Review and Evaluation of the Separation Factor Approach for Structural Reliability

ISABELLE M. JOLLYMORE, KYLE TOUSIGNANT, and JEFFREY A. PACKER

ABSTRACT

Studies have indicated that resistance factors calculated using a first-order second moment reliability method that uncouples the load effect from the resistance, termed the "separation factor approach" (SFA), differ considerably from those calculated using more accurate methods that also consider statistical variations in both the load effects and resistance. This can be attributed, in part, to the SFA implementing a separation factor, α, equal to 0.55, which was determined from tentative loading criteria and statistics in the 1970s. This paper amalgamates the disparate literature/background on the SFA and investigates its sources of error to illustrate its inherent assumptions and limitations. Three studies are conducted whose results are used to recommend appropriate separation factors (with associated bounds) for use in the SFA when determining resistance factors for steel components.

It is found that $\alpha = 0.55$ was calibrated to an atypical range of live-to-dead load ratios and small values of V_R, which undermines its applicability when used in conjunction with modern-day statistics. Despite this, $\alpha = 0.55$ is found to perform well when the reliability index, β, is equal to 3.0. For $\beta = 3.5$ and $\beta = 4.0$, $\alpha = 0.70$ and $\alpha = 0.80$, respectively, give results that agree with more accurate reliability methods at a live-to-dead load ratio of 3.0.

KEYWORDS: structural reliability, first-order second moment method, reliability index, resistance factors, separation factor.

INTRODUCTION

The load and resistance factor design (LRFD) philosophy uses load and resistance factors to achieve an acceptably small notional probability of failure for steel members, connections, connectors, and whole structures. Load factors are used to increase the load effect, and resistance factors are used to decrease the resistance to emulate a worst-case scenario. Load and resistance factors are chosen such that, if the LRFD design criterion is satisfied under these worst-case conditions, then the notional probability of failure is deemed acceptably small. LRFD is an alternative to the allowable strength design (ASD) philosophy, which aims for the resistance to be greater than the load effect by a factor of safety.

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Various reliability methods are used to calculate resistance factors for steel components. A resistance distribution is developed for the component using a representative sample of adequate size, and the statistics pertaining to this are extracted; the same is done for the loading if it is assumed to contribute to reliability. Then, for a target level of reliability (embodied by the reliability index, which corresponds to a specific probability of failure over the component's design life), the resistance factor can be calculated. These methods can also be employed to calculate and compare reliability indices for a given resistance factor.

The separation factor approach (SFA) is a reliability method that arose from the work of Galambos and Ravindra (1973). They derived an equation for the resistance factor, ϕ, that conveniently simplifies the calculation by uncoupling the load effect from the resistance in the ultimate limit state design equation, thereby establishing the inherent assumption that resistance alone determines reliability according to the SFA. This simplification was achieved by introducing a separation factor, α, as proposed by Lind (1971), for the resistance and each load component. Historically, the separation factor has been taken to be 0.55, as per the recommendation of Galambos and Ravindra.

As recently as 2010, use of $\alpha = 0.55$ was endorsed by the Structural Stability Research Council (SSRC) in Appendix B.10 of the SSRC Guide (Ziemian, 2010). This factor continues to be used in conjunction with the SFA for reliability analyses in academia and by some material standards

technical committees for resistance factor calculations (CSA, 2011). It has come to light more recently that resistance factors calculated using this approach can differ from those calculated using more accurate methods, particularly for the case where $\alpha = 0.55$ is adopted. As a result, CSA S408-11 (2011) stipulates that this approach should not be used for the calculation of resistance factors. On the other hand, the SFA equation is specified in Commentary Section C2.3.5 of ASCE 7 (2022) to derive resistance factors, using a separation factor of approximately 0.7.

This paper addresses the need for a comprehensive review of the SFA by gathering significant literature on the topic since its inception and explores the implications of the governing assumptions and simplifications of the SFA to establish its merits, shortcomings, and potential sources of numerical error. Following this, three studies are carried out: (1) a recalibration of separation factors using modern statistics, (2) a comparison of the performance of the SFA in modern reliability studies to a more accurate reliability method that considers the loading in addition to resistance, and (3) a numerical experiment in which the SFA using the optimal separation factors determined from the previous two studies and the more-accurate reliability method are equated. The results of these three studies are then used to propose new separation factors with associated bounds for use in the SFA; this paper focuses specifically on reliability of steel components.

LOAD AND RESISTANCE FACTOR DESIGN

The LRFD philosophy may be expressed by the equation (Ziemian, 2010):

$$\phi R_n \geq \sum \gamma_i Q_i \tag{1}$$

which states that the factored resistance (or available strength), ϕR_n, must equal or exceed the effect of the factored loads, $\sum \gamma_i Q_i$.

The nominal resistance, R_n, is the strength of an element computed based on nominal material and geometrical properties, and the resistance factor, ϕ, is a dimensionless factor ≤ 1.0. The product ϕR_n embodies the uncertainties associated with the resistance, R, of a structural component. The products of each specified load effect, Q_i, and its corresponding load factor, γ_i, are summed to constitute the load effect side of the design criterion expressed by Equation 1. The term $\sum \gamma_i Q_i$ reflects the possibility of overloading and the uncertainties inherent in the calculation of the load effect(s), Q. The probability of failure of a structural component (i.e., the probability that the component will reach a specified ultimate limit state) is equal to the probability that the cumulative load effect, Q, exceeds the resistance, R (i.e., $R < Q$), as shown in Figure 1(a).

LRFD in North America can be traced to the work of Cornell (1969), and the postulate that the probability of failure, p_F, may be accurately determined from the probability density curve for the safety margin (or limit state function), $Z = R - Q$, where p_F is equal to the area under the curve where $Z < 0$, as shown in Figure 1(b). If R and Q are considered random variables and the arbitrary probability density curve of the safety margin Z is known, then a standardized variate, U (of Z), can be defined as:

$$U = \frac{Z - Z_m}{\sigma_Z} \tag{2}$$

where Z_m is the mean of Z and σ_Z is the standard deviation of Z. The expression that describes the probability of failure, p_F, can then be written as (Melchers and Beck, 2018):
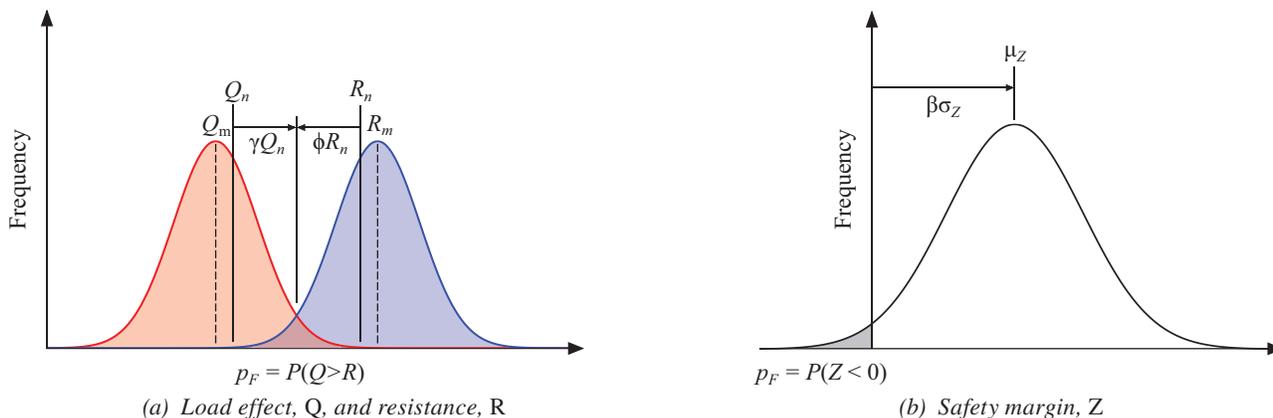

(a) Load effect, Q, *and resistance,* R

(b) Safety margin, Z

Fig. 1. Symbol, variable, and model definitions.

$$p_F = P\left[U < -\frac{Z_m}{\sigma_Z}\right] \tag{3a}$$

$$p_F = F_U\left[-\frac{Z_m}{\sigma_Z}\right] \tag{3b}$$

where F_U is the cumulative distribution function of U.

The above-mentioned postulate, combined with the principle of constant reliability (Lind, 1971), comprises the basis of first-order second moment reliability methods. Such reliability methods involve (1) linear (i.e., first-order) limit state functions and (2) computing a notional reliability measure, which is a function only of the means and variances (first and second moments, respectively) of the random variables (e.g., R and Q) rather than their probability distributions (Ellingwood et al., 1980). First-order reliability methods are often used because of their simplicity and ability to treat all uncertainties in a design problem in a consistent manner; hence, first-order methods have been used to develop LRFD criteria for steel components.

The ratio Z_m/σ_z in Equations 3a and 3b is referred to as the reliability index, β; it is a unitless measure of safety (or reliability) that represents the number of standard deviations between the mean value of Z (Z_m) and the failure condition ($Z = 0$) (Cornell, 1969). The reliability index has also been called the "safety index" or "performance index"; the term "reliability index" is adopted herein (Ang and Cornell, 1974; Galambos and Ravindra, 1973). The greater the value β, the greater the reliability, where the reliability is quantified as $1 - p_F$ (Allen, 1991). In other words, if σ_z remains constant, then a positive (right-hand) shift in Z_m will reduce p_F; this holds true for practically all probability distributions used for Z.

Equation 3b can therefore be written as:

$$p_F = F_U\left[-\beta\right] \tag{4}$$

According to Equation 4, β provides a direct measure of p_F (or, conversely, reliability) if the probability distribution of Z (and hence, the real values of Z_m and σ_z) is known. In practice, however, the distributions of R and Q (and Z) are invariably estimated. When this is done, p_F (calculated from Equation 4) is referred to as the "notional" probability of failure, indicating that it should be interpreted, at best, in a comparative sense (e.g., to evaluate the relative safety of various design alternatives) (Ellingwood et al., 1980).

To estimate the statistics for $Z = R - Q$ (i.e., Z_m and σ_Z) from corresponding estimates of those statistics for R and Q (i.e., R_m and Q_m; the mean values of R and Q, respectively; and σ_R and σ_Q, the corresponding standard deviations), the following statistical laws must be realized (Benjamin and Cornell, 1970):

1. If R and Q are random variables, $Z_m = (R - Q)_m = R_m - Q_m$.

2. Provided that R and Q are independent random variables, $\sigma_Z^2 = \sigma_R^2 + \sigma_Q^2$.

3. V_Z cannot be directly represented as a function of V_R and V_Q.

V_R and V_Q are the coefficients of variation (COVs) of R and Q, respectively. The COV of a random variable is a normalized measure of the dispersion of the probability distribution about the mean value and is equal to the ratio of the standard deviation to the mean. A relatively high COV is undesirable in the context of reliability because it is indicative of greater variation and, thus, lower certainty.

