Another Approach to Simplified Design of a Curved Steel Girder

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REVIEW OF SIMPLIFIED ANALYSIS OF A HORIZONTALLY CURVED GIRDER

In recent years several publications discussing an approximate analysis of a curved girder grid system were presented. In the first paper[1] by Richardson, Gordon and Associates, a two-girder system connected with diaphragms was discussed. Further development for the multi-girder system connected by diaphragms was published by U. S. Steel Corporation.[2] The presentation of W. T. Robertson at the ASCE meeting in October, 1965[3] and the article by James W. Gillespie[4] also can be mentioned, among other publications concerned with the same subject.

The principal assumption of analysis used in all the above mentioned publications is based on the well established fact that the change in torque between two points of a curved girder subjected to bending is equal to the area of bending moment diagram between these points, divided by the radius of curvature of the girder. If the area of bending moment diagram between points **C** and **D** of a curved girder **AB** (Fig. 1) is M_a, the torsional moment accumulated between points **C** and **D** is

$$\Delta T = \frac{M_a}{R} \tag{1}$$

Assuming, further, that the diaphragms which connect the girders are providing the primary resistance to torsion, and indicating the distance between diaphragms as d, the torsional moment acting on the diaphragm at points of connection with the girder will be

$$T = \frac{Md}{R} \tag{2}$$

where M is the average value of moment for the portion of the girder attached to the diaphragm.

On the other hand, considering that the torsional moments acting along the girder are equal to the hori-

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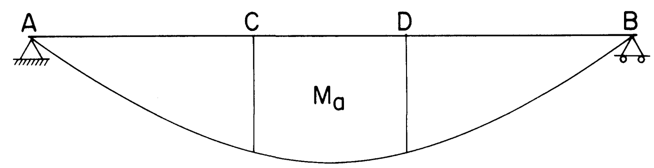

Fig. 1. Bending moment diagram

zontal components, H, of longitudinal forces due to stresses in the flanges, multiplied by the distance between their centers of gravity, it can be written (Fig. 2):

$$T = Hh \tag{3}$$

Assuming that the shear, V, at the diaphragm ends counter-balances these torsional moments, and substituting for T the value from Eq. (2), the following equation for a two-girder system can be obtained:

$$VD = (H_1 + H_2)h = (M_1 + M_2)\frac{d}{R}$$

or

$$V = \frac{(M_1 + M_2)d}{RD} \tag{4}$$

For the multiple girder system this equation is modified as:

$$V = \frac{\Sigma Md}{CRD} \tag{5}$$

where D is the distance between fascia girders, and C is a factor governed by the number of girders. These addi-

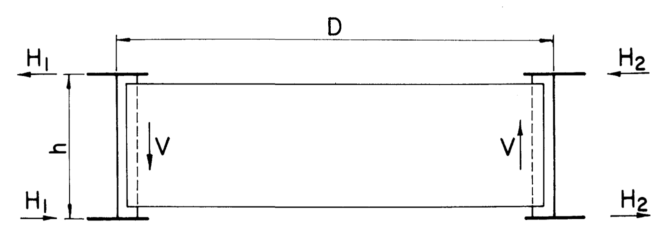

Fig. 2. Force distribution at diaphragm connections

tional vertical forces, V, calculated by Eq. (4) or (5), are applied to the girder as secondary loads due to curvature, and stresses produced by these forces should be added to the stresses due to principal loads.

However, some additional influence due to curvature is taken into account by the approximate methods mentioned above. From Fig. 2 it can be seen that each flange of the girder acts as a continuous beam supported by diaphragms and loaded by horizontal forces H which, from Eqs. (2) and (3), can be written as:

$$H = \frac{T}{h} = \frac{Md}{Rh} \tag{6}$$

Stresses due to these forces should be analyzed also and added to the stresses due to vertical loads.

In analyzing this method of computation for torsional effect of loads on a curved structure, it should be noted that additional forces V and H are calculated as secondary forces created by main tensile and compressive stresses acting in beam flanges due to bending moment. Being a by-product of bending moments, secondary forces can be evaluated for the uniformly distributed and concentrated loads, which are the main sources of the bending. The direct applied torsional moments, uniformly distributed or concentrated, do not induce primary bending moments* and, therefore, their torsional effect, which actually is essential, cannot be evaluated by the above described method. The other direct approach for evaluation of their effect will be used.

