Moment-Rotation Characteristics of Shear Connections

D. J. L. KENNEDY

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IN SIMPLE TYPE construction it is assumed that the ends of beams and girders are connected for shear only and are free to rotate under load in the plane of loading.

A standard connection used has been a pair of angles fastened to the web by welds and field bolted or welded to the supporting girder or column as shown in Fig. 1. This type of connection, even when designed for shear only, has proved most reliable and among its advantages are:

1. The longitudinal position of the angles can be adjusted slightly to allow for over- or under-cut in the length of the beam.
2. The projection of the angles beyond the end of the beam provide some clearance for erection.

END PLATE CONNECTION

A simpler shear connection consists solely of a vertical end plate welded to the end of the web transverse to the beam as shown in Fig. 2. In fabricating this connection, only one piece is handled, rather than two. The accurate cutting to length of the beam required can be obtained to $\pm 1/16$-in. with automatic sawing equipment which provides a square and true cut as well. Skewed connections are also easily made by positioning the end plate at the correct angle.

A question naturally arising is whether the end plate shear connections provide behavior similar to that for the connection angles. Are they as flexible? Can they provide the rotations required before fracture? Will larger moments be induced in supporting columns? What are the moment-rotation characteristics of such connections?

BEHAVIOR OF END PLATE CONNECTIONS

A limited series of tests[1] provided the answers to some of these questions. Figure 3 shows the test jig positioned in the 1200 kip testing machine with a test beam of G40.12 steel (σ_y = 44 ksi) bolted to the jig with A325 bolts.

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Two comparative tests were conducted on angle connections of comparable geometry. Loads were obtained from the testing machine and rotations from dial gages measuring the movement of the flanges of the beam with reference to the column. The general behavior of the shear connections in the tests established the following:

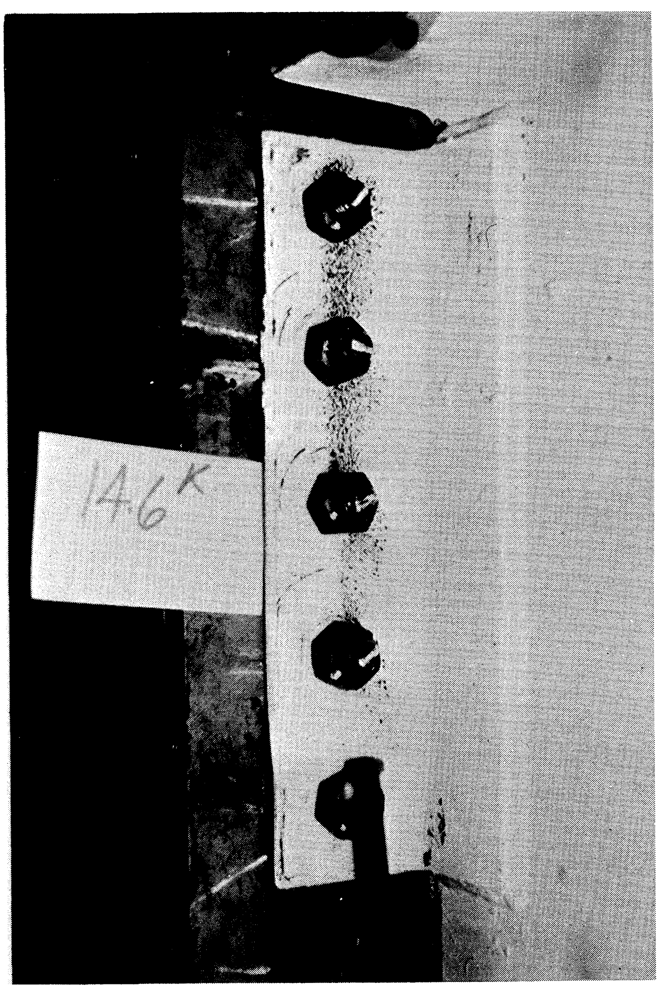

Fig. 1. Simple connection with pair of angles

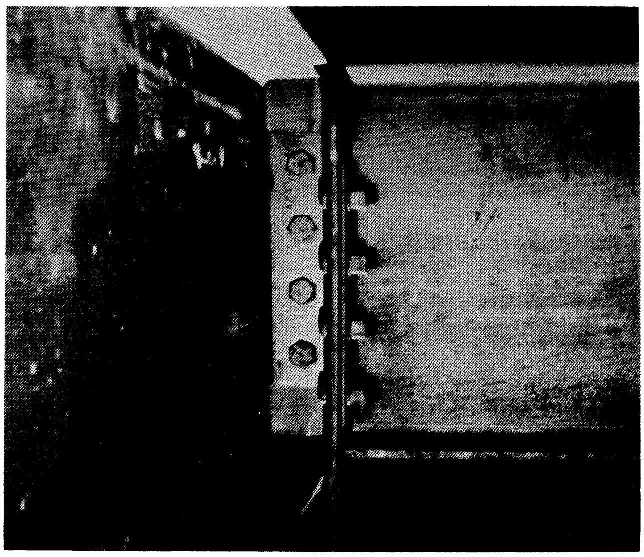

Fig. 2. End plate connection, National School Studios, Winnipeg, Manitoba

1. At relatively small rotations, but at loads exceeding the yield moment on the beam, the lower beam flange comes into bearing against the column flange. The connection then becomes much stiffer and the moment rotation curve rises sharply as shown in Fig. 4. The lower curve illustrates the behavior if the lower flange and part of the web is coped as shown in Fig. 5. The spalling of the whitewash indicates that a great amount of inelastic deformation beginning at very low loads has occurred.
2. The connections exhibit the same moment-rotation characteristics irrespective of the moment-shear ratio, as shown in Fig. 6, where curves are drawn for tests with lever arms varying from 0.75 to 5 ft.
3. The plate connections behave essentially in the same manner as the angle connections, although differences in detailed behavior exist.