If R and Q are considered independent normal random variables, the probability density curve for $Z = R - Q$ is a normal curve with mean $Z_m = (R - Q)_m = R_m - Q_m$ and standard deviation $\sigma_Z = \sqrt{\sigma_R^2 + \sigma_Q^2}$ (Allen, 1991; Thoft-Christensen and Baker, 1982). However, according to Allen, a better model for structural reliability is to replace the distributions of R and Q by the distributions of their natural logarithms, ln$R$ and ln$Q$, and to fit normal curves to these such that the two tails overlap (denoting the region in which failure may occur) in the same way as shown in Figure 1(a). Hence, if R and Q are log-normally distributed, $Z = \ln R - \ln Q$ is taken as normally distributed, and:

$$\beta = \frac{\ln R_m - \ln Q_m}{\sqrt{\sigma_{\ln R}^2 + \sigma_{\ln Q}^2}} \tag{5a}$$

$$\beta = \frac{\left[\ln(R/Q)\right]_m}{\sqrt{\sigma_{\ln R}^2 + \sigma_{\ln Q}^2}} \tag{5b}$$

$$\beta = \frac{\ln(R_m/Q_m)}{\sqrt{\sigma_{\ln R}^2 + \sigma_{\ln Q}^2}} \tag{5c}$$

where $\ln R_m$, $\ln Q_m$, and $\ln(R_m/Q_m)$ are the mean values of $\ln R$, $\ln Q$, and $\ln(R/Q)$, respectively, and $\sigma_{\ln R}$ and $\sigma_{\ln Q}$ are the corresponding standard deviations of $\ln R$ and $\ln Q$, respectively. Log-normally distributed random variables can take only positive real values, which is true-to-life for most engineering measurements, and perhaps the benefit implied by Allen (1991).

By converting the mean values and standard deviations of $\ln R$ and $\ln Q$ to those for R and Q (i.e., through mathematical manipulation), Equation 5c for β can be written as follows (Thoft-Christensen and Baker, 1982; Allen, 1975; Allen, 1991; Galambos and Ravindra, 1973):

$$\beta = \frac{\ln\left[(R_m/Q_m)\sqrt{(1+V_Q^2)/(1+V_R^2)}\right]}{\sqrt{\ln(1+V_R^2)(1+V_Q^2)}} \tag{6}$$

Equation 6 is currently used in CSA S6:19 (CSA, 2019) to calculate β for bridges and bridge components. However, if V_R and V_Q are small [i.e., according to Ellingwood et al. (1980), if V_R and V_Q are both less than about 0.30], then:

$$\beta = \frac{\ln\left(R_m/Q_m\right)}{\sqrt{V_R^2 + V_Q^2}} \tag{7}$$

Historically, this assumption has been referred to as the "small-variance assumption," despite referring to the COVs and not the variances of the variables R and Q; for the sake of clarity, it will be termed the "small-COV assumption" herein. The format according to Equation 7 has long been the basis of LRFD criteria in North America (ASCE, 2022; CSA, 2011), as studies have shown that R and Q follow an approximately log-normal distribution (Xi and Packer, 2021; Ellingwood et al., 1980).

Small-COV Assumption

The accuracy of the simplification in Equation 7 is dependent not only on the magnitudes of V_R and V_Q, but also on the ratio R_m/Q_m. For adequate reliability, R_m must be greater than Q_m; however, the distance between these two means, $Z_m = R_m - Q_m$, is not generally known. The effect of the ratio R_m/Q_m on the relative accuracy of the simplification in Equation 7 for predicting β (i.e., how the small-COV assumption affects the calculated β) is shown in Figures 2(a)–2(c), using R_m/Q_m values of 1.70, 2.00, and 2.50, respectively. These values are chosen because Appendix E of the 1969 version of the CSA S16 standard, which was based on ASD, indicates a basic safety factor (R_m/Q_m) of 1.67. This has been employed in structural steel design for several decades, and the factor 1.67 has been rounded off to 1.70 here to avoid unwarranted precision. Table E1 of CSA S16-69 indicates further variations in this factor (from 1.67 to 2.50) based on the type of stress undergone by various elements (CSA, 1969).

In Figures 2(a)–2(c), the solid lines represent Equation 6, long dashed lines represent Equation 7, and short dashed (vertical/horizontal) lines represent the upper limits on V_R and V_Q for the small-COV assumption. Plotting with V_Q on the x-axis and V_R on the y-axis displays the actual equations as somewhat elliptical curves and the approximate equations as circles.

It is evident that when $V_R < 0.30$ and $V_Q < 0.30$ there is a minimum level of reliability (i.e., value of β) associated with R_m/Q_m, which increases with increasing R_m/Q_m [and $\ln(R_m/Q_m)$] as this causes the curves to move further from the origin. It is also clear that for values of V_R that are small (i.e., approaching zero), the approximation given by Equation 7 will underestimate β for a fixed V_R and V_Q. Similarly, for values of V_Q that are small, the approximation will overestimate β for a fixed V_R and V_Q.

To investigate these trends further, a numerical experiment was performed wherein values between 0 and 0.30 were randomly selected for V_R and V_Q, and corresponding values of β and its approximation, denoted β_a, were calculated using Equations 6 and 7, respectively, to determine the percent error in the approximation of β, $\varepsilon_\beta = (\beta - \beta_a)/\beta \times 100\%$. This was performed 100,000 times for each of $R_m/Q_m = 1.70$, $R_m/Q_m = 2.00$, and $R_m/Q_m = 2.50$. The mean percent error, $\overline{\varepsilon}$, the variance of the percent error, σ_ε^2, the maximum percent error, ε_{MAX}, and the minimum percent error, ε_{MIN}, were calculated for each R_m/Q_m. These parameters are calculated to illustrate the distribution of ε_β, as well as best-case ($\varepsilon_{MIN} \leq \varepsilon_\beta \leq 0$) and worst-case ($\varepsilon_{MAX}$) scenarios. It is found that as R_m/Q_m increases, $\overline{\varepsilon}$ increases and σ_ε^2 decreases; the exact values from the experiment are shown in Table 1. Note that $\varepsilon_\beta > 0$ corresponds to an underestimate by Equation 7 and $\varepsilon_\beta < 0$ corresponds to an overestimate by Equation 7, and thus for all three cases, the approximation results in an underestimate on average.

Whenever a small value of V_R was selected, Equation 6 yielded a larger β than the approximation by Equation 7 (i.e., a positive error), proving the earlier point that the approximation underestimates β when V_R approaches zero. Similarly, whenever a small value of V_Q was selected, the actual equation yielded a smaller β than the approximation (i.e., a negative error), proving that the approximation overestimates β when V_Q approaches zero. Another trend that is discernable from Figures 2(a)–2(c) and from the numerical experiment is that the approximation improves as β increases. Whenever small values of both V_R and V_Q were selected, along with a large value of β, the percent error due to the approximation by Equation 7 was approximately zero.

DERIVATION OF THE SEPARATION FACTOR APPROACH

The governing criterion for LRFD can be expressed in the form of a design criterion (Cornell, 1969):

$$R_m \geq \theta Q_m \tag{8}$$

where the central safety factor, θ, combines the uncertainties of resistance and load effects.

If the small-COV assumption discussed previously (Ellingwood et al., 1980) is valid, Equations 7 and 8 can be used to express the central safety factor approximately as:

$$\theta = \exp\left(\beta\sqrt{V_R^2 + V_Q^2}\right) \tag{9}$$

where Equation 9 implies that $\theta = R_m/Q_m$. Otherwise, Equations 6 and 8 are used to write the equation accurately as:

Table 1. Mean, Variance, Maximum, and Minimum Error in the Approximation of β Using Equation 7				
R_m/Q_m	$\bar{\varepsilon}$ (%)	σ_ε^2 (%²)	ε_{MAX} (%)	ε_{MIN} (%)
1.70	0.866	11.7	9.49	−6.50
2.00	0.909	6.99	7.87	−4.34
2.50	0.948	4.14	6.54	−2.68

(a) For $R_m/Q_m = 1.70$

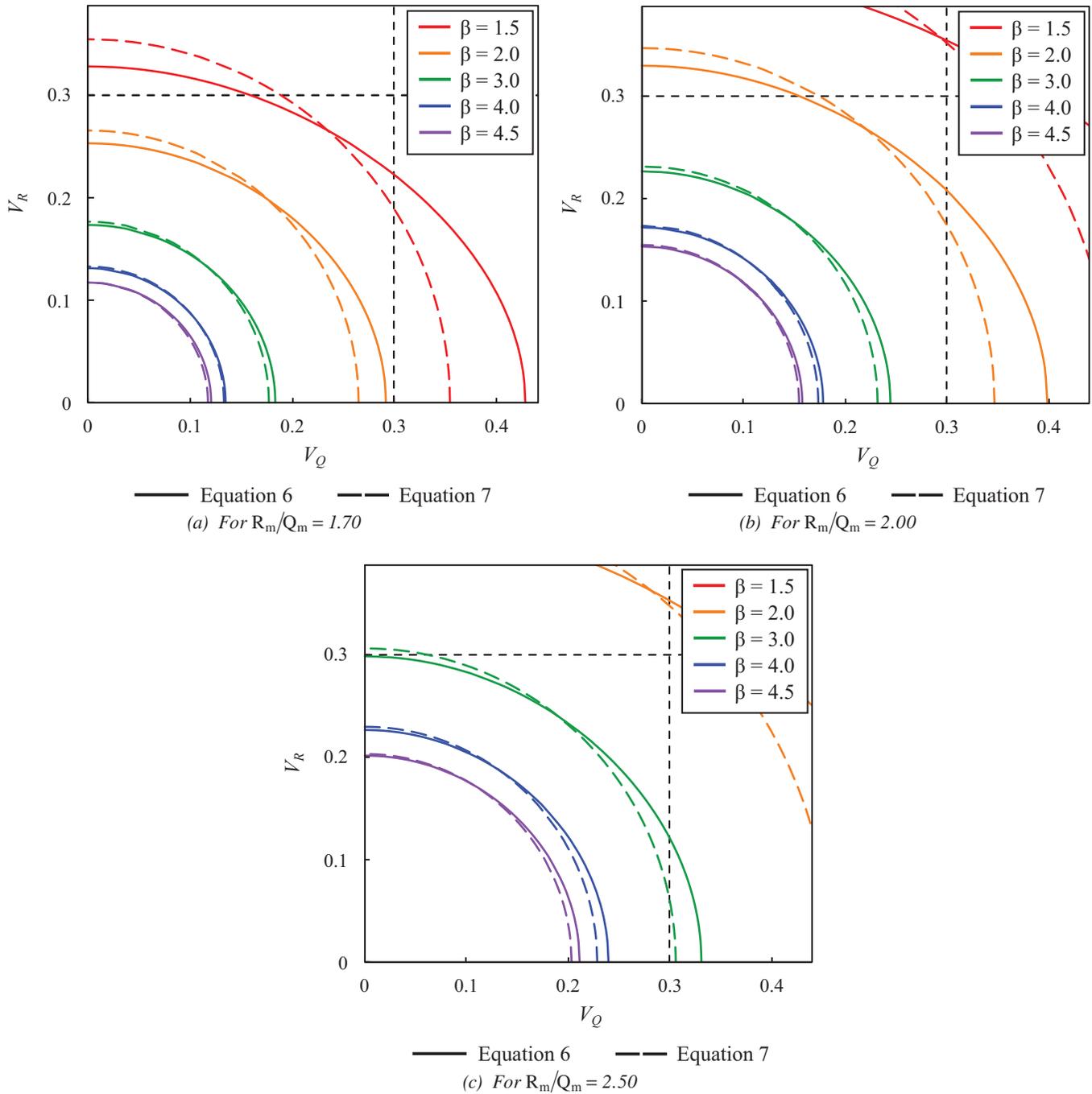*(b) For* $R_m/Q_m = 2.00$

(c) For $R_m/Q_m = 2.50$

Fig. 2. Approximations to various reliability indices, β.

$$\theta = \frac{\exp\left(\beta\sqrt{\ln(1+V_R^2)(1+V_Q^2)}\right)}{\sqrt{\dfrac{(1+V_Q^2)}{(1+V_R^2)}}} \quad (10)$$

The small-COV form of the central safety factor equation (i.e., Equation 9) is used in the SFA. If the COVs are not small, and if any of the particular cases addressed in the previous subsection hold true, use of this small-COV form can result in excessive error.