ACTING FORCES AND TORQUE AT THE ENDS FOR A CURVED GIRDER

Besides the uniformly distributed and concentrated loads, directly applied torsional moments (uniformly distributed and concentrated) are very common loads for any curved bridge.

A uniformly distributed load applied to a curved structure creates an accompanying uniformly distributed moment along the entire length of the structure because the center of gravity line of a uniform load does not coincide with the geometric center line of the structure (Fig. 3).

If the intensity of the uniform load is W and eccentricity is e (in ft), then the uniformly distributed torsional moment is

$$m_t = We \text{ kips/lin ft}$$

The presence of parapets and sidewalks, particularly of unequal size, creates additional uniformity distributed torsional moments.

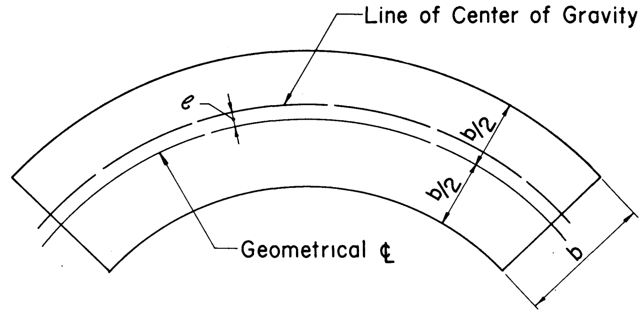

Fig. 3. Center of gravity of uniform load on curved span

Horizontal forces, such as wind on the superstructure or on the live load, centrifugal forces, and unsymmetrical lane loads also can contribute to the uniformly distributed moments. The magnitude of these moments, particularly for structures with a sharp curvature, is essential and cannot be disregarded. Concentrated torsional moments of different directions and magnitude are usually created by the eccentric position of live loads.

All four types of loads mentioned above are schematically shown in Fig. 4. It is obvious that the influence of direct torsional moments should be considered in the approximate methods of computations for curved structures. Additional forces in terms of V produced by these moments should be evaluated not as a function of bending moments, but independently. The following approach is proposed:

Consider a curved structure loaded with uniform or concentrated loads located eccentrically toward the center line of the structure. Each curved structure with two or multiple girder systems can be considered as torsionally restricted at the supports. From the geometry of the structure and values and position of loads, torsional moments at the restricted ends can be evaluated. From the value of these moments and the manner of their distribution among the individual bearings, additional loads on beams at the point of diaphragm connec-

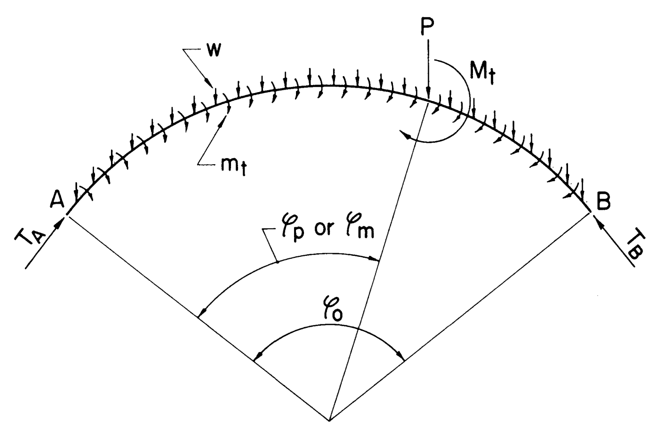

Fig. 4. Four types of loads acting on curved structure

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* *Actually, they induce small bending moments, which are disregarded by the approximate theory.*

tion are calculated. The first step in this procedure will be the evaluation of torsional moments at the restricted ends of the curved structure due to the applied loads and moments (Fig. 4).

