# DISCUSSION

## Design of Diagonal Cross-Bracings— Part 1: Theoretical Study; Part 2: Experimental Study

Papers by A. PICARD and D. BEAULIEU
(3rd Quarter, 1987 and 4th Quarter, 1988)

Discussion by **Sayed H. Stoman**

The authors have presented an interesting and very useful study on the behavior of cross bracings. Application of the experimental study can result in significant savings both in the weight of the primary bracing elements as well as the main structure.

However, as described in Part 1 of the authors' study, the results are predicated on a number of assumptions that must be fully realized in design application in order to ensure safety and economy. These assumptions are:

(a) Both braces are identical in length and sectional and material properties,
(b) One diagonal is always under tension while the other diagonal is in compression, and
(c) End conditions are simple.

Deviations from the above assumptions can lead to effective length factors that are substantially different from those recommended by the authors.

Brace effective length is a function of the axial loading in and stiffness of both diagonals. Hence, unless one diagonal is always under tension and the tensile diagonal is identical or superior to the compression diagonal, design of the compression brace on the basis of an effective length equal to 0.5 times the diagonal length, as recommended by Picard and Beaulieu, or 0.85 times the half diagonal length as recommended by El-Tayem and Goel,[1] may be unconservative for simply supported cross bracing systems.

Moreover, when both diagonals are in compression, the transverse stiffness that is furnished by the respective diagonals to each other is significantly reduced. Consequently, the resulting effective length factor can be much larger than the $K_{max}$ of 0.72 that is cited by the authors. However, if the extreme ends of the compression diagonal are rigidly attached to the framed structure, a design based on Picard and Beaulieu's recommendation can be overly conservative.

By employing the Raleigh-Ritz method of stationary potential energy, Stoman[2,3,4] formulated closed-form stability criteria for directly evaluating the transverse stiffness provided by the tension brace to the interconnected compression brace and for evaluating the compression brace critical load for any ratio of the axial force in the respective diagonals. The criteria addressed the general case where the two diagonals are different in section and material properties, stiffness and boundary conditions, and lengths. With permission from ASCE, figures and essential aspects of the formulation are excerpted here, as a supplement to the authors' study, to facilitate design:

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### Brace ends built-in

For the bracing system shown in Fig. 1, the transverse stiffness furnished by the tensile diagonal to the compression diagonal was evaluated to be[3]

$$K_s = \frac{\pi^2(4PE_t + Q)}{2L_t} \tag{1}$$

where

$$PE_t = \frac{\pi^2 E_t I_t}{L_t^2} \tag{2}$$

and $E_t$, $I_t$, $L_t$, and $Q$, respectively, represent modulus of elasticity, moment of inertia, length, and the axial force of the tensile diagonal. It was also shown[3] that the compression brace critical load is given by

$$P_{cr} = 4PE_c + \frac{2L_c}{\pi^2} K_s \leq 8.183\ PE_c \tag{3}$$

where

$$PE_c = \frac{\pi^2 E_c I_c}{L_c^2} \tag{4}$$

In these equations, $E_c$, $I_c$, $L_c$, and $P$, respectively, represent the modulus of elasticity, moment of inertia, length, and the axial force of the compression diagonal. Substituting for $K_s$ from Eq. 1 into Eq. 3 yields

$$P_{cr} = 4PE_c + \frac{L_c}{L_t}(4PE_t + Q) \leq 8.183\ PE_c \qquad (5)$$

in which only $Q$ is a variable for a given bracing.