The root term in Equation 9 prevents the independent treatment of the resistance and load terms. It is advantageous to separate the COVs so that when a target β is chosen, the resistance factor, ϕ, can be evaluated for different loading uncertainties. Additionally, this would enable γ_i for different load types to be evaluated independent of other load types and of resistance uncertainties.

Lind (1971) showed that the root expression in Equation 9, or any two-variable expression in that form, could be linearized by introducing a so-called separation factor, α:

$$\sqrt{V_R^2 + V_Q^2} \approx \alpha(V_R + V_Q) \quad (11a)$$

where

$$\alpha = \frac{\sqrt{1+(V_R/V_Q)^2}}{1+(V_R/V_Q)} \quad (11b)$$

As shown by the dotted line in Figure 3(a), if V_R/V_Q is near unity, α is relatively constant. This is not always practical; however, if V_R/V_Q is instead restricted to the range $\frac{1}{3} \le V_R/V_Q \le 3$, then α can be set equal to 0.75 with less than 6% error (Lind, 1971), as demonstrated in Figure 3(b).
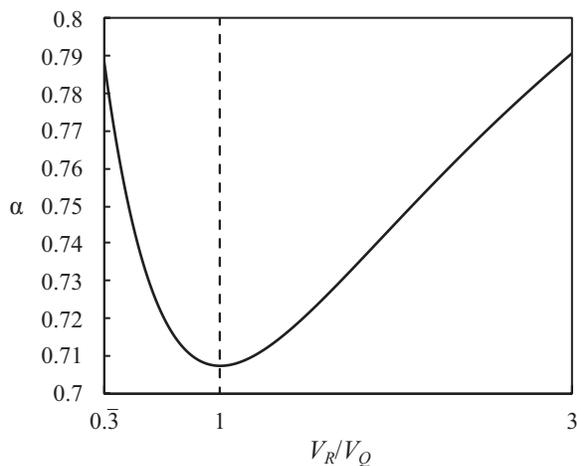
Therefore, if $V_R < 0.30$ and $V_Q < 0.30$ to satisfy the small-COV assumption, and $\frac{1}{3} \le V_R/V_Q \le 3$, Equation 8 can be written as

$$\exp(-\beta\alpha V_R)R_m \ge \exp(\beta\alpha V_Q)Q_m \quad (12)$$

or as

$$\delta_R\exp(-\beta\alpha V_R)R_n \ge \delta_Q\exp(\beta\alpha V_Q)Q_n \quad (13)$$

where the bias coefficient for the resistance, δ_R, is the ratio of the mean resistance to the nominal resistance, R_m/R_n, Q_n is the nominal (or specified) total load effect, and δ_Q is the bias coefficient for the total load effect.

Equation 13 takes a similar form as the criterion in Equation 1. Comparing the two, it follows that:

$$\phi = \delta_R\exp(-\alpha\beta V_R) \quad (14)$$

Equation 14 is the basic form of the equation used in many codes, standards, and guides (e.g., Ravindra and Galambos, 1978; Fisher et al., 1978; Ziemian, 2010; ASCE, 2022) to calculate resistance factors for a target reliability index (or reliability indices for a target resistance factor). This equation clearly demonstrates the philosophy that resistance is the sole determinant of reliability, which is central to the SFA. In the next section, the right-hand side of Equation 13 is separated further into constituent load effects.

Consideration of Different Load Types

Equation 13 was derived by treating the load effect, Q, as if it originated from a single load type acting on a structural steel component. However, in general, Q arises due to effects from dead load, D, live load, L, wind load, W, and other actions.

(a) Values of α from Equation 11a
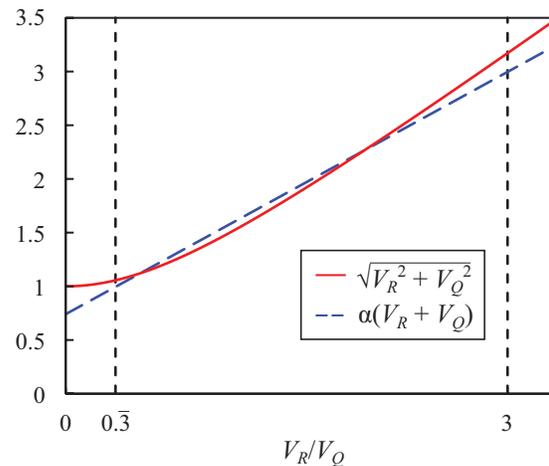
(b) Linearization with $\alpha = 0.75$

Fig. 3. Variation of α with V_R/V_Q, and accuracy of a single α-value.

In the following, the COV of the total load effect, V_Q, and the central safety factor approximation, θ_a, are derived as shown in Appendix B of Galambos and Ravindra (1973). From these workings, equations for ϕ and γ_i are also defined. Galambos and Ravindra considered two common cases for the load effect side of the first-order second moment design criterion: (1) dead and lifetime maximum live load and (2) dead, sustained live load and lifetime maximum wind load.

Dead and Lifetime Maximum Live Load

Galambos and Ravindra (1973) assumed the load effect, Q, for combined dead and live gravity load, to have the following form:

$$Q = ET \tag{15a}$$

where

$$T = c_D AD + c_L BL \tag{15b}$$

and where D and L are random variables representing dead and live load intensities, respectively, which embody uncertainties due to the idealization of loads that vary randomly in space and time with equivalent uniformly distributed or concentrated design loads; c_D and c_L are deterministic influence coefficients that transform the dead and live load intensities into dead and live load effects, respectively; A and B are random variables reflecting uncertainties in the transformation of the idealized design loads into dead and live load effects, respectively; T is a random variable denoting the total load undergone by a member; and E is a random variable representing the uncertainties in structural analysis.

Assuming E and T are uncorrelated, it follows from Equation 15a that:

$$V_Q = \sqrt{V_E^2 + V_T^2 + V_E^2 V_T^2} \tag{16}$$

where V_E and V_T are the COVs of E and T, respectively. If both V_E and V_T are less than 0.30, the error introduced by eliminating the product term does not exceed 2.2%. Thus, by neglecting the product term, Equation 16 becomes:

$$V_Q = \sqrt{V_E^2 + V_T^2} \tag{17}$$

The equation for V_T can be written as:

$$V_T = \frac{\sqrt{c_D^2 A_m^2 D_m^2 (V_A^2 + V_D^2) + c_L^2 B_m^2 L_m^2 (V_B^2 + V_L^2)}}{c_D A_m D_m + c_L B_m L_m} \tag{18}$$

where the numerator represents the standard deviation of T, denoted σ_T, and the denominator is the mean of T, denoted T_m. It is difficult to obtain information about the distributions of the random variables A and B, so it has

been common historically to take $A_m = B_m = 1.0$, which is valid for uniformly distributed dead and live loads (Jeong, 1981). Thus, the equation for V_Q can be rewritten by inserting Equation 18 into Equation 17, where $A_m = B_m = 1.0$ and $c_D = c_L = 1.0$:

$$V_Q = \sqrt{V_E^2 + \frac{D_m^2 (V_A^2 + V_D^2) + L_m^2 (V_B^2 + V_L^2)}{(D_m + L_m)^2}} \tag{19}$$

The governing criterion for the separation factor can be carried further to include the effects of multiple load types (in this case, D and L). Firstly, Equation 12 can be written as:

$$\exp(-\beta \alpha_R V_R) R_m \ge \exp(\beta \alpha_Q V_Q) Q_m \tag{20}$$

where two distinct separation factors—α_R for the resistance side and α_Q for the load side—are introduced to encourage a better approximation of Equation 8.

Noting from Equation 17 that $V_Q = \sqrt{V_E^2 + V_T^2}$, and using the concept illustrated by Equation 11a, Equation 20 can be rewritten as:

$$\exp(-\beta \alpha_R V_R) R_m \ge \tag{21}$$
$$\left[\exp(\beta \alpha_Q \alpha_E' V_E) E_m \right] \left[\exp(\beta \alpha_Q \alpha_T V_T) T_m \right]$$

The second exponential term on the right-hand side of Equation 21 can be approximated further by using the constant and linear terms of a Taylor series exponential expansion [i.e., $\exp(x) = 1 + x$]:

$$\exp(\beta \alpha_Q \alpha_T V_T) T_m \approx (1 + \beta \alpha_Q \alpha_T V_T)(D_m + L_m) \tag{22a}$$

$$\exp(\beta \alpha_Q \alpha_T V_T) T_m \approx \tag{22b}$$
$$\left(1 + \frac{\beta \alpha_Q \alpha_T \sqrt{D_m^2 (V_A^2 + V_D^2) + L_m^2 (V_B^2 + V_L^2)}}{D_m + L_m} \right)(D_m + L_m)$$

where Equation 18 was substituted into Equation 22a to get Equation 22b. An error is introduced with this simplification, which is investigated in the next subsection.

The square root term in Equation 22b can be separated as proposed by Lind (1971) (in Equation 23b), and after rearranging, the final approximation is achieved in Equation 23d:

$$\exp(\beta \alpha_Q \alpha_T V_T) T_m \approx$$
$$(D_m + L_m) + \beta \alpha_Q \alpha_T \sqrt{D_m^2 (V_A^2 + V_D^2) + L_m^2 (V_B^2 + V_L^2)} \tag{23a}$$

$$\exp(\beta \alpha_Q \alpha_T V_T) T_m \approx$$
$$D_m + L_m + \beta \alpha_Q \alpha_T \alpha' \left(D_m \sqrt{V_A^2 + V_D^2} + L_m \sqrt{V_B^2 + V_L^2} \right) \tag{23b}$$

$$\exp\left(\beta\alpha_Q\alpha_T V_T\right)T_m \approx D_m + L_m +$$
$$\beta\alpha_Q\alpha_T\alpha'_D D_m\sqrt{V_A^2 + V_D^2} + \beta\alpha_Q\alpha_T\alpha'_L L_m\sqrt{V_B^2 + V_L^2}$$

$$(23c)$$

$$\exp\left(\beta\alpha_Q\alpha_T V_T\right)T_m \approx \left(1 + \beta\alpha_Q\alpha_T\alpha'_D\sqrt{V_A^2 + V_D^2}\right)D_m +$$
$$\left(1 + \beta\alpha_Q\alpha_T\alpha'_L\sqrt{V_B^2 + V_L^2}\right)L_m$$

$$(23d)$$

If Equation 23d is substituted into Equation 21, and the products of the separation factor terms are combined into individual separation factors (i.e., $\alpha_E = \alpha_Q\alpha'_E$, $\alpha_D = \alpha_Q\alpha_T\alpha'_D$, and $\alpha_L = \alpha_Q\alpha_T\alpha'_L$), then:

$$\exp\left(-\beta\alpha_R V_R\right)R_m \geq \left[\exp\left(\beta\alpha_E V_E\right)E_m\right]$$
$$\left[\left(1 + \beta\alpha_D\sqrt{V_A^2 + V_D^2}\right)D_m + \left(1 + \beta\alpha_L\sqrt{V_B^2 + V_L^2}\right)L_m\right]$$

$$(24)$$

Given that the first-order second moment design criterion for dead and live load can be written as:

$$\phi R_m \geq \gamma_E E_m\left(\gamma_D D_m + \gamma_L L_m\right) \tag{25}$$

where γ_E is the load factor accounting for uncertainties in structural analysis, γ_D is the dead load factor, and γ_L is the live load factor, it is implied that:

$$\phi = \delta_R\exp\left(-\alpha_R\beta V_R\right) \tag{26a}$$

$$\gamma_E = \exp\left(\alpha_E\beta V_E\right) \tag{26b}$$

$$\gamma_D = 1 + \alpha_D\beta\sqrt{V_A^2 + V_D^2} \tag{26c}$$

$$\gamma_L = 1 + \alpha_L\beta\sqrt{V_B^2 + V_L^2} \tag{26d}$$

R_m is obtained by rearranging Equation 24 and writing it as an equality, as shown in Equation 27a. Assuming $\theta = R_m/Q_m$, dividing both sides by $Q_m = E_m T_m = E_m(D_m + L_m) = E_m(D_m/L_m + 1)$ gives the central safety factor approximation in Equation 27b:

$$R_m = \frac{\exp\left(\beta\alpha_E V_E\right)E_m\left[\left(1 + \beta\alpha_D\sqrt{V_A^2 + V_D^2}\right)\dfrac{D_m}{L_m} + \left(1 + \beta\alpha_L\sqrt{V_B^2 + V_L^2}\right)\right]}{\exp\left(-\beta\alpha_R V_R\right)}$$

$$(27a)$$

$$\theta_a = \frac{\exp\left(\beta\alpha_E V_E\right)\left[\left(1 + \beta\alpha_D\sqrt{V_A^2 + V_D^2}\right)\dfrac{D_m}{L_m} + \left(1 + \beta\alpha_L\sqrt{V_B^2 + V_L^2}\right)\right]}{\left(\dfrac{D_m}{L_m} + 1\right)\exp\left(-\beta\alpha_R V_R\right)}$$

$$(27b)$$

Note that both θ and θ_a are technically approximate. As θ_a introduces two additional assumptions on top of the initial small-COV assumption (i.e., simplification of the V_Q equation using another small-COV assumption, and use of the linear Taylor series approximation), it is deemed an approximation of θ.

The values of the four separation factors α_R, α_E, α_D, and α_L are selected to minimize the error in the approximation of the central safety factor, which is defined as $\varepsilon_\theta = (\theta - \theta_a)/\theta$.

Dead, Sustained Live Load, and Lifetime Maximum Wind Load

For this case, the load effect, Q, was assumed to take the same form as in the case of dead and live load (i.e., the form given by Equation 15a). The random variable T can be expressed as:

$$T = c_D AD + c_L BL + c_W CW \tag{28}$$

where W is a random variable representing wind load intensity, c_W is the deterministic influence coefficient that transforms the wind load intensity into wind load effect, and C is a random variable reflecting uncertainties in the transformation of the idealized design load into wind load effect.

Applying the same rationale and assumptions that were employed for the case of dead and live load to this case, the approximation to the central safety factor is derived as:

$$\theta_a = \frac{\exp\left(\beta\alpha_E V_E\right)\left[\left(1 + \beta\alpha_D\sqrt{V_A^2 + V_D^2}\right) + \left(1 + \beta\alpha_L\sqrt{V_B^2 + V_L^2}\right)\dfrac{L_m}{D_m} + \left(1 + \beta\alpha_W\sqrt{V_C^2 + V_W^2}\right)\dfrac{W_m}{D_m}\right]}{\left(1 + \dfrac{L_m}{D_m} + \dfrac{W_m}{D_m}\right)\exp\left(-\beta\alpha_R V_R\right)}$$

$$(29)$$

where

$$V_Q = \sqrt{V_E^2 + \frac{c_D^2 A_m^2 D_m^2\left(V_A^2 + V_D^2\right) + c_L^2 B_m^2 L_m^2\left(V_B^2 + V_L^2\right) + c_W^2 C_m^2 W_m^2\left(V_C^2 + V_W^2\right)}{\left(c_D A_m D_m + c_L B_m L_m + c_W C_m W_m\right)^2}}$$

$$(30)$$

and $A_m = B_m = C_m = 1.0$ and $c_D = c_L = c_W = 1.0$. The first-order second moment design criterion for dead, live, and wind load can be written as:

$$\phi R_m \geq \gamma_E E_m\left(\gamma_D D_m + \gamma_L L_m + \gamma_W W_m\right) \tag{31}$$

where γ_W is the wind load factor. By comparing the form of Equation 31 to that of Equation 29, it is implied that:

$$\gamma_W = 1 + \alpha_W\beta\sqrt{V_C^2 + V_W^2} \tag{32}$$

The equations for ϕ, γ_E, γ_D, and γ_L are unaltered by the inclusion of wind load (i.e., the same as Equations 26a–26d).

Linear Taylor Series Approximation

In the derivation of θ_a for both load cases, the exponential term for the load effect was approximated using the linear Taylor series expansion (for the dead and live load case, Equation 22a). Since the Taylor series is a summation of terms [where the series becomes equal to the function it is approximating as x (i.e., $\beta\alpha_Q\alpha_T V_T$) approaches infinity], this approximation will result in an underestimate for all x (i.e., $\beta\alpha_Q\alpha_T V_T$) greater than zero because it lacks most of the terms. From Figure 4, it is shown that as $\beta\alpha_Q\alpha_T V_T$ departs from zero (i.e., the point about which the Taylor series is centered), the underestimate becomes greater in magnitude.

As demonstrated in Figure 4, the linear function deviates from the exponential function rather quickly. It is useful to quantify this deviation with typical values of each of these parameters; these were chosen to be $0 \leq \beta \leq 5.0$, $0.5 \leq \alpha_Q \leq 0.7$, $0.5 \leq \alpha_T \leq 0.7$, and $0 \leq V_T \leq 0.3$ (i.e., $0 \leq \beta\alpha_Q\alpha_T V_T \leq 0.735$). The error does not change with $c_L L_m / c_D D_m$, as $c_L L_m / c_D D_m$ participates as a mere scaling factor in Equation 22b and is thereby unaffected by the introduction of the Taylor series approximation.

This investigation found that the maximum approximation error exceeds 16%. For a more typical range of reliability indices (i.e., $3.0 \leq \beta \leq 4.0$) and the same ranges of α_Q, α_T, and V_T as before, $\beta\alpha_Q\alpha_T V_T$ ranges from 0 to 0.588, and the maximum error is roughly 11.8%.

Error in the Approximation to the Central Safety Factor

As indicated previously, Galambos and Ravindra (1973) opted to minimize the error in the approximation to the central safety factor as a means of determining appropriate α-values for use in the SFA. To supplement their argument in Appendix B, Galambos and Ravindra (1973) presented Figures B-1 through B-6: Figures B-1–B-3 explored the dead and lifetime maximum live load case at different values of β, while Figures B-4–B-6 explored the dead, sustained live load, and lifetime maximum wind load case at different values of β. This subsection (1) outlines the process that led to the recommendation of a universal α-value ($\alpha = 0.55$) to be used in the SFA, and (2) investigates the Figures B-1–B-6 presented in Galambos and Ravindra (1973).

For their investigation, Galambos and Ravindra (1973) prescribed values (or values with associated frequencies of occurrence) to the variables which define the design situation for both loading cases; these are identified in Table 2.

The shorthand notations V_{AD}, V_{BL}, and V_{CW} were adopted to represent $\sqrt{V_A^2 + V_D^2}$, $\sqrt{V_B^2 + V_L^2}$, and $\sqrt{V_C^2 + V_W^2}$, respectively. Galambos and Ravindra (1973) used V_D, V_L, and V_W interchangeably with V_{AD}, V_{BL}, and V_{CW}, respectively, as it was assumed that $A_m = B_m = C_m = 1.0$; therefore, $V_A = V_B = V_C = 0$, which means that $V_D = V_{AD}$, $V_L = V_{BL}$, and $V_W = V_{CW}$.

For Figures B-1–B-3, $V_{Q,min} \approx 0.06$ (when V_E, V_{AD}, and V_{BL} are smallest and $c_D D_m / c_L L_m$ is largest) and $V_{Q,max} \approx 0.25$ (vice versa); because $V_Q < 0.30$, the small-COV assumption always holds. However, for Figures B-4 and B-6, $V_{Q,min} \approx 0.22$ (when V_E, V_{AD}, V_{BL}, V_{CW}, and $c_D D_m / c_W W_m$ are smallest and $c_D D_m / c_L L_m$ is largest) and $V_{Q,max} \approx 0.60$ (vice versa); in this case, the small-COV assumption does not always hold.

Galambos and Ravindra (1973) carried out an error minimization procedure, whereby various combinations of α_R, α_E, α_D, and α_L (and α_W for the wind case) were tested to determine the set that resulted in the most optimal (minimized) distribution of error. To do this, one of the three
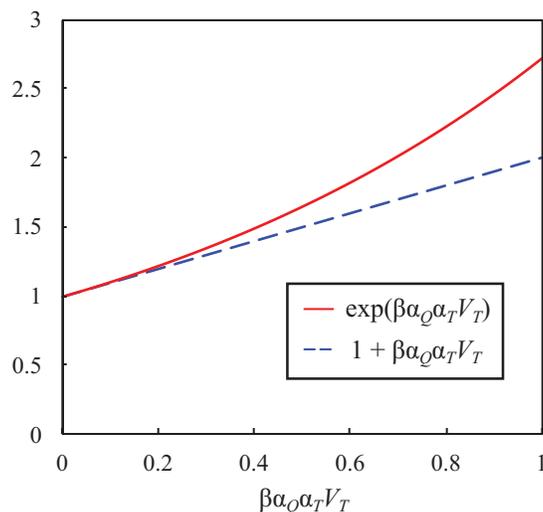

Fig. 4. Exponential function and the linear Taylor series approximation.

Table 2. Data Variables as Specified in Appendix B of Galambos and Ravindra (1973)				
Variable	**Figure(s)**	**Range**	**Value**	**Frequency of Occurrence**
V_R	B-1, B-2, B-3, B-4, B-5, B-6	0.10–0.15	0.13	1.0
V_E	B-1, B-2, B-3, B-4, B-5, B-6	0.05–0.15	0.05 0.10 0.15	0.30 0.50 0.20*
$\sqrt{V_A^2 + V_D^2} = V_{AD}$	B-1, B-2, B-3	0.02–0.10	0.02 0.06 0.10	0.20 0.60 0.20
	B-4, B-5, B-6		0.02 0.06 0.10	0.30 0.50 0.20
$\sqrt{V_B^2 + V_L^2} = V_{BL}$	B-1, B-2, B-3	0.10–0.40	0.10 0.25 0.40**	0.20 0.60 0.20
	B-4, B-5, B-6	0.40–0.70	0.40 0.55 0.70	0.50 0.30 0.20
$\sqrt{V_C^2 + V_W^2} = V_{CW}$	B-1, B-2, B-3	N/A	N/A	N/A
	B-4, B-5, B-6	0.30–0.40	0.30 0.35 0.40	0.50 0.30 0.20
$c_D D_m / c_L L_m$	B-1, B-2, B-3	1.00–4.00	1.00 1.60 4.00	0.20 0.50 0.30
	B-4, B-5, B-6	0.10–0.30	0.10 0.20 0.30	0.30 0.50 0.20
$c_D D_m / c_W W_m$	B-1, B-2, B-3	N/A	N/A	N/A
	B-4, B-5, B-6	0.25–1.00	0.25 0.50 1.00	0.50 0.30 0.20
β	B-1, B-4	3.0–4.0	3.0	1.0
	B-2, B-5		3.25	1.0
	B-3, B-6		3.5	1.0

* In the parameter list for Figure B-1 in Galambos and Ravindra, this was mistakenly written as 0.10.
** In the parameter list for Figure B-1 in Galambos and Ravindra, this was mistakenly written as 0.60.

prescribed values of V_E, V_{AD}, V_{BL}, and $c_D D_m / c_L L_m$ (and V_{CW} and $c_D D_m / c_W W_m$ for the wind case) were randomly sampled according to the frequencies indicated in Table 2; the values selected were used to calculate V_Q, θ, θ_a, and ε_θ in sequence. This process was repeated to an acceptable degree of convergence and for different values of β. For the dead and lifetime maximum live load case, Galambos and Ravindra found that $\alpha_R = 0.52$, $\alpha_E = 0.60$, $\alpha_D = 0.70$, and $\alpha_L = 0.60$ resulted in the least error, yielding a mean error of zero and a standard deviation of 3%; a confirmatory study by the authors found that the mean error was 0.075

and the standard deviation was 3.22%, which corroborates the claims by Galambos and Ravindra. The characteristics that signify an optimal error distribution were not explicitly defined in Galambos and Ravindra, so these are inferred to be (1) a mean error sufficiently close to zero and (2) a reasonably small standard deviation.

