{2}\,\frac{1}{I_i}$$

$$= \frac{H_nh^3}{12E}\sum_{i=1}^{n}\frac{c_i^2}{I_i} \tag{22}$$

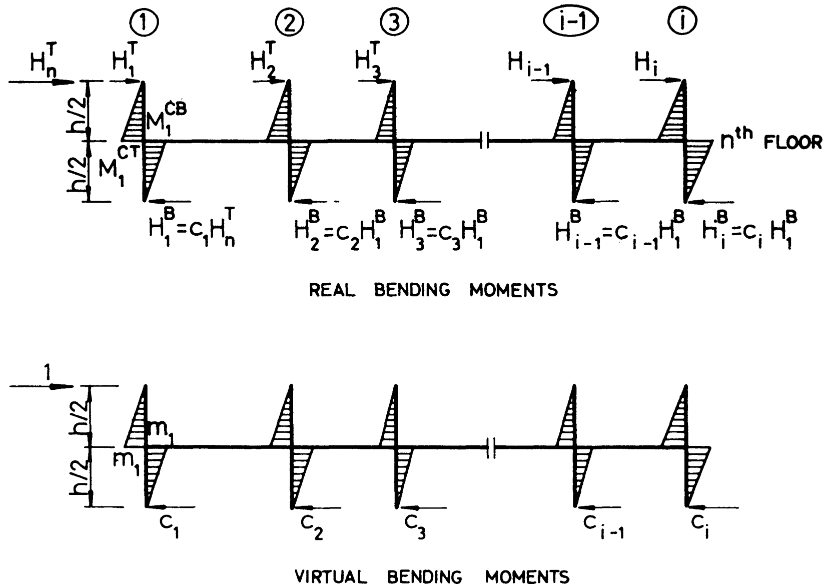

Fig. 14. Real and virtual bending moments

The angle of rotation ϕ is the quotient of the deflection and the story height:

$$\phi = \frac{H_n h^2}{12E} \sum_{i=1}^{n} \frac{c_i^2}{I_i} \tag{23}$$

Where the moment of inertia of the columns is proportional to the column shear coefficients $I_i = c_i I$, Eqs. (22) and (23) reduce to:

$$\left.\begin{aligned} \Delta_c &= \frac{H_n h^3}{12EI_1} \\ \phi &= \frac{H_n h^2}{12EI_1} \end{aligned}\right\} \tag{24}$$

Deflection due to bending of girders—The deflection component due to bending of the girders is obtained on the assumption that the columns are infinitely rigid. The virtual-work method in this case yields:

$$\left.\begin{aligned} \Delta_g &= \frac{1}{12EH_n} \sum \frac{V_i^2 L_i^3}{I_i} \\ \theta &= \frac{1}{12EH_n h} \sum \frac{V_i^2 L_i^3}{I_i} \end{aligned}\right\} \tag{25}$$

Substituting

$$V_i = g_i V_e$$

$$V_e = M_{FL}/B_E$$

$$L_i = \lambda_i L_1$$

$$M_{FL} = Hh$$

$$B_E = B + \sum K_i L_i$$

and assuming the moments of inertia of the girders proportional to the girder shears coefficients and the span ratio squared

$$I_i = g_i \lambda_i^2 I_1 \tag{26}$$

yields

$$\left.\begin{aligned} \Delta_g &= \frac{V_1 L_1^2 h}{12EI_1} \\ \theta &= \frac{V_1 L_1^2}{12EI_1} \end{aligned}\right\} \tag{27}$$

DESIGN EXAMPLE

The upper part of a 40-story, 4-bay symmetrical bent is shown in Fig. 15. The wind load acting on the upper part of the bent is $w = 0.8$ kips/ft. The drift limitation is 0.003. Determine the required moment of inertia of the girders in the 31st floor.

The calculations follow the procedure indicated in Parts I and II, and are set up in tabular form (Table 1). It is convenient to summarize first all geometrical data as shown in lines 1–8 of the table.