In accordance with the general theory[5] of curved girders, the torsional moments at restricted ends **A** and **B** for different loadings are:

(a) For uniformly distributed load:

$$T_A = -T_A = -WR^2\left(\frac{\varphi_0}{2} - \tan\frac{\varphi_0}{2}\right) \qquad (8)$$

(b) For uniformly distributed moment:

$$T_A = -T_B = -\frac{mR}{t}\tan\frac{\varphi_0}{2} \qquad (9)$$

(c) For concentrated forces P:

$$\begin{aligned} T_A &= -PR\left[\frac{\sin(\varphi_0 - \varphi_P)}{\sin\varphi_0} - \frac{\varphi_0 - \varphi_P}{\varphi_0}\right] \\ T_B &= PR\left[\frac{\sin\varphi_0}{\sin\varphi_0} - \frac{\varphi_P}{\varphi_0}\right] \end{aligned} \qquad (10)$$

(d) For concentrated moment (torsional):

$$\begin{aligned} T_A &= -M_t\frac{\sin\varphi_0 - \varphi_M}{\sin\varphi_0} \\ T_B &= M_t\frac{\sin\varphi_M}{\sin\varphi_0} \end{aligned} \qquad (11)$$

The value of one of these torsional moments is used in the next step for evaluating secondary forces, V.

EVALUATION OF SECONDARY FORCES DUE TO TORQUE

After establishing the value of torsional moment T at the restricted end of the curved structure by one of Eqs. (8) through (11), distribution of this moment among the girders can be performed, considering that the end reactions at bearings are proportional to their distance from the center of rotation.

For the center of rotation as the center of gravity of all bearings (see Fig. 5), it can be written:

$$\Sigma RX = R_1X_1 + R_2X_2 + \ldots R_nX_n = T \qquad (12)$$

with the assumption that:

$$\frac{R_1}{R_2} = \frac{X_1}{X_2}\ ;\ \frac{R_1}{R_3} = \frac{X_1}{X_3}\ ;\ \frac{R_1}{R_n} = \frac{X_1}{X_n} \qquad (13)$$

For the framing with equally spaced girders, Eq. (12) can be written in the form:

$$R_1CD = T \quad \text{or} \quad R_1 = \frac{T}{CD} \qquad (14)$$

where R_1 is the secondary reaction at the bearing of fascia girder, D is the distance between fascias, and C is a factor which depends on girder number. This factor was calculated for different numbers of equally spaced girders and given in U. S. Steel *Highway Structures Design Handbook*,[2] pg. 1/12.10. For the convenience of the reader, this information is given in Table 1.

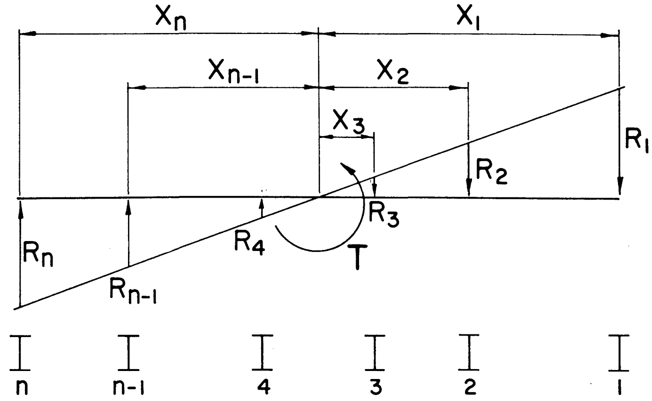

Fig. 5. Reactions distribution between supports

In the case of framing with unequal spacing, all reactions at the bearings can be calculated from Eqs. (12) and (13) individually.

After reactions at the end of each beam due to torque are established, it can be concluded that the total additional load on the beam will be equal to the sum of both reactions. However, the pattern of distribution of these secondary loads along the beam will depend upon the nature of actual loads acting on the structure and, in accordance with the general assumption, points of application of secondary loads will be at the points of diaphragm attachment.

PATTERN OF SECONDARY LOAD DISTRIBUTION

The following is assumed for the secondary load distribution pattern at the points of diaphragm attachment:

(a) For a uniformly distributed load acting on the structure, secondary loads shall be proportional to the area of bending moment diagram between points of diaphragm attachment.