In order to arrive at an expression for the effective length factor which is defined as

$$K = \sqrt{\frac{PE_c}{P_{cr}}} \qquad (6)$$

let

$$\alpha_1 = \frac{L_c}{L_t} \qquad (7)$$

$$\alpha_2 = \frac{PE_t}{PE_c} \qquad (8)$$

$$\gamma = \frac{Q}{P} = \frac{Q}{P_{cr}} \qquad (9)$$

Substituting $\alpha_1$, $\alpha_2$, $\gamma$ and $P_{cr}$ from Eq. 5 into Eq. 6 gives

$$K = (½)\sqrt{\frac{1 - \alpha_1\gamma}{1 + \alpha_1\alpha_2}} \geq 0.35 \qquad (10)$$

For the special case when both diagonals are identical, $\alpha_1 = \alpha_2 = 1.0$ and Eqs. 5 and 10, respectively, reduce to

$$P_{cr} = 8PE_c + Q \leq 8.183\ PE_c \qquad (11)$$

and

$$K = \sqrt{\frac{1 - \gamma}{8}} \geq 0.35 \qquad (12)$$

Equation 11 indicates that for identical diagonals as long as

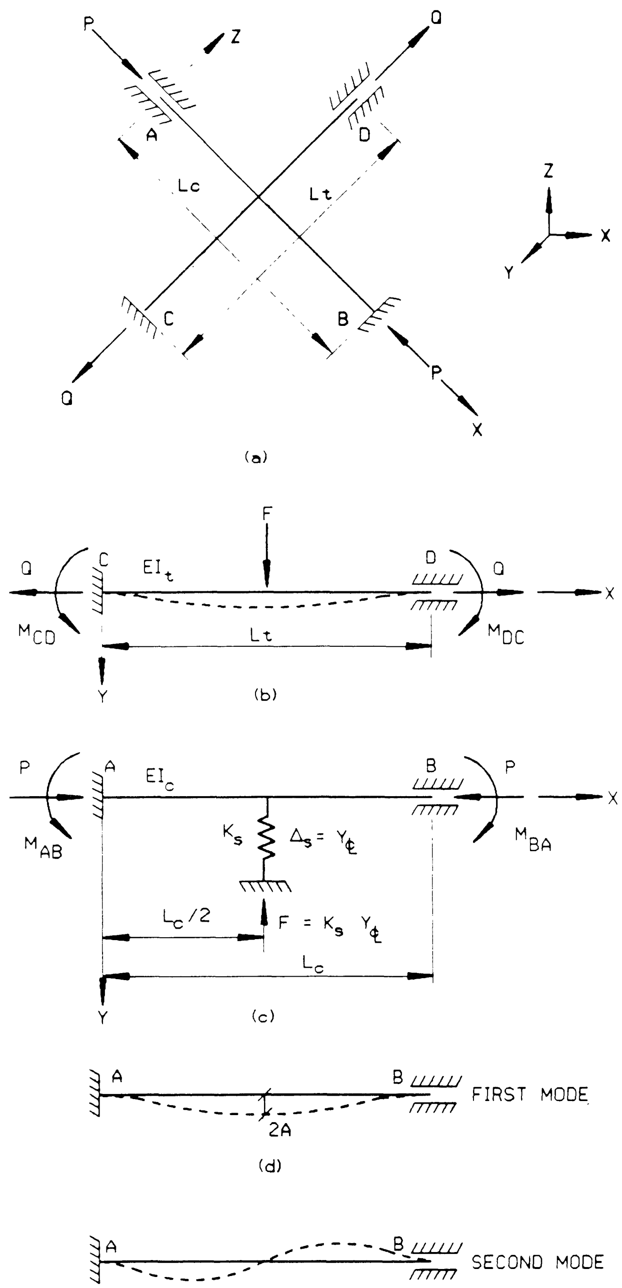


*Fig. 1. Built-in cross bracing system*

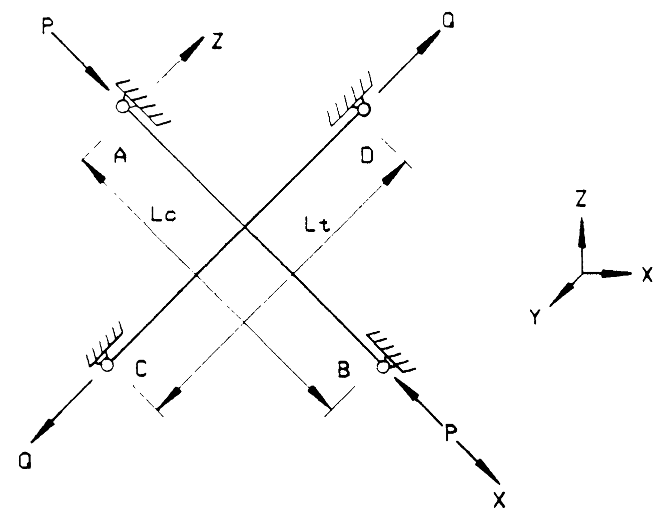

*Fig. 2. Simply supported cross bracing*

the tensile diagonal axial force $Q \geq 0.183\ PE_c$, buckling of the compression diagonal will be in the second mode. However, $Q$ is bounded by $Q_y = A_b \times \sigma_y$, i.e., brace area times its yield stress.