Galambos and Ravindra (1973) found that the separation factors were relatively insensitive to changes in the frequencies of occurrence of the varying parameters listed in Table 2, and used this observation as grounds for using a single constant separation factor, α, as opposed to multiple,

independent separation factors, in favor of simplicity. Thus, they explored the case where $\alpha = 0.55$ and produced histograms that depicted the distribution of ε_θ for different values of β. To confirm the results from Galambos and Ravindra, the authors repeated their error minimization process herein using 10,000 iterations, which resulted in acceptable convergence of the error distribution. Figures 5(a)–5(f) show comparisons of Figures B-1–B-6 as depicted in Appendix B of Galambos and Ravindra and the remakes of Figures B-1–B-6 resulting from the authors' error minimization process.

From Figures 5(a)–5(f), it becomes evident that Figure B-2 and Figures B-4–B-6 presented in Appendix B of Galambos and Ravindra (1973) do not align with the assigned values and frequencies of the parameters shown therein. The disparity between these histograms and their corresponding parameter lists was investigated by considering the following errors in the latter:

- For Figure B-2, the values of $c_D D_m / c_L L_m$ used to generate the histogram were the reciprocals of the $c_D D_m / c_L L_m$ values identified in the parameter list (i.e., instead of $c_D D_m / c_L L_m = \{1.00, 1.00, 1.60, 1.60, 1.60, 1.60, 1.60, 4.00, 4.00, 4.00\}$, $c_D D_m / c_L L_m = \{1.00, 1.00, 0.625, 0.625, 0.625, 0.625, 0.625, 0.25, 0.25, 0.25\}$ was used).

- For Figures B-4–B-6, the values of $c_D D_m / c_L L_m$ and $c_D D_m / c_W W_m$ used to generate the histograms were the reciprocals of the $c_D D_m / c_L L_m$ and $c_D D_m / c_W W_m$ values identified in the parameter lists; instead of $c_D D_m / c_L L_m = \{0.10, 0.10, 0.10, 0.20, 0.20, 0.20, 0.20, 0.20, 0.30, 0.30\}$ and $c_D \overline{D} / c_W \overline{W} = \{0.25, 0.25, 0.25, 0.25, 0.25, 0.50, 0.50, 0.50, 1.00, 1.00\}$, $c_D D_m / c_L L_m = \{10.0, 10.0, 10.0, 5.00, 5.00, 5.00, 5.00, 5.00, 3.33, 3.33\}$ and $c_D D_m / c_W W_m = \{4.00, 4.00, 4.00, 4.00, 4.00, 2.00, 2.00, 2.00, 1.00, 1.00\}$ were used.

- For Figure B-5, the values of V_L and V_W used to generate the histograms were completely different from the values identified in the parameter lists; through an iterative investigation by the authors, it was found that $V_L = \{0, 0, 0, 0, 0, 0.08, 0.08, 0.08, 0.14, 0.14\}$ and $V_W = \{0, 0, 0, 0, 0, 0.02, 0.02, 0.02, 0.16, 0.16\}$ produced a distribution much like that which Galambos and Ravindra presented.

Figures 6(a)–6(d) show that the aforementioned alterations result in error distributions that sufficiently match those of Galambos and Ravindra (1973).

Several of the statistical values that Galambos and Ravindra (1973) prescribed (i.e., the ranges and values in Table 2) are not in agreement with the accepted statistics of today; these differences are discussed in greater detail later. Most notably, the range of live-to-dead load ratios they assigned differ greatly from the values used in modern day. A typical live-to-dead load range for steel components at the ultimate limit state is $1.0 \le c_L L_m / c_D D_m \le 3.0$. The upper limit (i.e., $c_L L_m / c_D D_m = 3.0$) is of notable significance, as in AISC 360, the safety factors in ASD were calibrated to give the same structural reliability as resistance factors in LRFD at a live-to-dead load ratio of 3.0 (AISC, 2022).

Galambos and Ravindra (1973) assigned the range $1.0 \le c_D D_m / c_L L_m \le 4.0$ to Figures B-1 and B-3 and $3.33 \le c_D D_m / c_L L_m \le 10.0$ to Figures B-4–B-6; these correspond to $0.25 \le c_L L_m / c_D D_m \le 1.0$ and $0.10 \le c_L L_m / c_D D_m \le 0.30$, respectively, which are atypical ranges for steel components at the ultimate limit state. Figure B-2 was the only figure to which Galambos and Ravindra prescribed a reasonable live-to-dead load range: $1.0 \le c_L L_m / c_D D_m \le 4.0$ (or as they had written, $0.25 \le c_D D_m / c_L L_m \le 1.0$). However, the mean and variance of the error for Figure B-2 is much higher than the mean and variance of any of the other figures, which suggests that the selection of $\alpha = 0.55$ was based on the more unusual live-to-dead load cases (though still subject to the variability of the other parameters).

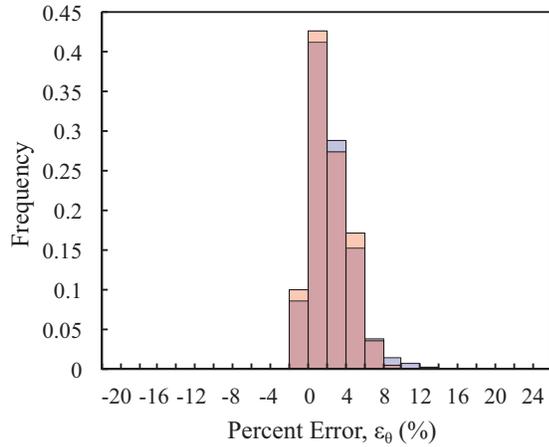## INVESTIGATING THE PERFORMANCE OF THE SEPARATION FACTOR APPROACH

In this section, a series of studies is performed on the SFA to investigate its performance with $\alpha_R = 0.55$ and when modified to reflect modern statistics. A recommendation is made based on the collective findings of these studies. These studies focus on the dead plus live load case only; counter to the dead, live, and wind load case, this case is not only prevalent in literature, but is generally more applicable to the steel components considered within the scope of this paper.
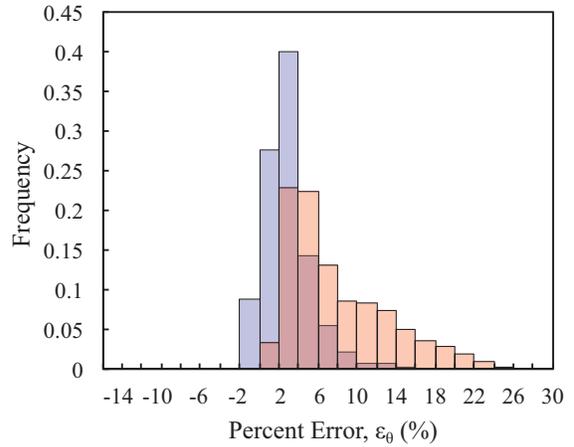
Separation Factor Recalibration

This study employs a methodology to recalibrate the separation factors used in the SFA. Using updated statistical values, optimal α-values are determined through a similar iterative process as was used by Galambos and Ravindra (1973), for $\beta = 3.0$ and $\beta = 4.0$, as well as $\delta_R = 1.00$ and $\delta_R = 1.30$ (i.e., four times for each combination of these).

Firstly, $V_E = 0.0$ (and similarly, $V_A = 0.0$ and $V_B = 0.0$) was chosen for the recalibration of α_R. Most modern reliability studies of steel members and components assume that the method of structural analysis is devoid of error (i.e., $E = 1.0$), manifested by no mention of V_E. Furthermore, according to the CSA S6:19 Commentary, statically determinate analysis has an associated bias coefficient and COV of 1.0 and 0.0, respectively (CSA, 2019).
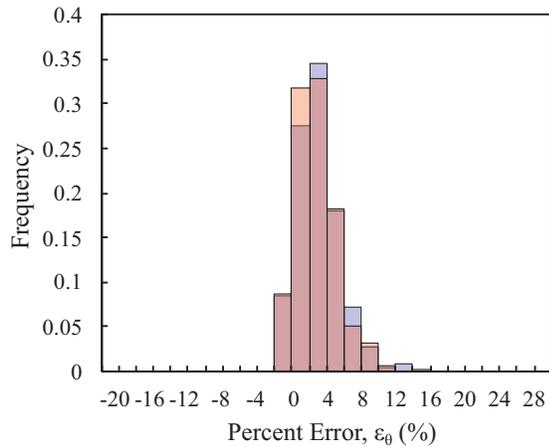
To begin the recalibration, the values or ranges of discrete values with associated frequencies of occurrence shown in Table 3 were assigned to each of the design parameters (COVs, bias coefficients, etc.). The distributions for V_R and δ_R were based on the results of 21 modern reliability studies
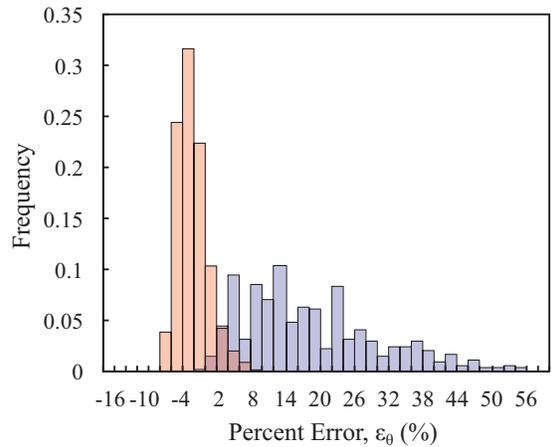
Fig. 5. Comparison of the current study results to original error distributions from Galambos and Ravindra (1973).

and statistical data collections, where $\delta_R = 1.30$ represents the mean of the δ_R distribution. Random sampling was then used to calculate ε_θ 50,000 times to produce an error distribution for all combinations of relevant α-values (i.e., α_D, α_L, and α_R) using increments of 0.01. The bounds of these are shown in Table 3. Here, 50,000 iterations were chosen because there are more possible combinations of design parameters than before, and thus more iterations were required for adequate convergence.