Fig. 3. End plate test jig and specimen

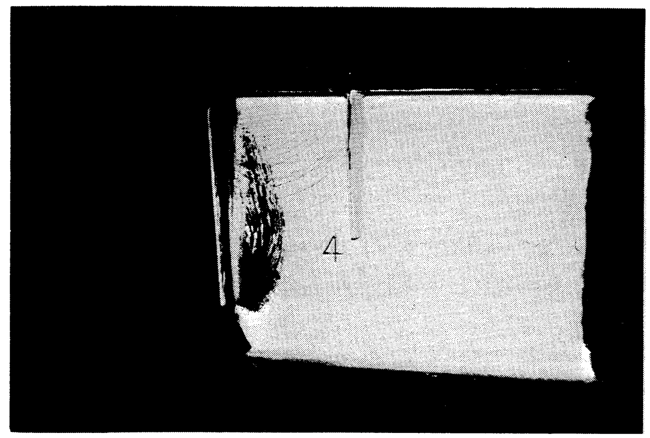

Fig. 5. End plate test with coped lower flange and web

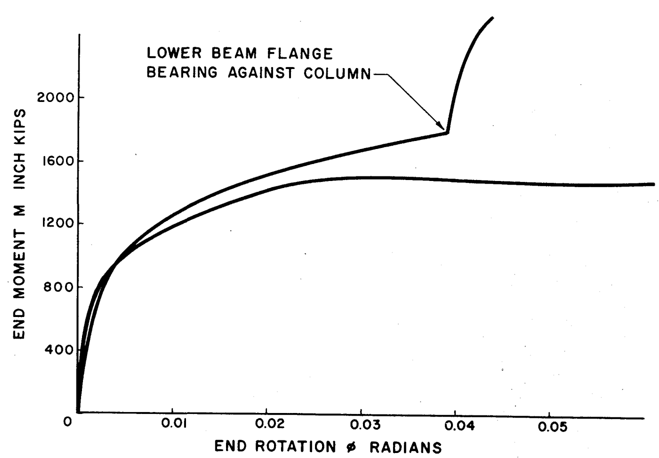

Fig. 4. M-ϕ curve for end plate connections

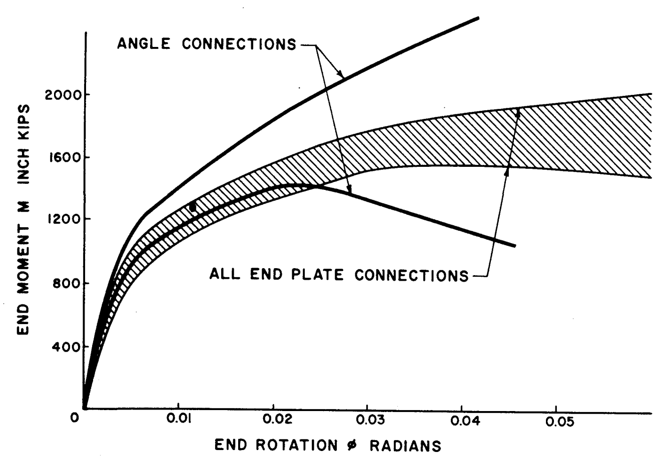

Fig. 6. M-ϕ curves for end plate and angle connections

A more extensive series of tests,[2] consisting of 24 ests, was carried out in 1967 to investigate the effect n the connection behavior of end plate thickness, beam lepth, number of rows of bolts and gage. With gages of · or 5½ in., plate thicknesses of ¼-, ⅜- or ½-in., and vith 2 to 6 rows of bolts, a large range in flexibility was .chieved. The steel was nominally G40.12 with measured ield points of 48 to 57 ksi and welds were made with :7018 electrodes. All bolts were ¾-in. diameter A325 olts.

As the tests were intended to simulate actual contruction practice, the lower flange of the beam was not oped. All tests were conducted at a constant lever arm f 4 ft. Based on visual observations, all end plate connecions exhibited the same behavior, which generally can e summarized as follows:

1. At very low loads, inelastic action begins and cracking of the whitewash on the plate near the upper end occurred.
2. Spalling of the whitewash on the web of the beam indicated yielding near the lower end of the connection.
3. The plates pulled away from the column at the top, with much yielding occurring in the plates adjacent to the welds and bolts, as shown in Fig. 7.
4. The end plate was pushed into the web, with the result that the bottom flange approached and finally hit the column as shown in Fig. 8.
5. After the bottom flange hit the column, subsequent rotation essentially occurred about the bottom flange of the beam, with resulting increased rotational stiffness of the connection. Figures 9 and 10 illustrate the large deformations obtainable at failure. Failure generally occurred with tearing of the plate in the heat-affected zone at the toe of the weld, starting at the top and progressing downward. Bolt failures occurred in two tests.