Table 1
Values of Factor C in Equation (14)

Number of Girders	Factor C
2	1.00
3	1.00
4	1.11
5	1.25
6	1.40
7	1.56
8	1.72
9	1.88
10	2.04

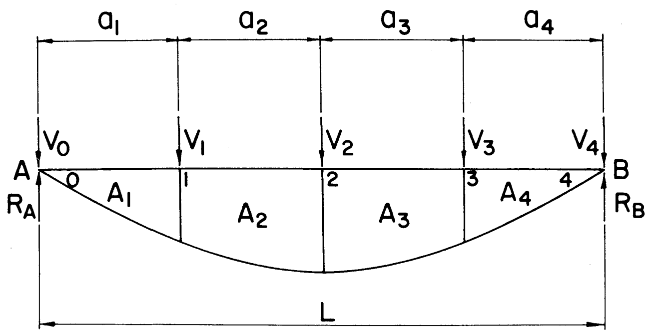

Fig. 6. Areas of M/R loads between diaphragms

For the beam **AB** (Fig. 6) with diaphragm attachment at points **1, 2** and **3,** values of the secondary vertical forces V_1, V_2, and V_3 are proportional to the reactions from areas A_1, A_2, A_3, and A_4. The vertical forces V_0 and V_4 are reactions directly transmitted to the supports. From the equation $\Sigma V = R_A + R_B$ and ratio of areas A_1, A_2, etc., forces V can be established. In Table 2 the values of V are given in terms of $R_A + R_B$ for framing with 1, 2, 3, 4 or 5 equally spaced diaphragms. In the case of framing with more than 5 diaphragms, or with diaphragms not equally spaced, the calculation of forces V in terms of $R_A + R_B$ should be performed individually.

(b) For the concentrated load, the same rule applies as for the uniformly distributed load.

For the position of the concentrated load, or several loads, which create maximum bending moment, the moment diagram should be drawn and secondary forces V should be taken as the reactions of the moment diagrams located between the points of diaphragm connections.

The rules described under (a) and (b) above are actually equivalent to the suggestion for load distribution assumed in Ref. 1.

Table 2

Number of Diaphragms	Values of Vertical Forces P in terms of $R_A + R_B$
1	$P_A = P_B = 0.1875$ $P_1 = 0.625$
2	$P_A = P_B = 0.092$ $P_1 = P_2 = 0.408$
3	$P_A = P_B = 0.055$ $P_1 = P_3 = 0.265$ $P_2 = 0.360$
4	$P_A = P_B = 0.036$ $P_1 = P_4 = 0.184$ $P_2 = P_3 = 0.28$
5	$P_A = P_B = 0.025$ $P_1 = P_5 = 0.135$ $P_2 = P_4 = 0.218$ $P_3 = 0.244$

For the moments applied directly to the structure, the following additional rules are suggested:

(c) For uniformly distributed moments, secondary loads are assumed proportional to the distances between diaphragms.

(d) For concentrated moments, secondary loads are assumed distributed between two diaphragms at the bay where moment is applied. Secondary loads should be proportional to the distances from the point of diaphragm attachment.

The four rules (a), (b), (c), and (d) offer a very simple procedure to establish additional vertical forces due to torque produced by the applied loads. After establishing values of additional vertical forces, additional bending moments and stresses can be calculated and added to the principal stress due to the bending.

SECONDARY STRESSES DUE TO HORIZONTAL FORCES

It has been previously mentioned that secondary forces considered for approximate analysis of curved framing consist of vertical forces V and horizontal forces H.

Horizontal forces H in the aforementioned approximate analysis were calculated by Eqs. (3) and (6), which assumed that torsional moment is equal to the horizontal components of the flange stresses multiplied by the distance between flanges.

Basically, this approach is sound. However, the value of H calculated from Eq. (3) as $H = T/h$ is correct only for a member which is symmetrical about its neutral axis. It would be more appropriate to calculate the horizontal component of stress, as follows:

If the flange area is A (in.^{2}) and average stress is f (psi), the total tensile or compressive force will be:

$$F = fA \tag{15}$$

The horizontal component of this force (Fig. 7) will be:

$$H = 2F \sin \alpha/2$$

Since $\sin \alpha/2 = ab/R$,

$$H = 2F(ab/R)$$

For a relatively small curvature, $2ab = S$, and

$$H = FS/R \tag{16}$$

For unit length, the horizontal component for a linear foot will be:

$$H = F/R \tag{17}$$

Using Eqs. (15) and (17), the unit horizontal force along the girder flange can be calculated more properly for any shape of a member.