Step 1: Determine wind shears and moments.

$$H_{31}^T = 0.8 \times 120 = 96.0 \text{ kips}$$

$$H_{31}^B = 0.8 \times 132 = 105.6 \text{ kips}$$

$$M_{31}^T = 96 \times 60 = 5760 \text{ kip-ft}$$

$$M_{31}^B = 105.6 \times 66 = 6970 \text{ kip-ft}$$

$$M_{FL} = 6970 - 5760 = 1210 \text{ kip-ft}$$

Table 1. Design Example

	Bent ① ② ③ ②' ①'	①	[1]	②	[2]	③ ℄
1	B = width of bent	100' - 0"				
2	L_i = girder spans		30.0		20.0	
3	a_i = column distances from neutral axis		50.0		20.0	
4	$\lambda_i = L_2/L_1$, span ratio		1.00		0.67	
5	λ_i^2		1.00		0.44	
6	column size	14WF = 127		14WF = 95		14WF = 95
7	A_i = column area	37.33		27.94		27.94
8	I_c = moment of inertia of columns	1477		1064		1064
9	$R_i = A_i/A_1$, relative column area	1.00		0.75		0.75
10	$D_i = a_i/a_1$, direct stress intensity	1.00		0.40		0
11	$K_i = R_i D_i$, relative column direct stress	1.00		0.30		0
12	$B_E = \Sigma k_i a_i$, equivalent base	2(50.0x1.0 + 20.0x0.3) = 112				
13	$g_s = 1+K_i$, girder shear coefficient		1.00		1.30	
14	$g_M = (1+K_i)\lambda$, girder moment coefficient		1.00		0.87	
15	$1 + g_M$	1.00		1.87		1.74
16	C_i = column shear coefficient	0.134		0.250		0.232
17	C_i^2	0.018		0.063		0.054
18	C_i^2/I	1.15×10^{-5}		5.95×10^{-5}		5.07×10^{-5}
19	$\Sigma (C_i^2/I)$	19.27×10^{-5}				
20	ϕ = column deformation angle	0.00122				
21	ψ = chord drift angle	0.00062				
22	θ = allowable girder deformation angle	0.0045 - (0.00122+0.00062) = 0.00266				
23	I_g = required moment of inertia of girders		1520		870	

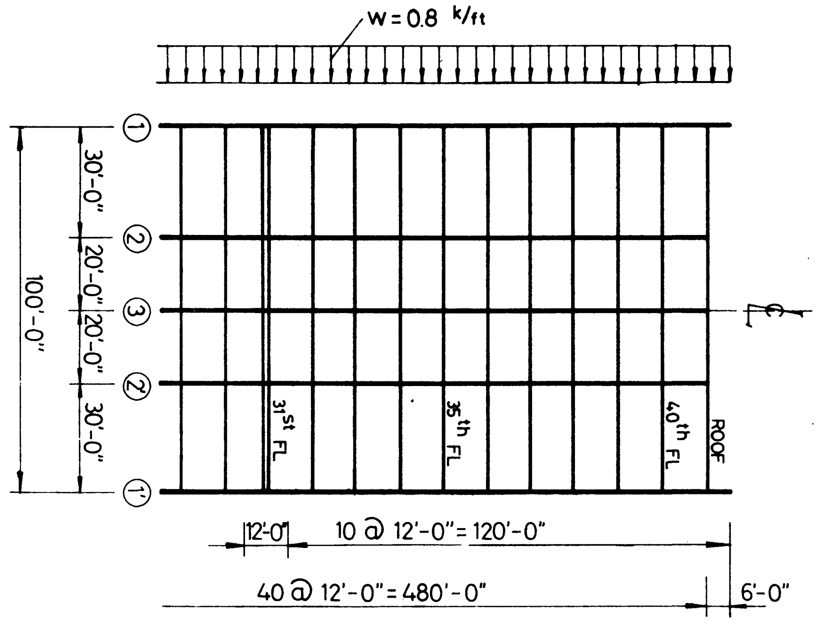

Fig. 15. The upper part of a 40-story bent (design example)

Step 2 Determine: relative column areas, direct stress intensities and relative column direct-stress coefficients (lines 9, 10, 11).

Step 3: Determine equivalent base, girder shear and moments coefficients (lines 12, 13, 14).

Step 4: Determine column shear coefficients (lines 15, 16):

$$C_1 = \frac{L_1}{2B_E} = \frac{30.0}{2 \times 112} = 0.134 \qquad (12)$$

$$C_2 = 1.87 \times 0.134 = 0.250$$

$$C_3 = 1.74 \times 0.134 = 0.232$$

Check: $2(0.134 + 0.250) + 0.232 = 1.000$

Step 5: Determine angle of rotation ϕ due to bending of columns:

$$\phi = \frac{Hh^2}{12E} \sum \frac{c_i^2}{I_i} \qquad (23)$$

The angle ϕ is obtained in line 20:

$$\phi = \frac{105.6 \times 12^2 \times 144}{12 \times 29000} 19.27 \times 10^{-5} = 1.22 \times 10^{-3}$$