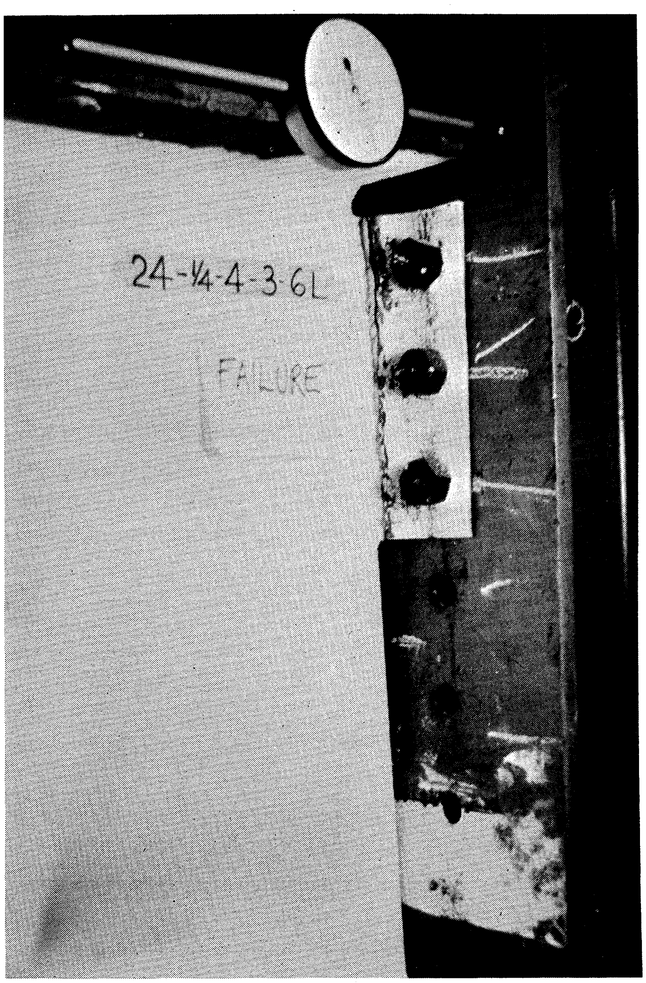

Fig. 7. Inelastic behaviour of end plate connection

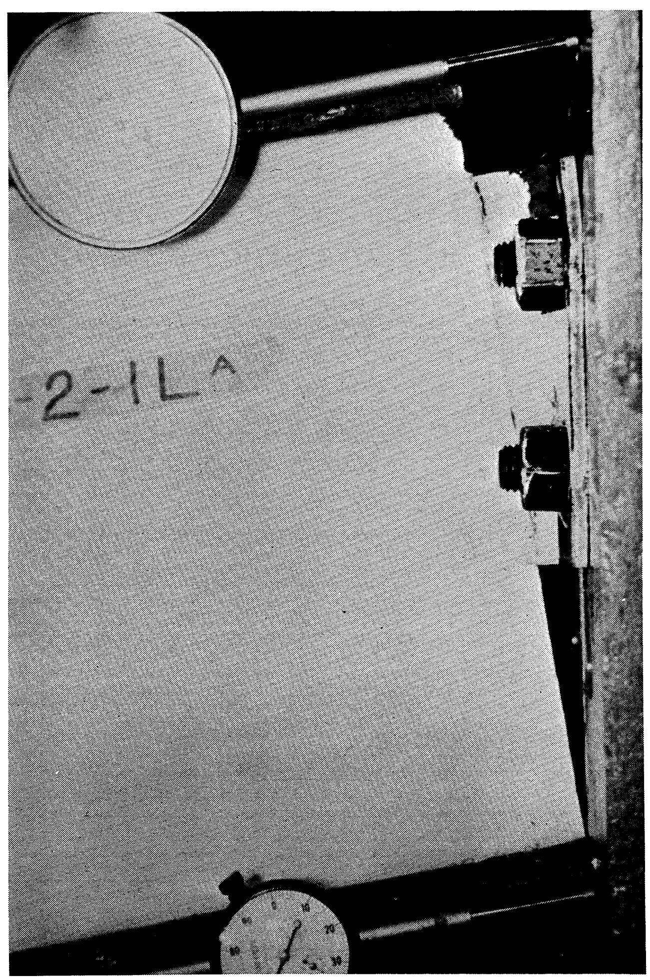

Fig. 8. End plate test, lower flange bearing against column

Fig. 9. Top view of end plate connection after test

In four tests of angle connections, the same general behavior was observed, with the angles pulling away from the column at the top and yielding of the beam web at the toe of the angles at the bottom causing the beam flange to bear against the column. As the angles distort at the top of connection, the heels move toward each other and the legs attached to the web tend to move away from the web. The weld on the web tears off. In two of the four tests, the severe prying action on the bolts led to bolt failure.

Figure 11 shows the condition of the angles at failure. The spalling of the whitewash, indicating extensive yielding, is evident, as is the tearing of the weld along the leg of the angle attached to the web.

Typical moment-rotation curves are shown in Figs. 12, 13, and 14 for end plate shear connections. The occurrence of inelastic action at low loads is evident from the curves, which have little or no linear portions. The moment at which the beam flange comes into bearing against the column is noted from the sharp increase in stiffness of the connections.