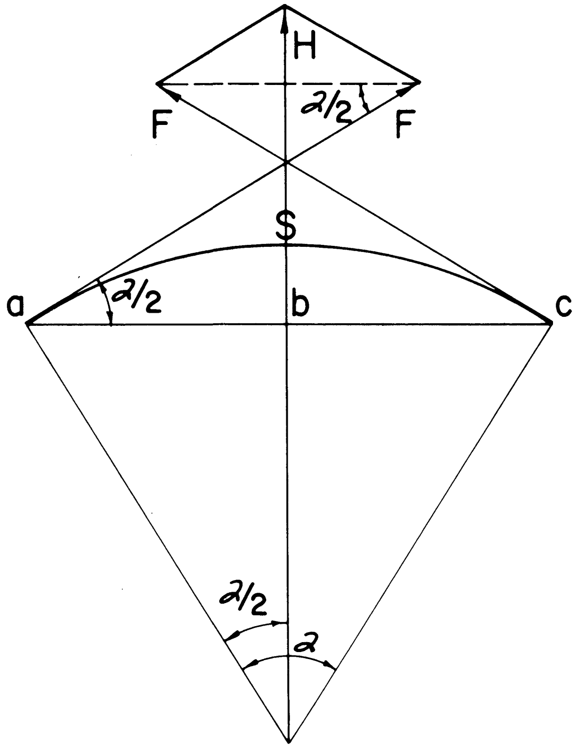

Fig. 7. Scheme of horizontal forces induced on flanges

After obtaining the horizontal forces acting on the flange, the corresponding stresses can be calculated considering the flange as a continuous beam supported at the point of diaphragm connection. It should be mentioned that for steel framing with concrete composite deck connected to the top flange by shear connectors, additional horizontal forces should be considered for the top flange for the loads acting on steel framing only. Dead loads superimposed on the composite structure and live loads can be disregarded because of high horizontal rigidity of the concrete deck. For the bottom flange, which remains unrestricted all forces shall be considered.

It also shall be mentioned that in Eq. (13), stress should be taken considering both principal and secondary vertical forces.

The illustrative examples show details of application for the proposed method.

Example 1

Given: Bridge framing with 5 girders spaced 7.5 ft c. to c. as shown in Fig. 8. Live load per linear foot = 7.5 kips. Radius of curvature at the center line = 400 ft. Span length = 64 ft. Diaphragms at 4 equal spaces.

Solution: Calculations are performed for secondary vertical forces due to dead load for fascia girder. Total torsional moment at supports in accordance with Eq. (8):

$$T_A = -T_B = -Wr^2\left(\frac{\varphi_0}{2} - \tan\frac{\varphi_0}{2}\right)$$

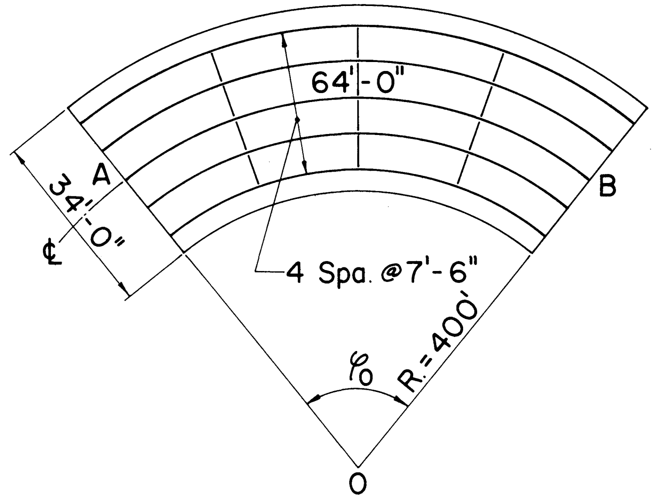

Fig. 8. Scheme of framing for Examples 1 and 2

For given data:

$$\varphi_0 = \frac{L}{R}\frac{64}{400} = 0.16 \text{ rad.} = 9.167° = 9°10'00''$$