**Brace ends pinned**

Likewise, if the extreme ends of the diagonals were pinned to the structure as shown in Fig. 2, the transverse stiffness provided by the tensile diagonal to the compression diagonal would be[4]

$$K_s = \frac{\pi^2\,(PE_t + Q)}{2L_t} \tag{13}$$

and

$$P_{cr} = PE_c + \frac{2L_c}{\pi^2} K_s \tag{14}$$

substituting for $K_s$ from Eq. 13 gives

$$P_{cr} = PE_c + \frac{L_c}{L_t}(PE_t + Q) \leq 4PE_c \tag{15}$$

Again, as in Eq. 5, for a given bracing the only variable in Eq. 15 is $Q$. The effective length factor as defined in Eq. 6 then becomes

$$K = \sqrt{\frac{1 - \alpha_1\gamma}{1 + \alpha_1\alpha_2}} \geq 0.50 \tag{16}$$

For identical diagonals Eqs. 15 and 16 reduce to

$$P_{cr} = 2PE_c + Q \leq 4PE_c \tag{17}$$

and

$$K = \sqrt{\frac{1 - \gamma}{2}} \geq 0.50 \tag{18}$$

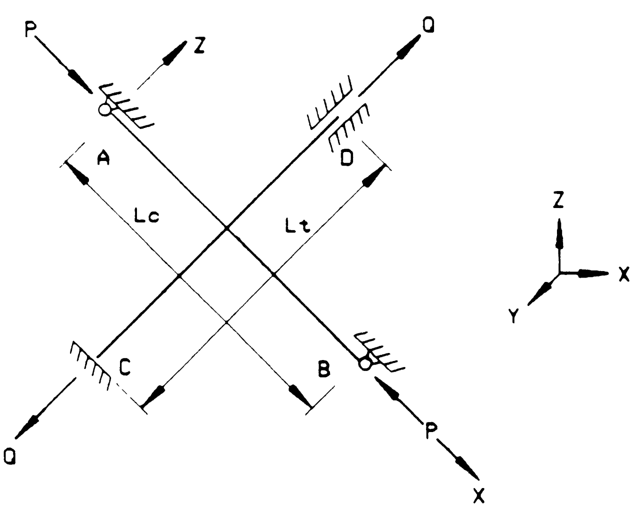


*Fig. 3. Cross bracing with tensile brace built-in*

Equation 17 indicates that as long as $Q \geq 2\ PE_c$, buckling of the compression brace will be in the second mode. However, $Q$ is bounded by $Q_y$.

