TECHNICAL NOTE

A Derivation of the Uniform Force Method for Analysis and Design of Gusset Plate Connections for Vertical Diagonal Bracing

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INTRODUCTION

The Uniform Force Method (UFM) for vertical diagonal bracing with gusset plate connections is a statically determinate analysis and design in which there are no moments at the interfaces between column, beam, and gusset, producing economical results. The UFM is characterized by the configuration shown in AISC *Steel Construction Manual* (AISC, 2023) Figure 13-2 and the following six equations from the AISC *Manual*.

$$\alpha - \beta \tan\theta = e_b \tan\theta - e_c \tag{13-1}$$

$$V_{rc} = \frac{\beta}{r} P_r \tag{13-2}$$

$$H_{rc} = \frac{e_c}{r} P_r \tag{13-3}$$

$$V_{rb} = \frac{e_b}{r} P_r \tag{13-4}$$

$$H_{rb} = \frac{\alpha}{r} P_r \tag{13-5}$$

where

$$r = \sqrt{(\alpha + e_c)^2 + (\beta + e_b)^2} \tag{13-6}$$

The necessary geometric constraints and interface shear and normal forces shown in Figures 1, 2, 3, and 4, herein, were defined by second author Thornton. In 1984, he originated the constraint equation, AISC *Manual* Equation 13-1, and the force equations, AISC *Manual* Equations 13-2 through 13-6. These six equations, with a derivation of Equation 13-1, but no derivation of the force equations, were first presented at the AISC National Steel Construction Conference (Thornton, 1991). They subsequently appear in Thornton's T. R. Higgins award-winning lecture paper (Thornton, 1995) and in every AISC *Manual* since the Second Edition LRFD *Manual* (AISC, 1995). A necessarily longer derivation of force equations for a less constrained generalized case with only one gusset control point, rather than two, is given in Appendix A of AISC Design Guide 29, *Vertical Bracing Connections* (Muir and Thornton, 2014).

By introducing the UFM constraint equation, the less constrained case in Design Guide 29 might be reduced to generate the UFM force equations, but that indirect proof has not been published, nor any proof until now. Herein, a short, simple, and direct derivation of the UFM force equations is presented.

GEOMETRIC CONSTRAINT

Figures 1, 2, 3, and 4 are elaborations of AISC *Manual* Figure 13-2. The connection geometry is shown in Figure 1. The slope-intercept form is used to write equations for the three lines passing through point GCP and passing through WP, CCP, and BCP. These lines are labeled (1), (2), and (3)

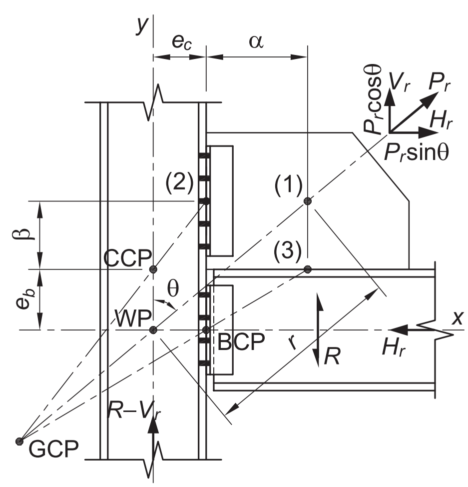

Fig. 1. Connection geometry.

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in Figure 1 and where they appear in the other figures of this paper. The origin of the coordinate system is at WP, and the equations are written as follows.

Line (1)
$$y = \frac{x}{\tan\theta} \tag{1}$$

Line (2)
$$y = \frac{\beta}{e_c} x + e_b \tag{2}$$

Line (3)
$$y = \frac{e_b}{\alpha} x - \frac{e_b}{\alpha} e_c \tag{3}$$

Eliminate y from Equations 1 and 2.

$$\frac{x}{\tan\theta} = \frac{\beta}{e_c} x + e_b \tag{4}$$

$$x = \frac{e_b}{(1/\tan\theta) - (\beta/e_c)} \tag{5}$$

Eliminate y from Equations 1 and 3.

$$\frac{x}{\tan\theta} = \frac{e_b}{\alpha} x - \frac{e_b}{\alpha} e_c \tag{6}$$

$$x = \frac{e_b e_c}{e_b - \alpha/\tan\theta} \tag{7}$$

Eliminate x from Equations 5 and 7.

$$\frac{e_b e_c}{e_b - \alpha/\tan\theta} = \frac{e_b}{(1/\tan\theta) - (\beta/e_c)} \tag{8}$$

$$\alpha - \beta\tan\theta = e_b \tan\theta - e_c \tag{9}$$

Thus, Equation 13-1 is proven.