$$\frac{\varphi_0}{2} = 0.08 \text{ rad.} = 4°35'00''$$

$$\tan\frac{\varphi_0}{2} = 0.080165$$

$$T_A = -T_B = -7.5 \times 400^2\,(0.0800 - 0.080165) = 198 \text{ kips}$$

$$D = 4 \times 7.5 = 30 \text{ ft}$$

Reactions R_A and R_B at end of fascia, for a 5-beam system: Using Eq. (14) and value of C from Table 1,

$$R_A = R_B = \frac{T}{CD} = \frac{198}{1.25 \times 30} = 5.3 \text{ kips}$$

With the factors from Table 2, secondary forces for the 3-diaphragm system (Fig. 9):

$$V_1' = V_3' = 0.265 \times 2 \times 5.3 = 2.81 \text{ kips}$$

$$V_2' = 0.36 \times 2 \times 5.3 = 3.82 \text{ kips}$$

These vertical forces V_1', V_2', and V_3' are secondary forces created by a uniformly distributed load applied at the center of framing. Actually, however, due to the curvature this uniformly distributed load creates a uni-

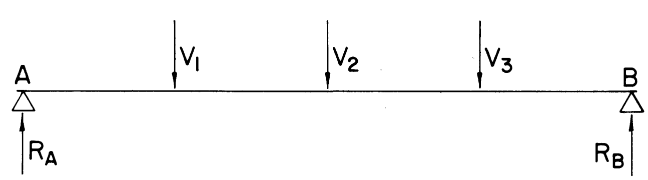

Fig. 9. Secondary forces at points of diaphragm connections

formly distributed moment which can be calculated as follows:

Considering one linear foot of bridge with $R = 400$ ft (Fig. 10) and width of deck = 34 ft; establish center of gravity at segment by taking area moment about exterior edge of deck:

$$\text{Area 1: } 0.957 \times 34 \times 17 \quad = 553.15$$

$$\text{Area 2: } 2 \times 0.043 \times \frac{34^2}{3 \times 2} = \frac{16.57}{569.72}$$

$$S = \frac{569.72}{34} = 16.75$$

Eccentricity = 17.0 − 16.75 = 0.25

D. L. moment about center line per one linear foot:

$$m = 7.50 \times 0.25 = 1.875 \text{ kip-ft}$$

With reference to Eq. (9),

$$T_a = -T_b = -1.875 \times 400 \times 0.080165 = -60 \text{ kip-ft}$$

Reactions for fascia girder from secondary forces:

$$R_A = R_B = \frac{60}{30 \times 1.25} = 1.6 \text{ kips}$$

Distributing this reaction in accordance with assumption (c) for secondary load distribution patterns:

$$V_1'' = V_2'' = V_3'' = \frac{2 \times 1.6 \times 16}{64} = 0.80 \text{ kips}$$

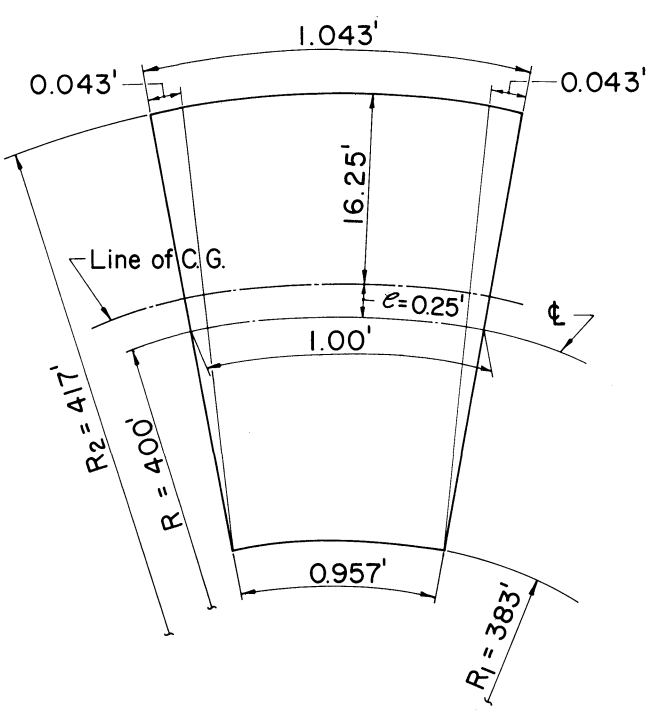

Fig. 10. Scheme of one lin. ft. of deck for establishing eccentricity

Total secondary forces acting on fascia at point of diaphragm connections:

$$V_1 = V_3 = 2.81 + 0.80 = 3.61 \text{ kips}$$

$$V = 3.82 + 0.80 = 4.62 \text{ kips}$$

The same secondary forces calculated by the approximate method presented in Ref. 2 (assuming average span = 64 ft) are:

$$\Sigma M = \frac{Wl^2}{8} = \frac{7.5 \times 64^2}{8} = 3840 \text{ kip-ft}$$

$$K = \frac{RD}{d} = \frac{400 \times 30}{16} = 750$$

$$V = \frac{\Sigma M}{CK} = \frac{3840}{1.25 \times 750} = 4.10 \text{ kips}$$

Similarly, for points **1** and **3:**

$$V_1 = V_3 = 0.75 \times 4.10 = 3.08 \text{ kips}$$

It can be seen that for a uniformly distributed load with eccentricity created purely by the circular shape of the structure, the difference between secondary forces is about 15 per cent. This difference may be considerably increased if the direct torsional moment due to unsymmetrical dead or live load is applied to the structure. This will be illustrated in the next example.

Example 2

Given: For the framing in Example 1, consider the torsional influence of two concentrated loads of 36 kips each in position as shown in Fig. 11.

Solution:

Bending moment: $M_B = 36 \times 24 = 864$ kip-ft

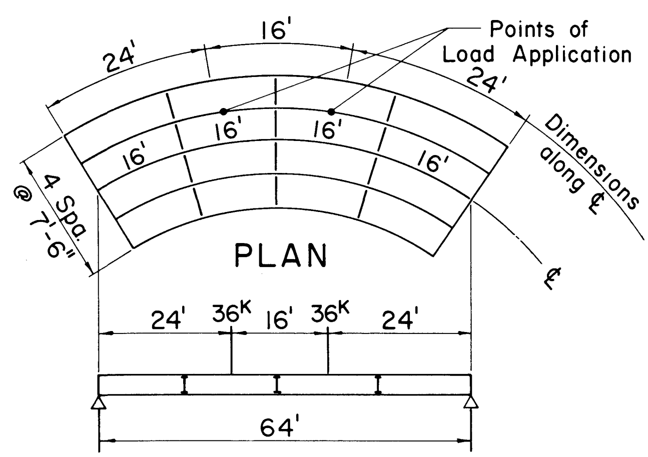

Fig. 11. Scheme of framing with eccentric live loads

Acting torsional moments at points of force application:

$$T = 36 \times 7.50 = 270 \text{ kip-ft}$$

$$\varphi_P = \varphi_T = \frac{24}{400} = 0.06 \text{ rad} = 3°26'16''$$

$$\varphi_0 - \varphi_P = 9°10'00'' - 3°26'16'' = 5°43'44''$$

$$\sin \varphi_P = 0.05996455; \sin(\varphi_0 - \varphi_P) = 0.09982145$$

$$\varphi_0 - \varphi_P = 0.16 - 0.06 = 0.10 \text{ rad}$$

$$\sin \varphi_0 = 0.15930687$$

From Eq. (10), torsional moments at supports due to concentrated loads are:

$$T_A' = 36 \times 400 \left[\frac{0.09982145}{0.15930687} - \frac{0.10}{0.16}\right] = -23.04 \text{ kip-ft}$$

$$T_B' = 36 \times 400 \left[\frac{0.05996455}{0.15930687} - \frac{0.06}{0.16}\right] = 20.16 \text{ kip-ft}$$