Step 6: Determine maximum wind stress in end column at base of bent:

$$f_{max} = N_1/A_1; \; N_1 = M/B_E$$

The maximum wind moment M is determined by multiplying the wind forces acting at different levels by the appropriate lever arms. The wind forces acting on the bent are subject to the wind pressure code requirement. Let us assume that the above calculations yield:

$$f_{max} = 4 \text{ ksi}$$

Step 7: Determine angle of rotation ψ at 31st floor due to chord drift:

$$x = 360'\text{-}0''$$

$$\psi = \frac{2f_{max}}{EB} x \left(1 - \frac{x}{2H}\right) \qquad (20)$$

$$= \frac{2 \times 4{,}000 \times 360 \times 12}{29{,}000{,}000 \times 100 \times 12}\left(1 - \frac{360}{2 \times 480}\right)$$

$$= 0.62 \times 10^{-3}$$

Step 8: Determine the allowable deformation angle for the floor under consideration. The drift limitation being 0.003, the allowable deformation angle α can be obtained from the proposed deflection line (Fig. 10):

$$\alpha_{7-8} = 1.5 \times 0.003 = 0.0045 = 4.5 \times 10^{-3}$$

Step 9: Determine the allowable girder deformation angle θ:

$$\theta = \alpha - (\phi + \psi)$$

$$= 4.5 \times 10^{-3} - (1.22 \times 10^{-3} + 0.62 \times 10^{-3})$$

$$= 2.66 \times 10^{-3}$$

Step 10: Determine the required moment of inertia of the girders:

$$V_1 = \frac{M_{FL}}{B_E} = \frac{1{,}210}{112} = 10.8 \text{ kips} \qquad (10)$$

$$I_{g_1} = \frac{V_1 L_1^2}{12\theta E}$$

$$= \frac{10.8 \times 30^2 \times 144}{12 \times 2.66 \times 10^{-3} \times 29{,}000} = 1{,}520 \text{ in.}^4 \qquad (27)$$

$$I_{g_2} = g\lambda^2 I_1 = 1.3 \times 0.44 \times 1{,}520 = 870 \text{ in.}^4 \qquad (26)$$

NOMENCLATURE

A_e = cross-sectional area of end column
A_i = cross-sectional area of i-th column
a_e = distance of end column from neutral axis
a_i = distance of i-th column from neutral axis
B = width of bent
B_E = equivalent base
c = column shear coefficient
D_i = direct stress intensity of i-th column
E = modulus of elasticity = 29,000,000 psi
f = unit stress in end column due to wind forces
g_M, g_s = girder moment and shear coefficients
H = total height of bent
H_n^B = total shear force acting below n-th floor
H_n^T = total shear force acting above n-th floor
h_n = story height of n-th floor
I = moment of inertia of column group
I_c, I_g = column and girder moments of inertia
K_i = relative column direct stress coefficient of i-th column
L_i = distance between i-th and $(i + 1)$-th columns
M_n^B = wind moment acting below n-th floor
M_n^T = wind moment acting above n-th floor
M_{FL} = floor moment
M_i^{CT} = column moment at column i above floor level
M_i^{CB} = column moment at column i below floor level
M_i^{GR} = girder moment at right-hand face of column i
M_i^{GL} = girder moment at left-hand face of column i
M_x = real bending moment due to wind forces at x
m_x = virtual bending moment due to unit virtual force applied at x
N_i^B = column direct force in i-th column acting below the floor
N_i^T = column direct force in i-th column acting above the floor
R_i = relative column area of i-th column
r_g = flexural rigidity of girders
r_c = flexural rigidity of columns
S_i^B = column shear force in i-th column acting below the floor
S_i^T = column shear force in i-th column acting above the floor
V_e = girder shear force acting in the end bay
V_i = girder shear force acting in the i-th bay
W_n = concentrated wind load at n-th floor
w = uniformly distributed wind load
x = height from base of structure
y_n = distance from top of structure
α_n = allowable deformation angle in the n-th story
Δ = total deflection
Δ_c = deflection due to bending of columns
Δ_c' = deflection due to axial deformation of columns
Δ_g = deflection due to bending of girders
Δ_x = horizontal deflection at distance x
λ = span ratio
θ = deformation angle due to bending of girders
ϕ = deformation angle due to bending of columns
ψ = deformation angle due to axial deformation of columns

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