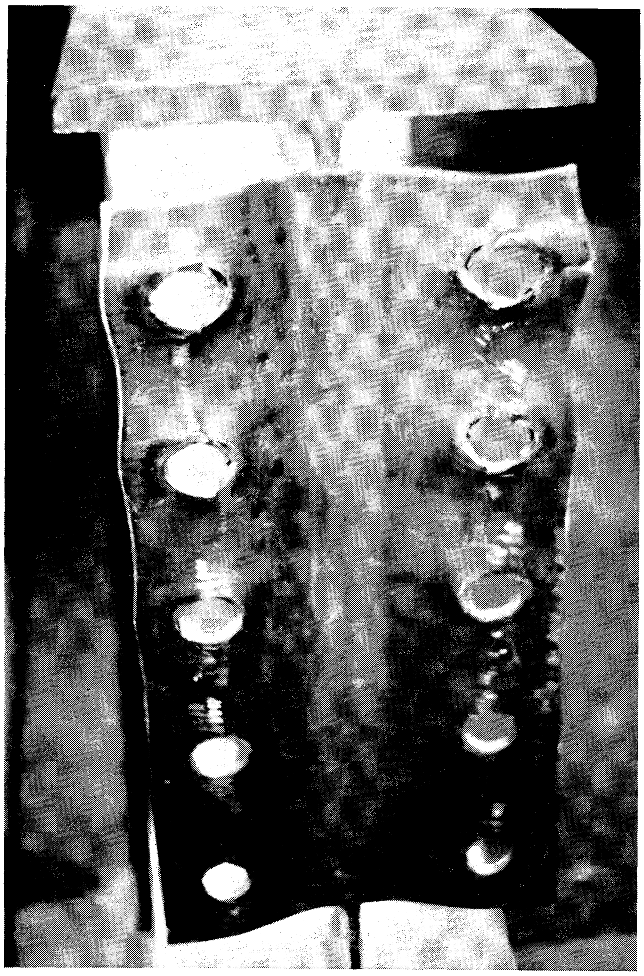

Fig. 10. Rear view of end plate connection after test

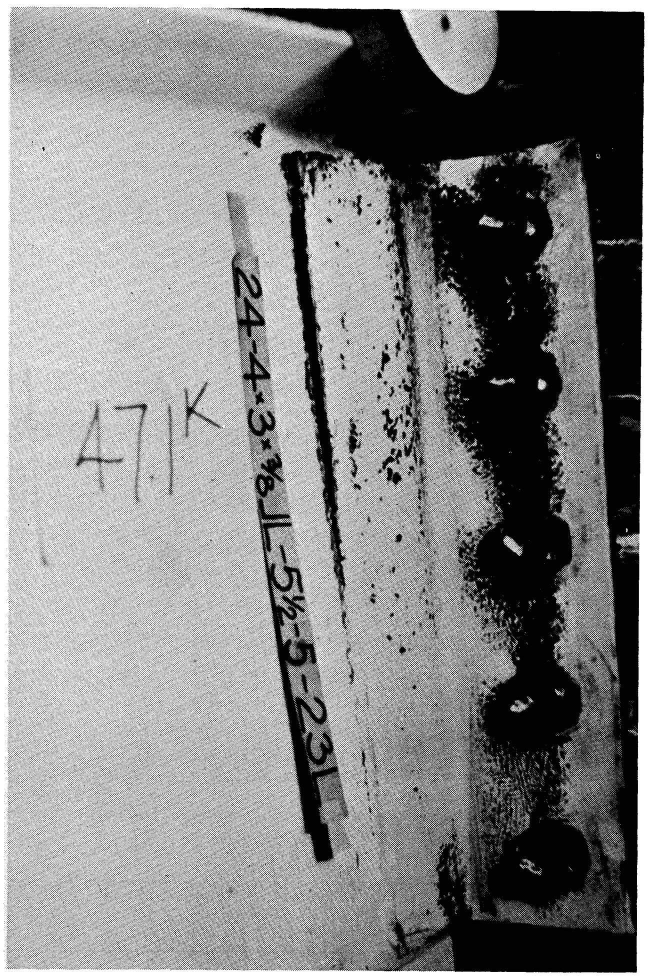

Fig. 11. Angle connection after test

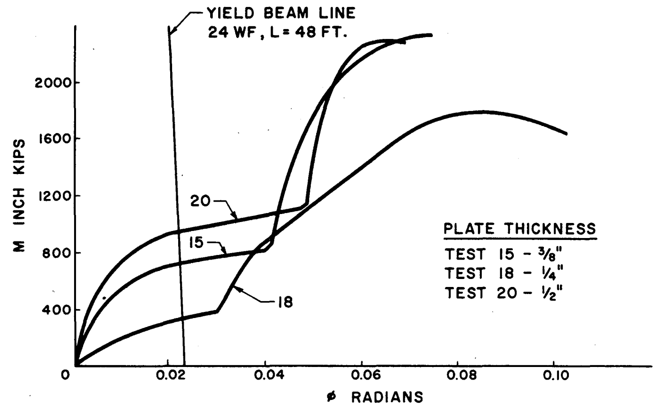

Fig. 12. M-ϕ curves for end plate tests 15, 18, and 20

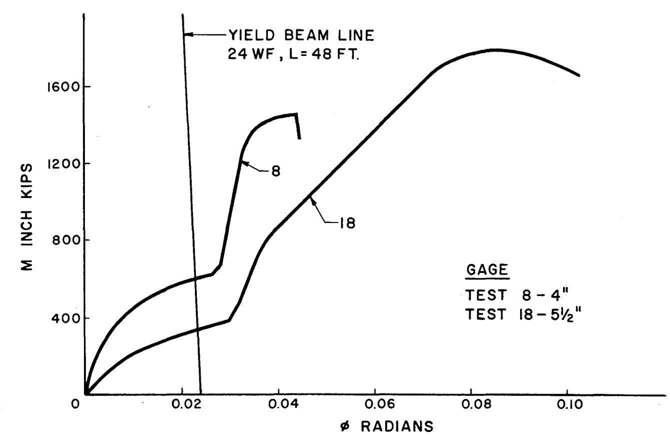

Fig. 13. M-ϕ curves for end plate tests 8 and 18

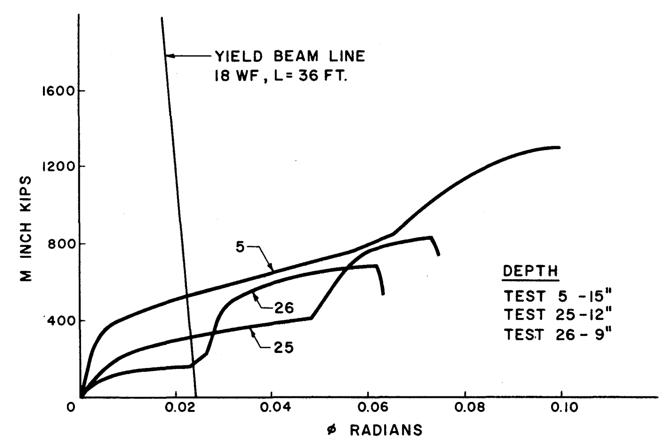

Fig. 14. M-ϕ curves for end plate tests 5, 25, and 26

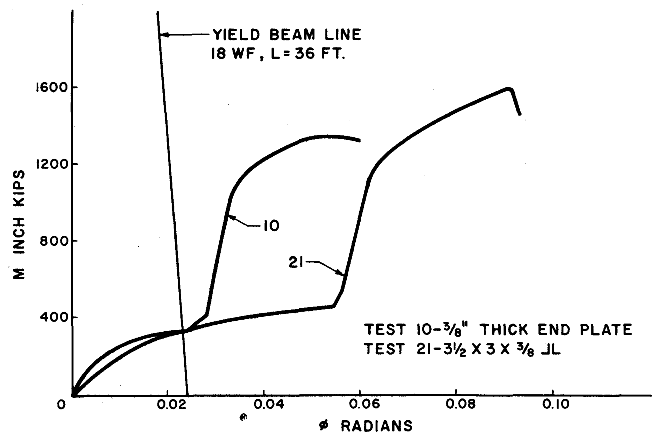

Fig. 15. M-ϕ curves for angle and end plate connection tests 10 and 21

Figure 12 shows the effect of increasing thickness of the connection plate from $\frac{1}{4}$- to $\frac{3}{8}$- to $\frac{1}{2}$-in. As the thickness is increased, the moment capacity is increased and the rotation at which the beam flange hits the column is also greater because of increased distance of the beam flange from the column. The yield moment of a 24WF76 for a yield value of 44 ksi is 7,700 kip-in. and the free end rotation ϕ_0 on a span of 24 times the depth, i.e., 48 ft, when loaded uniformly to produce the yield moment at the center line, is 0.024 radians.

Figure 13 shows that increasing the gage increased the flexibility. The test curves in Fig. 14 for an 18WF with a $\frac{1}{4}$-in. thick connection plate show that increasing the depth of the connection increased its stiffness and the rotation required for the flange to bear on the column.

The curves for Test Nos. 10 and 21, Fig. 15, compare the behavior of a $\frac{3}{8}$-in. connection plate and a $\frac{3}{8}$-in. connection angle. The chief difference is the rotation when the flange hits the column. This is ascribed to the greater distance from the end of the beam to the back of the angles — $\frac{1}{2}$-in. versus the plate thickness of $\frac{3}{8}$-in., and also to the more concentrated yielding of the web at the bottom of the plate connection. The curves indicate that the two types of connections give essentially the same behavior. Other test curves gave even better correlation.

STANDARDIZED MOMENT-ROTATION CURVE

By comparing sets of curves, as illustrated in the previous figures, it was possible to determine, within the limits of the experiments, the effect of variation of each particular parameter on the moment-rotation curve. This comparison was made for the portion of the curve up to the point at which the flange hit the column. This point, as discussed later, is beyond the rotation at which the beam would have yielded for reasonable span-to-depth ratios.