It is worth noting the difference between the values used by Galambos and Ravindra (1973) (i.e., the values in Table 2) and those in Table 3, as these generally demonstrate how the statistical values that are widely accepted among researchers have changed over the past 50 years. It was established that Galambos and Ravindra adopted an atypical range for $c_D D_m / c_L L_m$ in their calibration of the SFA and that $1.0 \le c_L L_m / c_D D_m \le 3.0$ (i.e., $0.33 \le c_D D_m / c_L L_m \le 1.0$) is a typical range; this is adopted in the recalibration study herein. Another notable difference can be found in V_R. Modern literature revealed a distribution of V_R akin to the simplified distribution adopted in Table 3, which contains values much higher than the upper limit adopted by Galambos and Ravindra. As a high V_R is indicative of a relatively poor correlation between nominal and measured resistance, Galambos and Ravindra essentially adopted a somewhat optimistic range of V_R in their calibration of
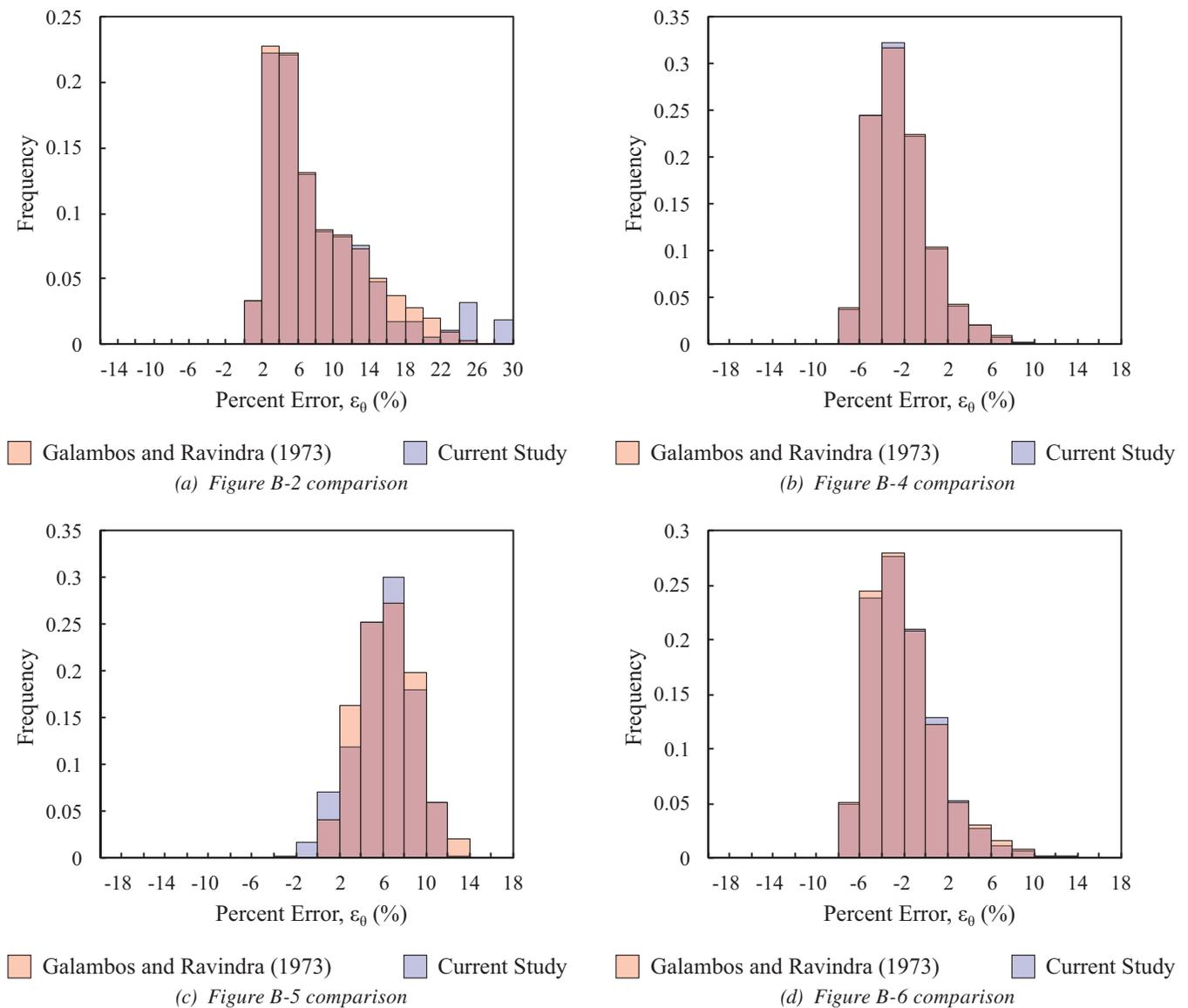

(a) Figure B-2 comparison

(b) Figure B-4 comparison

(c) Figure B-5 comparison

(d) Figure B-6 comparison

Fig. 6. Comparison of the modified current study results to original error distributions from Galambos and Ravindra (1973).

Table 3. Data Variables for Recalibrating the Separation Factor(s)		
Variable	**Value**	**Frequency of Occurrence**
V_R	0.06 0.10 0.14 0.18 0.22 0.26 0.30 0.34	0.02 0.20 0.24 0.10 0.16 0.18 0.05 0.05
V_E	0.0	1.0
V_A	0.0	1.0
V_D	0.10	1.0
V_B	0.0	1.0
V_L	0.27[†] 0.32[††]	0.33 0.67
δ_R	1.00 1.30	1.0 1.0
δ_D	1.05	1.0
δ_L	0.78[††] 0.90[†]	0.67 (when $V_L = 0.32$) 0.33 (when $V_L = 0.27$)
$c_D D_m / c_L L_m$	0.33 0.50 1.00	0.40 0.20 0.40
[†] Value from Bartlett et al. (2003) [††] Value from Schmidt and Bartlett (2002)		

the SFA, which may contribute to why $\alpha_R = 0.55$ produces less conservative (more optimistic) values of ϕ compared to more modern methods.

Combinations of α-values that meet the criteria for mean error, $\overline{\varepsilon}$, and coefficient of skewness (COS) of the error, υ, shown in Table 4 were chosen during the iterative process. Then, of the qualifying α-values, optimal combinations were selected based on those that yielded the minimum variance.

The values and thresholds shown in Table 4 are based on extensive numerical experiments performed on the data/statistics used by Galambos and Ravindra (1973) in the initial calibration of the SFA. With these data, it was found that a relatively small negative $\overline{\varepsilon}$ (i.e., slight overestimate on average), a limit on the positive skew (i.e., data forming a rightward tail), and sufficiently minimized variance were optimal distribution characteristics, which resulted in approximately <25% probability of positive error values (i.e., underestimates). Thus, using adjusted ranges of

α-values, an optimal set of α-values that repeated these results for modern statistics was investigated.

The amount of positive skew was limited by defining an upper threshold for the COS, which is a measure of skew in a set of data, where a positive COS indicates positively skewed data. The COS is defined as follows:

$$\upsilon = \frac{3(\overline{\varepsilon} - \dot{\varepsilon})}{\sigma_\varepsilon} \quad (33)$$

where $\dot{\varepsilon}$ is the median of the percent error and σ_ε is the standard deviation of the percent error. As seen in Table 4, only a slight positive skew was considered permissible, as greater skew to the right increases the magnitude of underestimates, which is unconservative. Recall that Galambos and Ravindra (1973) did not consider the COS as a criterion for error minimization, only mean error and standard deviation (variance). The results of implementing the above process are shown in Table 5.

Table 4. Thresholds for Separation Factors and Statistical Parameters

Bias Coefficient, δ_R	Parameter	Lower Limit (Inclusive)	Upper Limit (Inclusive)
1.00	α_R	0.60	0.90
	α_D	0.10	1.50
	α_L	0.10	1.50
	Mean, $\bar{\varepsilon}$	−3.00	−1.00
	COS, υ	N/A	0.50
1.30	α_R	0.80	1.20
	α_D	0.50	2.00
	α_L	0.50	2.00
	Mean, $\bar{\varepsilon}$	−9.00	−7.00
	COS, υ	N/A	0.50

Table 5. Optimal Separation Factors

Reliability Index, β	Bias Coefficient, δ_R	α_R	α_D	α_L	Variance
3.0	1.00	0.70	0.45	0.95	7.65
	1.30	0.80	1.00	1.65	11.8
4.0	1.00	0.70	0.30	1.05	13.1
	1.30	0.85	0.50	1.55	33.3

The results in Table 5 can be illustrated in histograms, like those used by Galambos and Ravindra (1973), to depict the distribution of error; these are shown in Figures 7(a)–7(d). As before, the statistical data in Table 3 and 50,000 iterations were used.

For each of the experiments used to produce Figures 7(a)–7(d), the percentage of iterations that gave an error value greater than zero was 16.4%, 1.91%, 17.3%, and 13.9%, respectively. Thus, in all cases, the percentage of positive error values (i.e., underestimates) was confirmed to be <25%.

Evaluating the SFA with Different Separation Factors

This study evaluates the performance of the SFA using different values of α_R when applied to four unique, published reliability studies (henceforth referred to as case studies). The aim of this study is to discern how the results vary between methods to hypothesize the scenarios in which the SFA deviates from the "true" value of ϕ, as well as the scenarios in which it converges on an acceptable value. Three calibrations of the SFA are utilized in this study:

1. Equation 26a using $\alpha_R = 0.55$ (i.e., the SFA as it was originally conceived).

2. Equation 26a with $\alpha_R = 0.70$ when $\beta = 3.0$ and when $\beta = 4.0$ (i.e., the α-values for $\delta_R = 1.00$).

3. Equation 26a with $\alpha_R = 0.80$ when $\beta = 3.0$ and $\alpha_R = 0.85$ when $\beta = 4.0$ (i.e., the α-values for $\delta_R = 1.30$).

Another existing reliability method is also presented in this study: the "approximate first-order reliability method" (or approximate FORM), which is outlined in Appendix B of CSA S408-11 (CSA, 2011). Contrary to the SFA, approximate FORM couples the load and resistance terms when calculating ϕ (CSA, 2011). The general equation for calculating ϕ using the approximate FORM methodology is:

$$\phi = \delta_R \frac{\sum \gamma_i Q_i}{Q_m} \exp\left(-\beta_T \sqrt{V_R^2 + V_Q^2}\right) \quad (34)$$

where β_T is the target reliability index (CSA, 2011). The preceding equation can be rewritten for the case of dead and live load, in terms of the live-to-dead load ratio, L_m/D_m (Schmidt and Bartlett, 2002):

$$\phi = \delta_R \frac{\gamma_D + \gamma_L (L_m/D_m)}{\delta_D + \delta_L (L_m/D_m)} \exp\left(-\beta_T \sqrt{V_R^2 + V_Q^2}\right) \quad (35)$$

V_Q is calculated using Equation 36, and like Equation 35, it also varies with L_m/D_m (CSA, 2024):

$$V_Q = \frac{\sqrt{(\delta_D V_D)^2 + [\delta_L V_L (L_m/D_m)]^2}}{\delta_D + \delta_L (L_m/D_m)} \quad (36)$$

Work by others (e.g., Rudman et al., 2022) has demonstrated that approximate FORM and Monte Carlo simulation (MCS) yield a similar plot of ϕ versus L_m/D_m. MCS is widely regarded as the most accurate method for determining resistance factors, but it comes at the cost of added complexity and computation time. Thus, it makes sense to compare the SFA using different α-values to approximate FORM, and for the sake of this study, to assume that approximate FORM gives "correct" resistance factors.

The case studies are derived from Driver et al. (2006), Tayyebi and Sun (2021), Rudman et al. (2022), and Fidalgo and Packer (2022). These publications contain reliability

analyses of specific steel components, and select values were extracted from each to be used in this study. The selected values are shown below in Table 6. Using the approximate FORM and the SFA as described above, ϕ-values were calculated for each column of parameters in Table 6, and the results are shown in Table 7.