INTERNAL FORCES BETWEEN THE GUSSET PLATE, BEAM, AND COLUMN

Once again from the configuration in Figure 1, the distance, r, from the origin to the point $(e_c + \alpha,\ e_b + \beta)$ is expressed by Equation 13-6, rewritten here:

$$r = \sqrt{(\alpha + e_c)^2 + (\beta + e_b)^2} \tag{10}$$

$$\sin\theta = \frac{\alpha + e_c}{r} \tag{11}$$

$$\cos\theta = \frac{\beta + e_b}{r} \tag{12}$$

The horizontal and vertical components of force P_r are:

$$H_r = P_r \sin\theta = \frac{\alpha + e_c}{r} P_r \tag{13}$$

$$V_r = P_r \cos\theta = \frac{\beta + e_b}{r} P_r \tag{14}$$

Referring to the column diagram in Figure 2, sum moments about BCP and use Equation 14 to substitute for V_r.

$$(\beta + e_b) H_{rc} = e_c V_r = \frac{e_c(\beta + e_b)}{r} P_r \tag{15}$$

$$H_{rc} = \frac{e_c}{r} P_r \tag{16}$$

Thus, Equation 13-3 is proven.

Referring to the gusset diagram in Figure 3, sum the horizontal forces and use Equations 13 and 16 to substitute for H_r and H_{rc}.

$$H_{rb} = H_r - H_{rc} = \frac{\alpha + e_c}{r} P_r - \frac{e_c}{r} P_r \tag{17}$$

$$H_{rb} = \frac{\alpha}{r} P_r \tag{18}$$

Thus, Equation 13-5 is proven.

Referring to the beam diagram in Figure 4, sum moments about BCP and use Equation 18 to substitute for Hr_b.

$$\alpha V_{rb} = e_b H_{rb} = \frac{e_b \alpha}{r} P_r \tag{19}$$

$$V_{rb} = \frac{e_b}{r} P_r \tag{20}$$

Thus, Equation 13-4 is proven.

Referring again to the gusset diagram in Figure 3, sum the vertical forces and use Equations 14 and 20 to substitute for V_r and V_{rb}.

$$V_{rc} = V_r - V_{rb} = \frac{\beta + e_b}{r} P_r - \frac{e_b}{r} P_r \tag{21}$$

$$V_{rc} = \frac{\beta}{r} P_r \tag{22}$$

Thus, Equation 13-2 is proven.

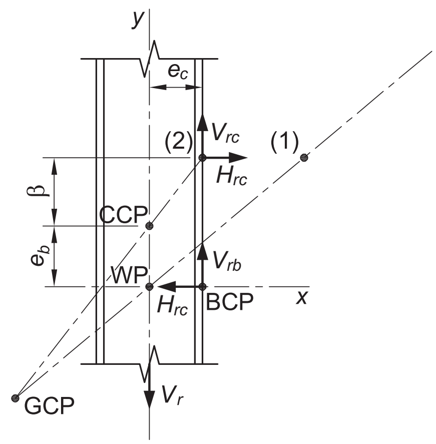

Fig. 2. Column diagram.

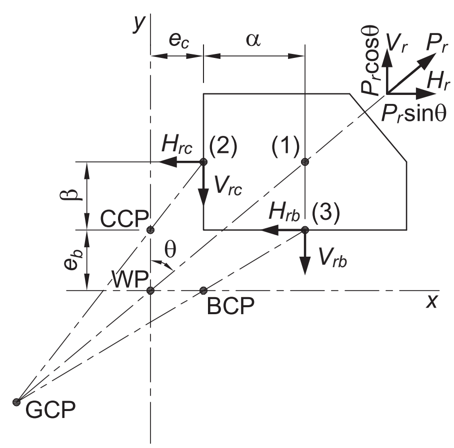

Fig. 3. Gusset diagram.

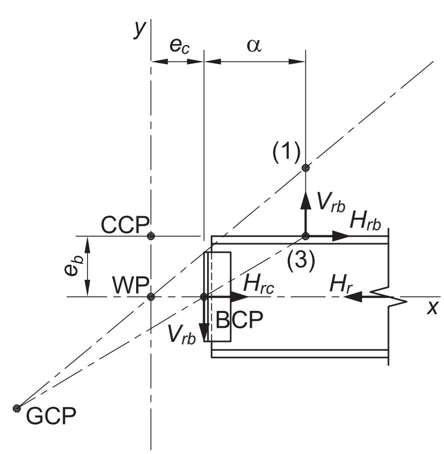

Fig. 4. Beam diagram.

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