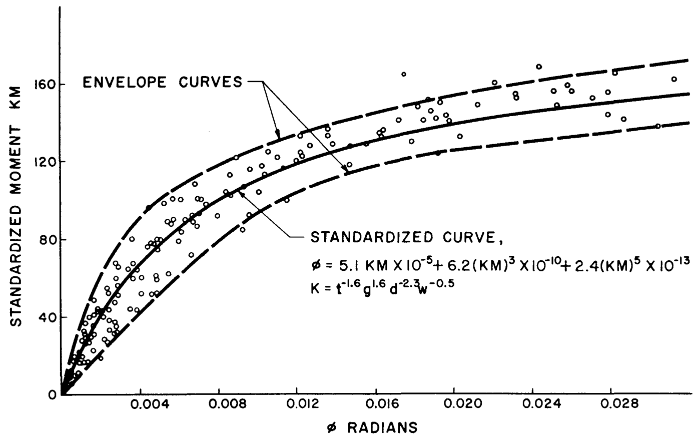

Fig. 16. Standardized moment-rotation curve

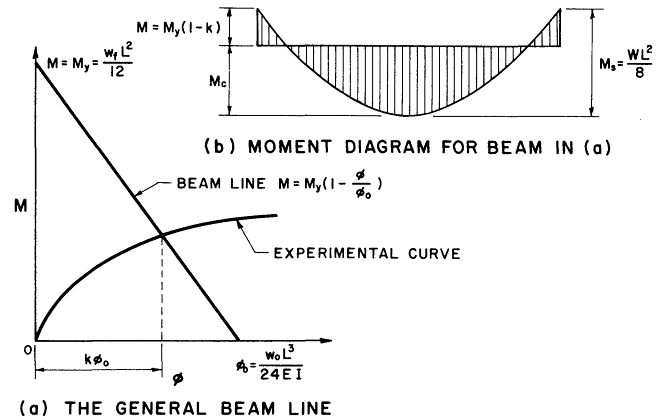

Fig. 17. The beam line

Comparisons were made to determine quantitatively the effect of plate thickness, gage, depth of connection (number of rows of bolts), and web thickness of the beam. Thus a standardization constant, K, was determined:

$$K = t^{-1.6}g^{1.6}d^{-2.3}w^{-0.5}$$

where t, g, d, w are respectively the thickness, gage, depth, and web thickness. By multiplying the moment obtained on any test by the above value K, a band of standardized moment-rotation curves is obtained. If perfect correlation were obtained, the band would reduce to a single line.

In Fig. 16 the standardized moment KM, with the moment in kip-in., is plotted versus the rotation ϕ, in radians, for the tests on the plate connections. The equation of the mean curve as drawn is given by the polynomial

$$\phi = 5.1KM \times 10^{-5} + 6.2(KM)^3 \times 10^{-10} + 2.4(KM)^5 \times 10^{-3}$$

with a spread of ± 10 per cent.

Thus for any connection, at least within the limits of the tests, knowing t, g, d, w, the constant K can be determined, and for any value of moment the rotation can be found from the above expression.

THE BEAM LINE

The adequacy of the shear connections for use depends on whether or not they can develop the required shear capacity and whether the rotations reached exceed those when the beam has itself reached its ultimate load. The latter point is first discussed.

In Fig. 17 a moment-rotation curve is sketched for some relatively flexible connection, and the linear beam line relationship between the end moment M developed at the end of a uniformly loaded beam and the corresponding rotation as given by the expression

$$\phi = wL^3/24EI - ML/2EI$$

From the above expression, for a fixed beam with zero end rotation the end moment M is determined to be $wL^2/12$, and for a simply supported beam with zero end moment the end rotation is

$$\phi_0 = wL^3/24EI$$

If the load w is the same for all points on the beam line, then the maximum stress for the fixed ended beam due to a moment $wL^2/12$ is only $2/3$ that for a simply supported beam with a moment of $wL^2/8$.

Beam lines can be drawn, as noted in many texts,[3, 4, 5] for working conditions or yield conditions.

The point of intersection of the beam line with the experimental moment-rotation curve for a particular connection indicates the moment and rotation that would result from a beam of length L with some uniform load, when the beam is fabricated with this connection at each end. It has commonly been accepted that, if the beam line has been drawn for working conditions, this point of intersection corresponds to the attainment of the working stress at some point on the beam, or, if it has been drawn for yield conditions, the yield stress is attained at some point. Thus, having drawn the beam line for yield conditions, any connection displaying a moment-rotation curve which crosses the yield beam line would be considered satisfactory and represent the condition where the beam yields before the connection fails. If the beam line has been drawn for the yield condition from a fixed end moment M_y to a free end rotation corresponding to the attainment of the yield moment at the center, then different points on the line do not represent the same total load on the beam. In fact, the load to cause yielding of the simply supported beam is $w_0 = 8\,\sigma_y S/L^2$, and for yielding of a fixed ended beam the load is $w_f = 12\sigma_y S/L^2$. Thus $w_f = 1.5w_0$.

CONNECTION CURVE CROSSING THE BEAM LINE AT A ROTATION $k\phi_0$

END MOMENT $M = M_y(1-k)$

℄ MOMENT $M_c = \frac{wL^2}{8} - M_y(1-k)$

LOAD ON BEAM $w = \frac{w_0(3-k)}{2}$

MAXIMUM STRESS AT END $\sigma_e = \sigma_y(1-k)$

MAXIMUM STRESS AT CENTRE $\sigma_c = \frac{\sigma_y(k+1)}{2}$

FOR $k=0$: $w = 1.5w_0 = w_f$; $\sigma_e = \sigma_y$; $\sigma_c = \frac{\sigma_y}{2}$

FOR $k=1$: $w = w_0$; $\sigma_e = 0$; $\sigma_c = \sigma_y$

FOR $0<k<1$: $w = \frac{w_0(3-k)}{2}$; $\sigma_e < \sigma_y$; $\sigma_c < \sigma_y$

Fig. 18. Moment, load, and stress relationships

Moreover, the yield beam line as drawn does not represent the attainment of yield stress on the beam for any condition other than for a simply supported beam and a fixed ended beam, i.e., the two end points of the line.

In general, consider the beam line in Fig. 17a, drawn from a fixed end moment, $M_f = M_y =$ the yield moment, to the free end rotation, $\phi_0 = w_0L^3/24EI$. The equation of the beam line may be written as $M = M_y(1 - \phi/\phi_0)$ where ϕ and M are the coordinates of any point on the line. For any connection possessing a curve crossing the beam line at a rotation $k\phi_0$, as shown, the end moment is

$$M = M_y(1 - k\phi_0/\phi_0) = M_y(1 - k)$$

Figure 17b shows the moment diagram for the beam corresponding to this condition with a total static moment $M_s = wL^2/8$. The expression for the corresponding load w is found to be $w = w_0(2 - k)/2$.

This is a linear expression in w and k. For $k = 1$, the beam is simply supported and $w = w_0$. For $k = 0$, representing the fixed ended condition, $w = w_f = 1.5w_0$, as stated above.

The maximum stress at the end of the beam is given by

$$\sigma_e = M_y(1 - k)/S = \sigma_y(1 - k)$$

That is, the maximum stress at the end of the beam varies linearly from the yield value to 0, as k varies from 0 to 1.

The maximum stress at the center line of the beam is

$$\sigma_c = \sigma_y(k + 1)/2$$

From the above, for $k = 0$, $\sigma_e = \sigma_y$; for $k = 1$, $\sigma_c = \sigma_y$; for $0 < k < 1$, $\sigma < \sigma_y$ as given in Fig. 18.