Driver et al. (2006) and Tayyebi and Sun (2021) had $\beta_T = 4.5$ and $\beta_T = 2.6$, respectively, which are β-values for which optimal α-values were not calculated in the preceding recalibration. Thus, the optimal α-values for $\beta = 4.0$ and $\beta = 3.0$ were used for Driver et al. and Tayyebi and Sun, respectively. It should also be emphasized that $\beta = 2.6$ as used by Tayyebi and Sun corresponds to small-size, light-gage HSS and should not be used as a reliability index in applications pertaining to overall structural reliability.

Recall that the approximate FORM ϕ-values (henceforth denoted ϕ_{FORM}) are assumed to be the true ϕ-values
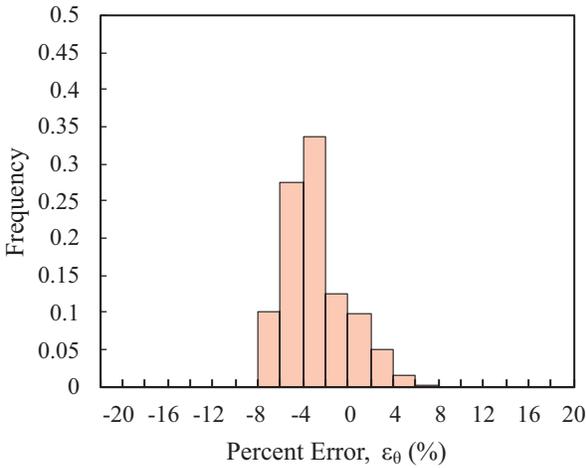
(a) Error distribution for $\beta = 3.0$ and $\delta_R = 1.00$
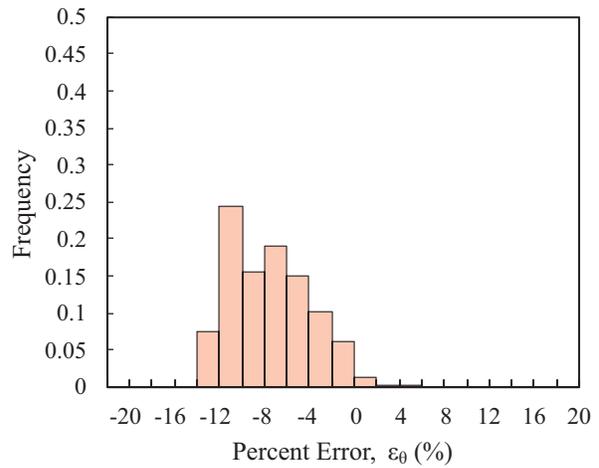
(b) Error distribution for $\beta = 3.0$ and $\delta_R = 1.30$
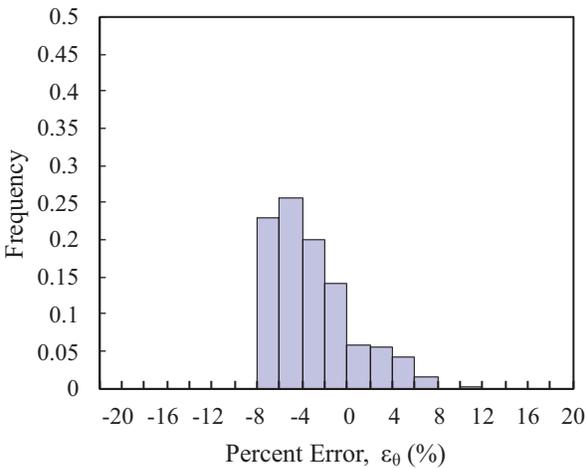
(c) Error distribution for $\beta = 4.0$ and $\delta_R = 1.00$

(d) Error distribution for $\beta = 4.0$ and $\delta_R = 1.30$

Fig. 7. Error distributions using new statistics and optimal α-values.

Table 6. Parameters from Case Studies

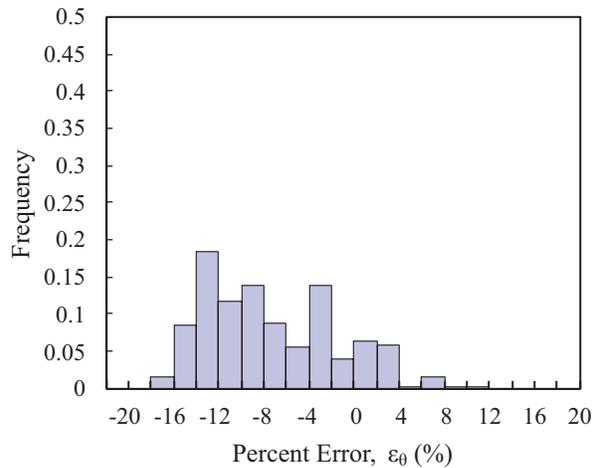| Variable	Driver et al. (2006)	Tayyebi and Sun (2021)	Rudman et al. (2022)	Fidalgo and Packer (2022)
V_D	0.10*	0.10*	0.10	0.10*
V_L	0.32*	0.32*	0.32	0.32*
V_R	0.091	0.237	0.247	0.335
β_T	4.5	2.6	3.0	4.0
γ_D[†]	1.2*	1.2	1.2	1.2*
γ_L[†]	1.6*	1.6	1.6	1.6*
δ_R	1.32	1.20	1.32	1.75
δ_D	1.05*	1.05*	1.05	1.05*
δ_L	0.78*	0.78*	0.78	0.78*

* Value was assigned for this study as it was not specified in the original study
[†] All values from ASCE 7 (ASCE, 2022)

Table 7. Resistance Factor Comparison

Method	Driver et al. (2006)	Tayyebi and Sun (2021)	Rudman et al. (2022)	Fidalgo and Packer (2022)
Approx. FORM ($L_m/D_m = 1.0$)	0.924	0.888	0.851	0.619
Approx. FORM ($L_m/D_m = 2.0$)	0.843	0.910	0.865	0.625
Approx. FORM ($L_m/D_m = 3.0$)	0.790	0.911	0.861	0.619
SFA(i)	1.054	0.855	0.878	0.838
SFA(ii)	0.991	0.780	0.786	0.685
SFA(iii)	0.932	0.733	0.730	0.560

in this study. A ϕ-value calculated using the SFA (henceforth denoted ϕ_{SFA}) is unconservative if it exceeds ϕ_{FORM} and is conservative if it falls below ϕ_{FORM}. All uses of conservative and unconservative will pertain to these definitions. Note that "slightly unconservative" isn't necessarily problematic, and "more conservative" does not necessarily equate to "better."

The results in Table 7 reinforce that the SFA generally requires a greater value of α_R to give conservative ϕ-values for higher reliability indices, which was originally demonstrated in the recalibration study. Both Driver et al. (2006) and Fidalgo and Packer (2022) had higher β_T and demonstrated that SFA(i) (i.e., $\alpha_R = 0.55$) yielded the least conservative ϕ_{SFA} of the three calibrations. Conversely, when $\beta_T \leq 3.0$, SFA(i)–SFA(iii) generally gave conservative results compared to approximate FORM (i.e., $\phi_{SFA} < \phi_{FORM}$), with SFA(i) giving the closest agreement.

V_R had a noticeable impact on the results, particularly on ϕ_{FORM}. When V_R is small (less than 0.10), ϕ_{FORM} varies noticeably with L_m/D_m, and when V_R is large (approaching or exceeding 0.30), ϕ_{FORM} varies minimally with L_m/D_m.

Because ϕ_{SFA} does not vary with loading, this variation makes it more difficult for ϕ_{SFA} to agree with ϕ_{FORM} over a range of L_m/D_m when V_R is smaller.

To ensure that these observed trends are not anomalous, it is important to proceed with a more general approach to identify whether these conclusions hold over an array of cases.

Equating the SFA and Approximate FORM

The SFA and approximate FORM both serve the same purpose but are built on differing key assumptions: the former assumes that resistance alone determines reliability, while the latter assumes that both loading and resistance impact reliability. As such, ϕ_{FORM} and ϕ_{SFA} tend to differ for a given set of statistics; this was shown quite clearly in the previous subsection.

In this final study, the value of α_R is adjusted continuously such that $\phi_{SFA} = \phi_{FORM}$ over a specified range of V_R-values. The values of the other parameters utilized in this study are shown in Table 8. Note that the only significant difference between these values and the values used

Table 8. Data Variables for Equating the SFA and Approximate FORM	
Variable	**Value or Range**
V_R	0.00–0.40
V_D	0.10
V_L	0.32[†]
γ_D[††]	1.2
γ_L[††]	1.6
δ_R	1.00
δ_D	1.05
δ_L	0.78[†]
[†] Value from Schmidt and Bartlett (2002)	
[††] Values from ASCE 7 (2022)	

in the recalibration study (i.e., the Table 3 values) is that a wider range of V_R is utilized in this case, with the intent of capturing the behavior of the reliability methods outside the typical range (i.e., for values of V_R exceeding 0.30 and approaching zero).

As noted before, the typical range for the live-to-dead load ratio in many steel applications is $1.0 \leq L_m/D_m \leq 3.0$ (some studies extend the lower bound to zero). This range is often used to calculate and plot ϕ for a chosen β_T (or vice versa) using approximate FORM in modern reliability studies. Hence, this study considers $L_m/D_m = 1.0$, $L_m/D_m = 2.0$, and $L_m/D_m = 3.0$ as three discrete cases. Given the values in Table 8, V_Q is equal to 0.148, 0.195, and 0.223 for each L_m/D_m, respectively.

To determine values of α_R that establish equality between the SFA and approximate FORM, Equation 26a and Equation 35 (i.e., the equations for ϕ_{SFA} and ϕ_{FORM}) are equated and rearranged for α_R:

$$\alpha_R = \frac{-\ln\left[\dfrac{\gamma_D + \gamma_L\left(L_m/D_m\right)}{\delta_D + \delta_L\left(L_m/D_m\right)}\exp\left(-\beta\sqrt{V_R^2 + V_Q^2}\right)\right]}{\beta V_R} \quad (37)$$

Using Equation 37 and the values in Table 8, α-values were calculated over the specified range of V_R. Figures 8(a)–8(c) show the results for $\beta = 3.0$, $\beta = 3.5$ and $\beta = 4.0$, respectively, and the aforementioned L_m/D_m ratios.

In the Figure 8 plots, the dotted vertical line represents the limit on V_R indicated by Ellingwood et al. (1980), while the dashed horizontal line represents the indicated value of α_R (or α as indicated in the figures). For the value of V_R where the dashed line intersects one of the curves, the α-value corresponding to the dashed line will give equality between the SFA and approximate FORM at that L_m/D_m. When one of the curves falls above the dashed line, this

means that a higher α-value than that indicated on the line is required for equality, and thus the SFA using the α-value on the line (0.55 for $\beta = 3.0$, 0.70 for $\beta = 3.5$, and 0.80 for $\beta = 4.0$) will give *unconservative* results (i.e., higher values of ϕ). When a curve falls below the line, the opposite is true: A lower α-value is required for equality, and the SFA using the α-value on the line gives *conservative* results (i.e., lower values of ϕ).

The α-values that represent the dashed lines displayed in Figures 8(a)–8(c) were chosen based on the results of the previous studies and by inspection of the plot trends. They represent the α-values that would somewhat minimize the magnitude of the error between ϕ_{SFA} and ϕ_{FORM} for each β, prioritizing the case of $L_m/D_m = 3.0$ (given by the blue curve) at which LRFD and ASD give equivalent structural reliability (AISC, 2022). This rationale assumes that the goal is to achieve the best agreement with approximate FORM, which is not necessarily the ideal result within the philosophy of SFA, as the aim of the SFA is to minimize the error in θ_a. However, considering the greater accuracy of approximate FORM, this approach is appropriate.