**Only tensile diagonal ends built-in**

If the tensile diagonal ends are built-in, then the transverse stiffness furnished by it to the simply supported compression brace (see Fig. 3) is defined by Eq. 1, which upon substitution into Eq. 14 yields

$$P_{cr} = PE_c + \frac{L_c}{L_t}(4PE_t + Q) \leq 4PE_c \tag{19}$$

and

$$K = \sqrt{\frac{1 - \alpha_1\gamma}{1 + 4\alpha_1\alpha_2}} \geq 0.50 \tag{20}$$

For identical diagonals ($\gamma_1 = \alpha_2 = 1.0$) these two equations simplify to

$$P_{cr} = 5PE_c + Q \leq 4PE_c \tag{21}$$

and

$$K = \sqrt{\frac{1 - \gamma}{5}} \geq 0.50 \tag{22}$$

**Only compression diagonal ends built-in**

Similarly, the transverse stiffness provided by the simply supported tensile diagonal to the built-in compression diagonal in Fig. 4 is defined by Eq. 13. Substitution of this $K$ value into Eq. 3 yields

$$P_{cr} = 4PE_c + \frac{L_c}{L_t}(PE_t + Q) \leq 8.183\ PE_c \tag{23}$$

and

$$K = \sqrt{\frac{1 - \alpha_1\gamma}{4 + \alpha_1\alpha_2}} \geq 0.35 \tag{24}$$

For identical diagonals ($\alpha_1 = \alpha_2 = 1.0$) the two equations reduce to

$$P_{cr} = 5PE_c + Q \leq 8.183\ PE_c \tag{25}$$

and

$$K = \sqrt{\frac{1 - \gamma}{5}} \geq 0.35 \tag{26}$$

## RESULTS

Equations 10, 16, 20, and 24 represent general expressions for evaluating the effective length factor for any diagonal lengths, section or material properties, and boundary conditions. These equations can be utilized for developing effective length spectra for all values of the parameters $\gamma$, $\alpha_1$, and $\alpha_2$. It is evident that the effective length is very sensitive to the relative stiffness of the two diagonals, which can be adjusted by a proper selection of individual brace size and/or end conditions.

The effective length factors thus obtained are subsequently plotted for illustration purposes and design application. Figure 5 represents the effective length spectra for a built-in bracing system based on Eq. 10, assuming $\alpha_1 = 1.0$ and $\alpha_2$ ranging from 0.2 to 1.0. With $\alpha_2$ increasing, $K$ values keep decreasing—with a range of 0.35 to 0.50. Likewise, the spectra obtained using Eq. 16 is plotted in Fig. 6 for the simply supported cross bracing. In this case, $K$ values range from a minimum of 0.50 to a maximum of 1.0 depending on the ratios defined by $\gamma$, $\alpha_1$, and $\alpha_2$.

Equations 12 and 18, which respectively represent special cases of Eqs. 10 and 16, are for identical diagonals under identical boundary conditions and are plotted in Fig. 7. Superimposed on this figure is a plot of Eqs. 22 and 26, representing the effective length factor for identical diagonals of different boundary conditions—a special case of Eqs. 20 and 24. Except for the limits of applicability, the preceding two equations are similar, and in fact identical for identical diagonals, as described by Eqs. 22 and 26. Hence, attention must be paid to the limits of applicability when using these equations. Picard and Beaulieu's experimental solution points for simply supported, identical cross bracings are also shown in Fig. 7.

It is also clear from Eq. 21 that unless the built-in tensile diagonal carries a compressive load in excess of $PE_c$, buckling of the simply supported compression diagonal will be in the second mode. Similarly, Eq. 25 indicates that, as long as the axial force $Q \geq 3.183\ PE_c$ in the simply supported tensile diagonal, buckling of the built-in compression diagonal will be in the second mode. Thus, with either brace ends built-in, stability enhancement of the system over a simply supported cross bracing is quite evident, noting that $Q$ is bounded by $Q_y$.

Caution must be exercised in the use of the criteria and specta curves at regions where $Q$ approaches $Q_y$ or $P_{cr}$ when acting in compression. At these extremes, the "tensile" diagonal's flexural stiffness is at its minimum, and it may not be able to provide the necessary transverse stiffness to prevent a snap-through buckling of the compression brace to its lower mode; see Stoman[2] for order of magnitude of bending stresses in the tensile diagonal. (The positive direction for $Q$ and $P$ is as shown in Figs. 1–4).