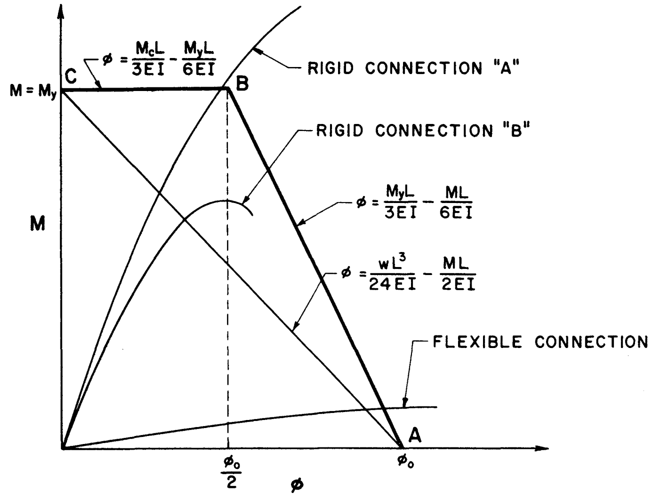

Fig. 19. Moment and rotation relationships

A beam line, to represent the condition of attainment of yielding at some point on the beam, at the center line or the end, must consist of two parts.

Considering first the case where *yielding* occurs at the center line of the beam, from the general equation for the beam line, and setting the center line moment $M_c = M_y$, the end rotation ϕ is given by

$$\phi = M_yL/3EI - ML/6EI$$

For $M = 0$, the end rotation is

$$M_yL/3EI = (w_0L^2/8)(L/3EI) = \phi_0$$

For $M = M_y$, the maximum value the end moment can take for the condition of attainment of yielding,

$$\phi = M_yL/6EI = \phi_0/2$$

Thus, this equation represents a straight line as shown in Fig. 19. For all points on this line from point **A**, representing a simply supported beam, to point **B**, representing a beam with end moment equal to the yield value, the center line moment equals the yield value.

Point **B** represents the case where the end connection has just sufficient rigidity to develop the yield moment at the end simultaneously with the development of the yield moment at the center line of the beam.

For connections with greater stiffness than represented by point **B**, the yield moment will be developed at the ends while the center line moment will be less. From the general beam line equation

$$\phi - wL^3/24EI - ML/2EI$$

for an end moment $M = M_y$, is obtained the expression

$$\phi = (M_cL/3EI - M_yL/6EI)$$

This states that for a constant value of end moment equal to M_y, the end rotation varies directly with the center line moment M_c.

For the center line moment equal to M_y, the rotation becomes

$$M_yL/3EI = \phi_0/2$$

This defines point **B**, as also established in Fig. 19 by the equation for the line **AB** and point **B** is common to both relationships.

For the center line moment equal to $M_y/2$, i.e., one-half the fixed end moment, the rotation is 0, corresponding to the fixed end condition at point **C**.

Thus, a beam line, to represent a condition where yielding is attained at some point on the beam, must consist of two phases: one for yielding at the center and the other for yielding at the ends.

The moment rotation curve for rigid connection **B** shows that this connection considered acceptable with a beam line drawn from **A** to **C** would suffer failure before yielding of the beam.

Similarly, for a beam line to represent the condition where the working stress is attained at some point on the beam, it must consist of two phases: one applicable to the center and the other to the ends.

Hechtman and Johnston[6] developed a "constant-maximum-stress" beam line for working stress conditions which is equivalent to portion **AB** of the two-phase beam line given in Fig. 19. They further pointed out that this was limited to cases in which the connection stiffness produces end moments no greater than the moment at mid-span, because with greater fixity the critical moment would occur at the ends.

The end plate connections generally developed from 0.03 to 0.16 of the yield moment at the beam line drawn for the yield condition and an L/d of 24.

This considerable variation indicates the wide range in flexibility that can be obtained, depending on the combination of the various parameters.

Considering the low ratio of the yield moment developed by these connections at the beam line, the standard engineering practice of treating the connections as shear connections only appears to be justified.

The angle connections developed end moments at the beam line within the same range as that developed by the end plates.

The rotation obtained at the beam line for end plate connections amounted to 0.84 to 0.97 of ϕ_0.

Since the rotational stiffness increased markedly when bearing on the column occurred, it would be desirable for the curve to exceed the beam line before the flange comes into bearing. In one test, bearing occurred at the beam line; in all other tests, the rotation exceeded the beam line before bearing occurred. Thus the beam would be carrying the yield moment before the lower flange hit the column and coping of the flange would not be required to limit the development of higher moments.

After hitting the column flange, the connections became much stiffer and moments ranging from 0.14 to 0.46 of the yield of the beam were obtained at rotations of 1.7 to 5.2 times the rotation corresponding to that of a simple beam with an L/d of 24. For the limited tests on angle connections, the moments and rotations developed were of the same order.

SHEAR CAPACITY OF CONNECTIONS

In the tests discussed, with the load applied at a lever arm of 4 ft, the shears developed were relatively small. For example, in one test on a 24W76 with 4 rows of bolts, the shear before column bearing occurred was only about 8 kips. The maximum web shear corresponding to yield on the web is 267 kips, and the maximum shear on the welds and bolts for a load factor of 3 would be about 250 and 230 kips, respectively.

The tests showed that the moment-rotation diagram obtained was independent of the lever arm used, i.e., independent of the moment-shear ratio. At least for the portion of the curves up to bearing on the flange, it appears that the rotational flexibility of the connections limits the moment developable and hence the shear capacity of the connection is not appreciably reduced. When cantilever tests are conducted with relatively large lever arms because of this flexibility, the shear capacity is never reached.

For a real beam with this type of connection, because of the flexibility of the end connection, the end moments developed for a given shear will be small and the point of contraflexure is close to the supports. For the 24W76 cited, a beam about 20 ft long, when loaded to produce yield moment, would have 1.7 times the working shear on the bolts. At this length the end moment developed at the beam line is about 252 kip-in. or 0.033 of the yield moment of 7,700 kip-in. The point of inflection is determined to be only about 2 in. from the end of the beam.

ANALYSIS OF CONNECTIONS

The behavior of the plates suggests two methods of analysis. At relatively low loads, as the top flange moves away from the column, the plate bends at the bolts and near the welds, suggesting that a flexural analysis based on the yield line concept could be made. At higher loads, the plate tended to act as a tensile membrane, suggesting a membrane analysis. The two modes of behavior are not separable and, indeed, when membrane action would be predominant at higher load, the portion of the plate near the neutral axis with relatively small displacements would be acting flexurally.