Figures 8(a)–8(c) both show that, as V_R approaches zero, the required α for equality diverges, beginning at around $V_R = 0.05$ for Figure 8(a) and $V_R = 0.10$ for Figures 8(b) and 8(c). Because the divergence approaches $+\infty$, the SFA becomes increasingly unconservative when using the α-value corresponding to the dashed line. This aligns with the observation made in the previous study: Because $V_R < 0.10$, the SFA gave unconservative values compared to approximate FORM.

The most prominent observation from the two previous studies is strongly supported by the results found here: that higher reliability indices warrant the use of higher values of α_R for use of the SFA. This and other observations are reflected in the following recommendation.

Recommendation

Based on the preceding findings, use of the SFA to calculate resistance factors for steel members and components is recommended as follows:

$$\phi = \begin{cases} \delta_R \exp(-0.55\beta V_R); & 0.05 \leq V_R \leq 0.30, \ \beta \approx 3.0 \\ \delta_R \exp(-0.70\beta V_R); & 0.10 \leq V_R \leq 0.40, \ \beta \approx 3.5 \\ \delta_R \exp(-0.80\beta V_R); & 0.10 \leq V_R \leq 0.40, \ \beta \approx 4.0 \end{cases} \quad (38)$$

The limits imposed on V_R ensure that the SFA will give satisfactory agreement with approximate FORM and prevent unconservative results. The scope of this work is limited to steel component reliability and a load case containing only dead and live loads, so the recommendation should be utilized accordingly. Beyond these limits, it is recommended to rely on more accurate reliability methods, such as approximate FORM or Monte Carlo simulation.

CONCLUSIONS

A comprehensive overview of the origin, initial derivation, and later development of the separation factor approach (SFA) has been summarized herein. From the amalgamation of this literature, it is found that the SFA was built on a few key simplifying assumptions that resulted in numerical
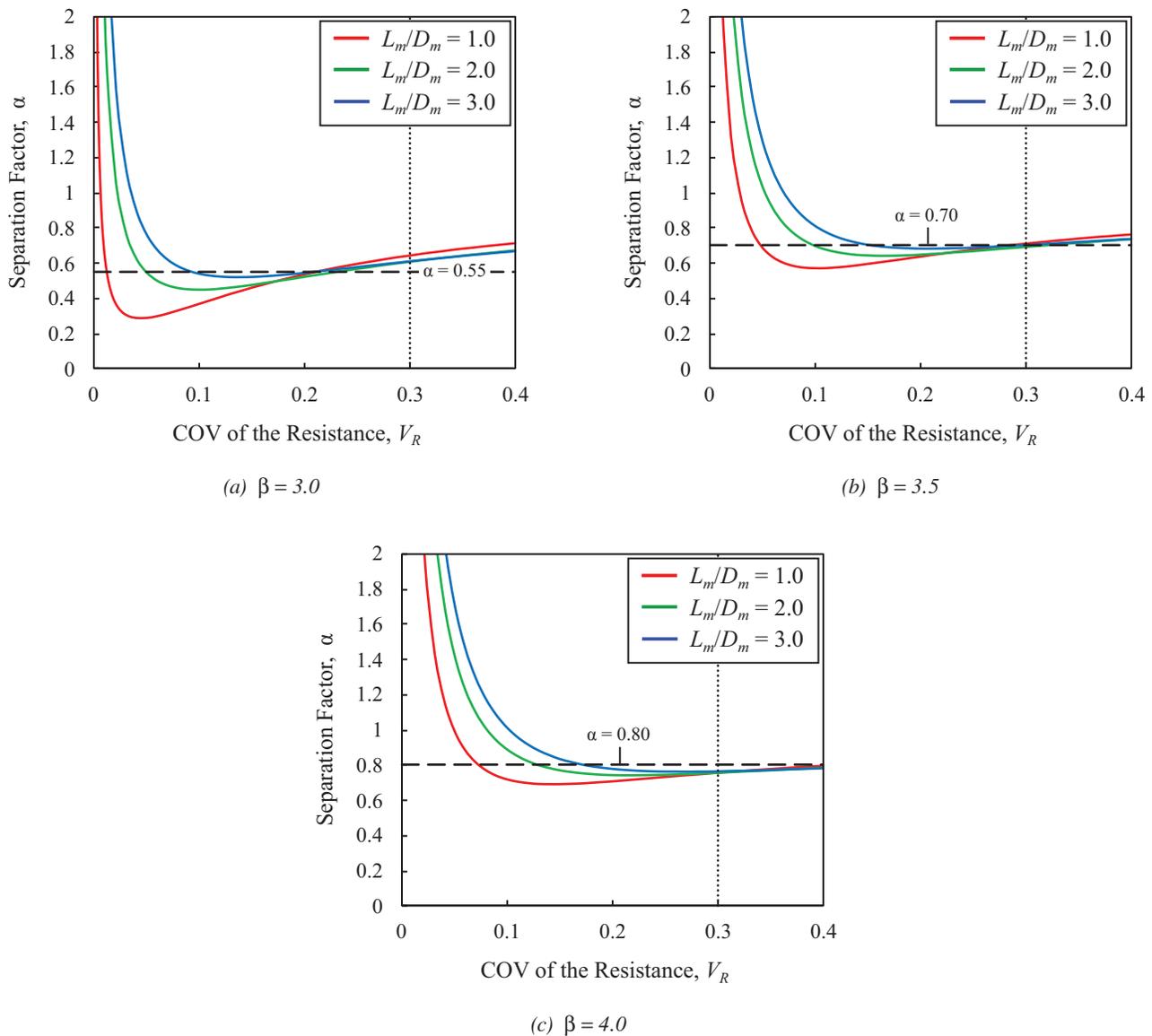

(a) $\beta = 3.0$

(b) $\beta = 3.5$

(c) $\beta = 4.0$

Fig. 8. Required separation factor for equality of the SFA and approximate FORM.

error of as much as 16% (underestimate) in the central safety factor, which contributes to the observed inaccuracy of the SFA in application. It is observed that the single constant separation factor, $\alpha = 0.55$, proposed by Galambos and Ravindra (1973) was not the optimal value according to the definition adopted by the SFA (i.e., minimization of the error in the approximation of the central safety factor) because it was calibrated to an atypical range of live-to-dead load ratios. Furthermore, it is found that the error in the approximation of the central safety factor can be better minimized if all separation factors (α_D, α_L, α_E, α_R) are varied independently of one another to determine an optimal combination. When this is performed using more up-to-date load and resistance statistics combined with a realistic range of live-to-dead ratios, it is found that a value of α_R ranging from 0.70 to 0.85 most effectively reduces the approximation error distribution.

Because the SFA uncouples the load and resistance terms while more accurate reliability methods do not, it should be expected that these methods will yield different resistance factors, even when both are used under optimal circumstances defined by their own assumptions. Despite this differing philosophy, separation factors can be calibrated such that the SFA will be in relative agreement with more accurate reliability methods under certain conditions. If agreement with more accurate reliability methods at $L_m/D_m = 3.0$ is the favored result, it is found that $\alpha_R = 0.55$, $\alpha_R = 0.70$, and $\alpha_R = 0.80$ give adequate results for reliability indices of 3.0, 3.5, and 4.0, respectively, over specified ranges of the coefficient of variation (COV) of the resistance, V_R, that prevent unconservative (overestimated) resistance factors. If accurate use of the SFA for calculating resistance factors for steel components is desired, the equation given in Equation 38 is recommended when operating within the specified limits of validity. This recommendation provides a basis for more definitive guidance on resistance factor determination in accordance with AISC 360 Section B3.1 Commentary.

ACKNOWLEDGMENTS

SYMBOLS

A Random variable accounting for uncertainties in the transformation of dead load intensities into dead load effects

A_m Mean of A

B Random variable accounting for uncertainties in the transformation of live load intensities into live load effects

B_m Mean of B

C Random variable accounting for the uncertainties in the transformation of wind load intensities into wind load effects

C_m Mean of C

D Dead load effect

D_m Mean dead load effect

D_n Nominal dead load effect

E Random variable accounting for uncertainties in structural analysis

E_m Mean of E

F_U Cumulative distribution function of U

L Live load effect

L_m Mean live load effect

L_n Nominal live load effect

Q Total load effect

Q_i Constituent load effect

Q_m Mean of the total load effect

Q_n Nominal total load effect

R Resistance

R_m Mean resistance

R_n Nominal resistance

T Random variable denoting the total load on a member

T_m Mean of the total load on a member

U Standardized variate of Z

V Coefficient of variation (COV)

V_A COV of A

V_{AD} COV of the dead load effect accounting for uncertainties in load transformation

V_B COV of B

V_{BL} COV of the live load effect accounting for uncertainties in load transformation

V_C COV of C

V_{CW} COV of the wind load effect accounting for uncertainties in load transformation

V_D COV of the dead load effect

V_E COV of E

V_L COV of the live load effect

V_Q COV of the total load effect

V_R COV of the resistance

V_T COV of the total load on a member

V_W COV of the wind load effect

V_Z COV of the safety margin

W Wind load effect

W_m Mean wind load effect

Z Safety margin

Z_m Mean of the safety margin

c_D Deterministic influence coefficient that transforms dead load intensities into dead load effects

c_L Deterministic influence coefficient that transforms live load intensities into live load effects

c_W Deterministic influence coefficient that transforms wind load intensities into wind load effects

i Subscript that can take on different load types (e.g., D, L, W)

p_F Probability of failure

Φ Cumulative distribution function of the standard normal distribution

α Separation factor

α' Intermediate separation factor

α_D Separation factor for determining the dead load factor

α'_D Intermediate separation factor for determining the dead load factor

α_E Separation factor for determining the load factor accounting for the uncertainties in structural analysis

α'_E Intermediate separation factor for determining the load factor accounting for the uncertainties in structural analysis

α_L Separation factor for determining the live load factor

α'_L Intermediate separation factor for determining the live load factor

α_Q Separation factor for the total load effect

α_R Separation factor for determining the resistance factor

α_T Separation factor for the total load on a member

α_W Separation factor for determining the wind load factor

β Reliability index

β_T Target reliability index

γ Load factor

γ_i Constituent load factor

γ_D Dead load factor

γ_E Load factor accounting for uncertainties in structural analysis

γ_L Live load factor

γ_W Wind load factor

δ Bias coefficient

δ_D Bias coefficient for the dead load effect

δ_L Bias coefficient for the live load effect

δ_Q Bias coefficient for the total load effect

δ_R Bias coefficient for the resistance

ε_β Percent error in the approximation of β

ε_θ Percent error in the approximation of θ

ε_ϕ Percent error between ϕ using the approximate FORM and ϕ using the SFA

$\bar{\varepsilon}$ Mean percent error

$\dot{\varepsilon}$ Median percent error

ε_{MAX} Maximum percent error

ε_{MIN} Minimum percent error

θ Central safety factor

θ_a Approximation of the central safety factor

σ Standard deviation

$\sigma_{\ln Q}$ Standard deviation of the natural logarithm of the total load effect

$\sigma_{\ln R}$ Standard deviation of the natural logarithm of the resistance

σ_Q Standard deviation of the total load effect

σ_R Standard deviation of the resistance

σ_T Standard deviation of the total load on a member

σ_Z Standard deviation of the safety margin

σ_ε Standard deviation of the percent error

υ Coefficient of skewness (COS)

ϕ Resistance factor

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