## CONCLUSION

General effective length criteria for cross bracing systems are formulated. Effective length spectra curves are generated to elucidate the criteria and to facilitate design application. Results indicate that the effective length is very sensitive to the relative stiffness of the two interconnected diagonals and, depending on brace end conditions, the effective length factors values range from 0.35 to 1.0. Refereed experimental data available in the range agree well with the proposed solution. Application of the criteria will lead to an optimum design and enhance overall structural stability.

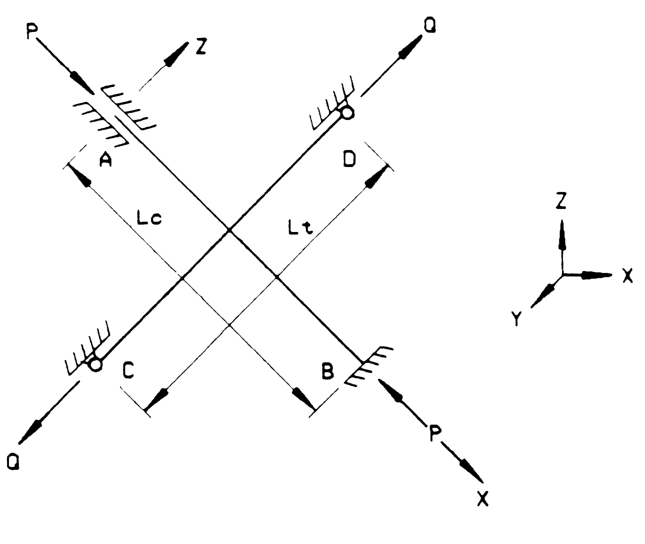


*Fig. 4. Cross bracing with compressive brace built in*

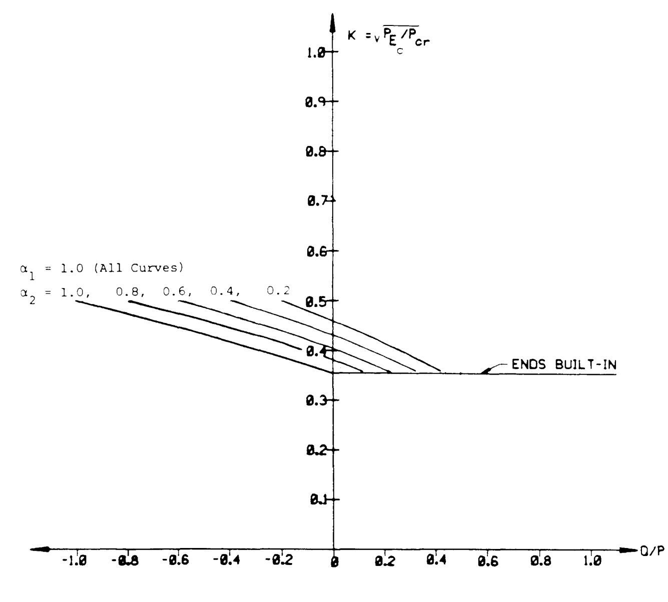


*Fig. 5. Effective length factor spectra for built-in cross bracing*

## REFERENCES

1. EL-Tayem, A. A. and S. C. Goel, "Effective Length Factor for the Design of X-bracing Systems," AISC *Engineering Journal,* (lst Quarter 1986) pp. 41–45.
2. Stoman, S. H., Nonlinear Collapse Load Analysis of Braced, Framed Structures. Thesis presented to Case Western Reserve University, Cleveland, May 1982, in partial fulfillment of the requirements of the Ph.D.
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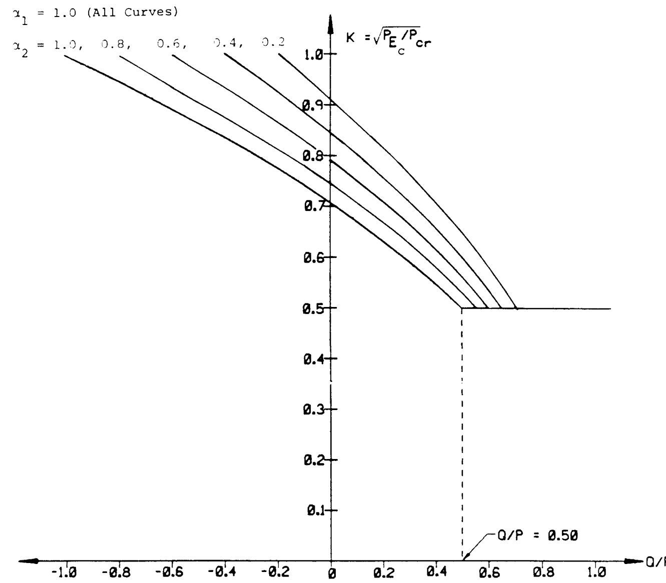


*Fig. 6. Effective length factor spectra for simply supported cross bracing*

Addendum/Closure by **A. Picard and D. Beaulieu**

The authors appreciate the interest shown by Dr. Stoman in their papers and thank him for his useful and weighty contribution.