When the neutral axis lies within the depth of the end plate, the plate is pushed into the beam web until the bottom flange bears against the column. Yielding progresses into the web as this occurs, indicating that the

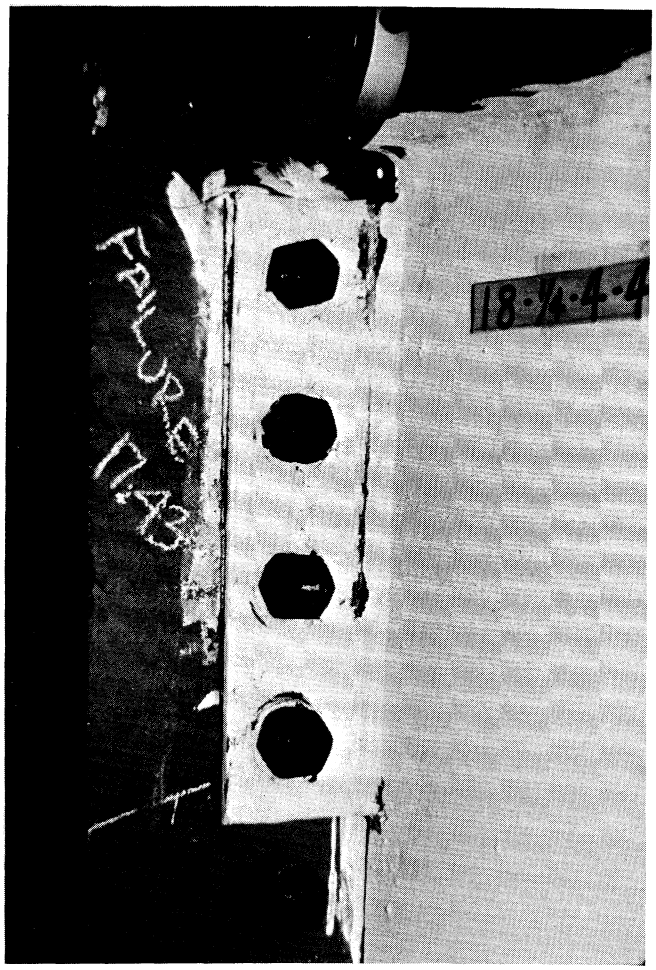

Fig. 20. End plate test

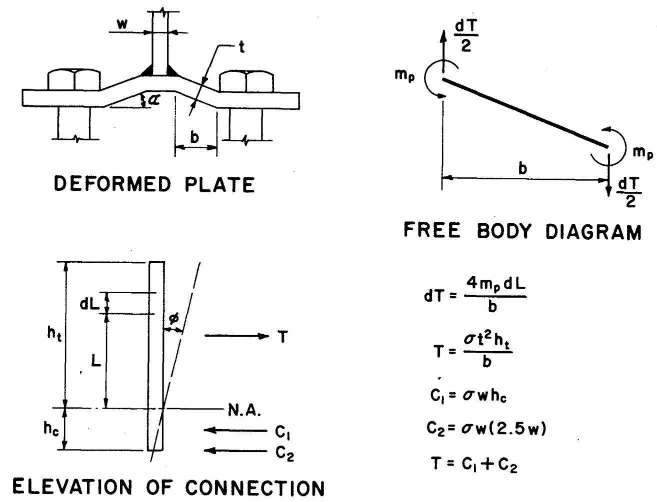

Fig. 21. Flexural analysis

compressive force developed here would equal some maximum stress integrated over the end area of the web bearing on the end plate below the neutral axis. The spreading out of the load in the beam web below the bottom of the connection gives use to an additional compressive force. After bearing of the bottom flange on the columns, sufficient compressive force to balance the tension developed by the top of the plate was obtained in bearing with negligible deformation of the beam flange, and rotation then occurred about the bottom flange. Figure 20 illustrates the condition at failure load.

FLEXURAL ANALYSIS

Under relatively small outward movements of the plate, full inelastic moments are assumed to develop at the toe of the fillet weld and, based on test observations, at the bolt line, as shown in Fig. 21. From the figure, for a length dL where m_p is the full inelastic moment per unit length,

$$dT = 4m_p dL/b$$

Hence, the tension developed is

$$T = \sigma t^2 h_t/b$$

The compressive force on the beam web is given by

$$C_1 = \sigma w h_c$$

The compressive force due to the load spreading out below the connection is taken as

$$C_2 = 2.5\sigma w^2$$

which is equivalent to spreading out over an additional vertical distance along the web of 2.5 times the web thickness.

This analysis does not give any information on rotations. Because of the severe local straining in the connections, it is suggested that calculations using the above expressions be based on the ultimate strength of the material.

With the development of membrane action, the connection would develop larger tensile forces than given by the above analysis.

MEMBRANE ANALYSIS

With large rotations, the top of the connection plate acts as a membrane. The outward movement at any point can be related to the increased distance from the bolt line to the weld, and from the straining of the plate the tension in the membrane can be determined. The component of the tension parallel to the weld, provided it does not exceed the strength of the beam web in tension, determines the tensile portion of the couple.

The compressive force below the neutral axis is determined as before. As the tensile force developed depends upon the outward movement of the top, the moment developed by membrane action is a function of the rotations.

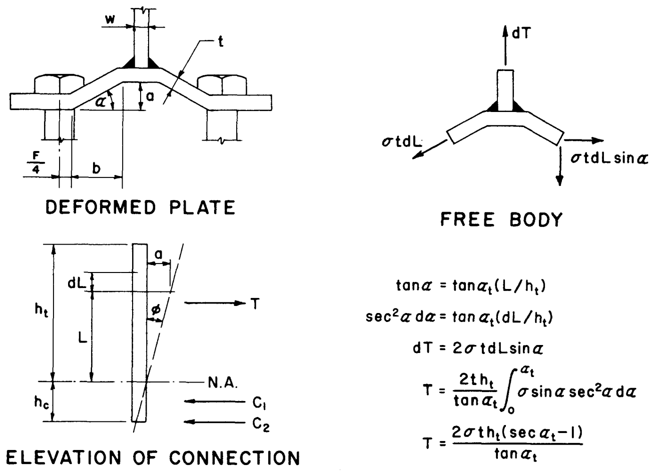

Fig. 22. Membrane analysis

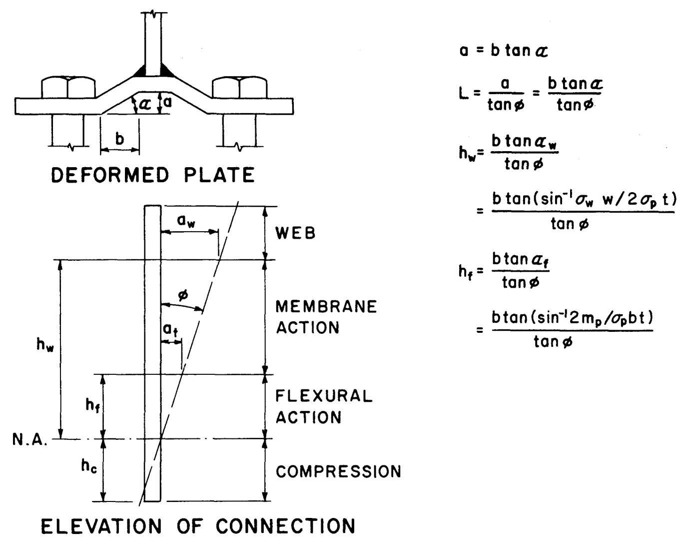

Fig. 23. Combined analysis

The tensile force developed at any distance L above the neutral axis depends upon the straining at that level and, therefore, the total stress-strain curve would have to be known up to the point of maximum straining. To simplify the analysis, an average stress could be assumed. In the following analysis it is assumed that

1. The outward movement of the web varies linearly with the distance from the neutral axis.
2. The stress developed in membrane action is constant.