Total torsional moments due to 2 forces:

$$T_A = -T_B = -23.04 - 20.16 = -43.20 \text{ kip-ft}$$

From Eq. (11), torsional moments at support due to concentrated torsional moments are:

$$T_A'' = -270 \times \frac{0.09982145}{0.15930687} = -169.2 \text{ kip-ft}$$

$$T_B'' = 270 \times \frac{0.05996455}{0.15930687} = 101.6 \text{ kip-ft}$$

Total torsional moments due to 2 acting moments:

$$T_A = -T_B = -169.2 - 101.6 = -270.8 \text{ kip-ft}$$

Calculate secondary forces separately from both loadings. Secondary forces due to concentrated loads:

$$R_A' = R_B' = \frac{43.2}{1.25 \times 30} = 1.155 \text{ kips}$$

$$V_1 + V_2 + V_3 = 2 \times 1.155 = 2.31 \text{ kips}$$

In order to distribute these forces in accordance with assumption (b) for secondary load distribution patterns, refer to the moment diagram shown in Fig. 12. Indicating the intensity of load as W and assuming that the total moment area shall be equal to 2.31 kips, then

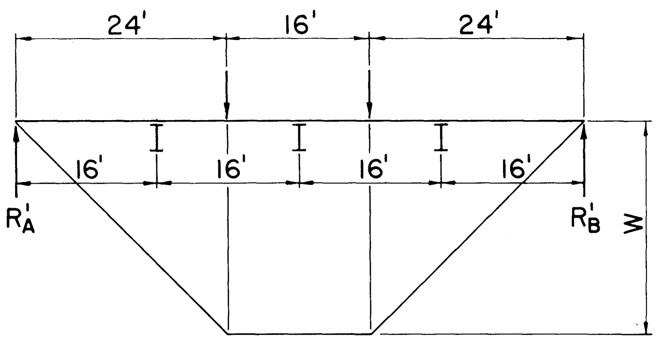

Fig. 12. Scheme of secondary loading due to concentrated forces

$$16W + \left(2 \times 24 \times \frac{W}{2}\right) = 2.31$$

$$W = 0.05775 \text{ kips}$$

Reaction:

$$V_2 = 2\,(0.75 \times 8 + 0.25 \times 0.75 \times 8 + 0.25 \times 0.5 \times 8 \times 0.3333)\,0.05775$$

$$= 2\,(6.0 + 1.50 + 0.333)\,0.05775$$

$$= 0.90 \text{ kips}$$

$$V_1 = V_3 = \frac{2.31 - 0.90}{2} = 0.705 \text{ kips}$$

Secondary forces due to concentrated torsional moment of 270.8 kip-ft:

$$R_A = R_B = \frac{270.8}{1.25 \times 30} = 7.22 \text{ kips}$$

$$V_1 + V_2 + V_3 = 2 \times 7.22 = 14.44 \text{ kips}$$

From assumption (d), the loads on diaphragms are proportional to the distance to the acting moment:

$$V_1 = V_3$$

$$V_2 = 2V_1 \text{ or } 4V_1 = 14.44 \text{ kips}$$

$$V_1 = \frac{14.44}{4} = 3.61 \text{ kips}; V_2 = 7.22 \text{ kips}$$

Compare these results with values which can be obtained using the approximate method as recommended in Ref. 2:

$$V = \frac{\Sigma M}{KC}$$

where

$$K = \frac{RD}{d} = \frac{400 \times 30}{16} = 750$$

For points **1** and **3:**

$$\Sigma M = 36 \times 16 = 576 \text{ kip-ft}$$

$$V_1 = V_3 = \frac{576}{750 \times 1.25} = 0.62 \text{ kips}$$

For point **2:**

$$\Sigma M = 36 \times 24 = 864 \text{ kip-ft}$$

$$V_2 = \frac{864}{750 \times 1.25} = 0.92 \text{ kips}$$

It can be seen that the secondary forces attributed to the concentrated load are alike by both methods. *However, the portion of secondary forces attributed to the torsional moments,* *which cannot be evaluated by previously discussed methods, is much more essential than the secondary loads induced by the concentrated loading.*

CONCLUSION

The proposed approximate method of calculation of additional vertical forces at point of diaphragm connections of curved steel framing is simple in concept and easy in its application. For loading applied along the center line of curved framing, this method produced results relatively close to the figures obtained by the *U. S. Steel Design Handbook* formula. *However, this method provides the possibility of revealing additional secondary forces due to directly applied moments which can be of considerable value and cannot be disregarded.*

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