The authors agree that conclusions and equations are valid only if the assumptions mentioned in the papers and emphasized by Dr. Stoman are satisfied.

The assumption that the connections at the ends of the diagonals are perfect hinges is on the conservative side. The assumption that one diagonal is always under tension has allowed to obtain Eqs. 9 to 12 of Part 1, by solving the differential equation of equilibrium of the tension member shown in Fig. 5. These equations, which are used to compute the transverse stiffness of the tension diagonal, are not valid if the diagonal is not in tension. Therefore, Eq. 20 is valid up to $T = 0$, that is $K_{max} = 0.72$.

The assumption that both braces are identical was made in order to simplify the theory. For instance, in Part 1, Eq. 11 becomes Eq. 12 because it is assumed that $I_t/I_c = 1.0$. This ratio could have been retained in the following equations. Similarly, two different lengths, for instance $L_1$ for the tension member and $L_2$ for the compression member, could have been used.

These assumptions and the assumption that the behavior of the tension diagonal is elastic up to buckling of the compression diagonal were made because they correspond to usual practical conditions. A complete finite element model has recently been developed to allow the simulation of elastic and inelastic behavior of cross-bracing systems including the effect of partial end restraints. The results of this study will be published later, and it is believed that they will support Dr. Stoman's elastic theory.

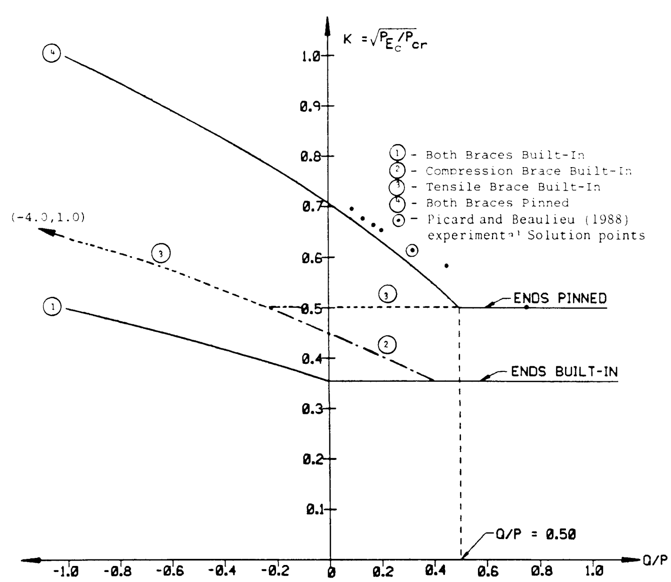


*Fig. 7. Effective length factor for identical cross bracing of various boundary conditions*