Consider the outward movement of the beam as shown in Fig. 22 and the plan view of the deformed end plate. The angle α varies from 0 at the neutral axis to α_t at the top and $\tan\alpha = a/b$ is proportional to L.

Therefore, $\tan\alpha = kL$ and $\tan\alpha_t = kh_t$.

Then, $\tan\alpha = \tan\alpha_t(L/h_t)$

Hence, $\sec^2\alpha \, d\alpha = \tan\alpha_t(dL/h_t)$

On an elemental length dL of the plate, the force developed $= \sigma t dL$ and the total component of this force parallel to the web developed by the plates on both sides $= 2\sigma t dL \sin\alpha$.

Hence, $T = \dfrac{2th_t}{\tan\alpha_t} \displaystyle\int \sigma \sin\alpha \sec^2\alpha \, d\alpha$

If σ is taken as a constant value,

$$T = \frac{2\sigma t h_t}{\tan_t} \int_0^{\alpha_t} \sin\alpha \sec^2\alpha \, d\alpha$$

$$T = \frac{2\sigma t h_t}{\tan\alpha_t} \cdot (\sec\alpha_t - 1)$$

By equating C and T, a value of α_t can be determined consistent with the average tensile stress σ developed in the membrane.

For the elemental force

$$dT = 2\sigma t h_t \sin\alpha \sec^2\alpha \, d\alpha/\tan\alpha_t$$

the lever arm about the neutral axis is

$$L = h_t \tan\alpha/\tan\alpha_t$$

Summing moments of the tensile and compressive forces about the neutral axis

$$M = \int L dt + \int L dc$$

$$= \{\sigma t h_t^2/(\tan\alpha_t)^2\}\{\sec\alpha_t \tan\alpha_t - \ln(\sec\alpha_t + \tan\alpha_t)\} + \sigma w h_c^2 + 2.5\,\sigma w^2 h_c$$

As the membrane action develops, the rectangular stress blocks assumed to have been present due to flexural action disappear under the tensile straining. The distribution of strains across the thickness of the plate and along the distance from the bolt line to the web are complicated by the presence of the portion strained inelastically in flexure. All vertical sections of the plate at any level do not have the same strain or stress, but by statics the same tensile force must be developed at a particular level for all vertical sections.

For a given rotation at some level, h_w, above the neutral axis the component of tensile force due to membrane action parallel to the web of the beam will equal the maximum value the web can sustain. Hence,

$$dT = 2\sigma_p t dL \sin\alpha_w = \sigma_w w dL$$

where σ_p and σ_w are the stresses in the plate and web. The height up to which membrane action does not exceed the web capacity is (see Fig. 23):

$$h_w = \frac{b \tan\alpha_w}{\tan\phi}$$

$$= \frac{b \tan(\sin^{-1}\sigma_w w/2\sigma_p t)}{\tan\phi}$$

TEST	M_{OBS} INCH KIPS	ϕ_{OBS} RADIANS	$\frac{M}{M_{OBS}}$	$\frac{\phi}{\phi_{OBS}}$
5	778	0.0560	1.01	0.94
6	190	0.0185	1.00	0.80
7	334	0.0227	0.93	1.01
8	699	0.0282	1.10	1.02
9	1235	0.0370	1.20	1.14
10	323	0.0405	0.98	1.47*
11	635	0.0510	0.92	1.11
13	453	0.0250	1.06	0.92
14	682	0.0301	0.97	0.96
15	852	0.0408	1.00	1.00
16	1480	0.0506	1.05	0.95
17	245	0.0305	0.92	1.15
18	458	0.0316	1.16	1.07
19	680	0.0412	1.00	1.00
20	1137	0.0480	1.02	1.00
25	432	0.0422	0.98	0.87
26	167	0.0318	0.98	1.21*
27	212	0.0767	1.12	2.12*
28	65	0.0486	0.81	1.22*
AVERAGE			1.01	

* FOR TESTS 10, 26, 27 AND 28, M/M_{OBS} USING FLEXURAL ANALYSIS WERE 1.08, 1.03, 0.78 AND 0.84

Fig. 24. Analyses of tests at column bearing

Similarly at some level, h_f, above the neutral axis the component of tensile force parallel to the web will exceed that due to flexural action. Hence,

$$dT = 2\sigma_p t dL \sin \alpha_f = 4m_p dL/b$$

from which

$$h_f = \frac{b \tan \alpha_f}{\tan \phi}$$

$$= \frac{b \tan (\sin^{-1} 2m_p/\sigma_p bt)}{\tan \phi}$$

Analyses of the end plate tests were made to attempt to predict the moment and rotation at the point of column bearing, using the combined analysis. To allow for the bolt holes, the maximum stress used in the membrane analysis was limited to that equivalent to the ultimate strength on the net section. Based on test observations, the membrane was considered to extend on the average from the edge of the fillet weld to the quarter point of the nut width. Test 12, which suffered an unexplained premature failure, but only after the beam line was exceeded and when column bearing was imminent, was not analyzed. The results tabulated in Fig. 24 show not unreasonable agreement between the tests and the analyses which were made using nominal dimensions. Where the calculated moment exceeds the test moment significantly, as in Test 9, the rotation is also greater. In four tests, Tests 10, 26, 27, and 28, the rotations are much greater than that at column bearing. For these tests with ¼-in. thick end plates, the flexural analysis gives moments close to those obtained in the combined analyses, indicating that at column bearing the connections were behaving essentially flexurally.

CONCLUSIONS

1. End plate connections can provide adequate shear connections when designed for shear only.
2. End plate connections display moment rotation characteristics similar to those of angle connections.
3. End plate connections can be designed for a wide range of flexibilities.
4. In deep connections the possibility of bolt fracture should be investigated.
5. End plate connections should provide a rotation in excess of the beam line when column bearing occurs.
6. A two-phase beam line is required to represent the attainment of some specified stress level in the beam.
7. A combined flexural membrane analysis can be used to predict the moment and rotation at column bearing.

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