A Modification to the Subassemblage Method of Designing Unbraced Multi-story Frames

OSCAR DE BUEN

A NEW METHOD of designing girders and columns of unbraced multi-story frames under the combined action of vertical and horizontal loads has been recently developed. The method is based on the concept of sway subassemblages and takes into account the elastic-plastic behavior of girders, the instability of columns, and second order effects, caused by $P\Delta$ moments, on the structure. It employs a semi-graphical solution using specially prepared charts, and its final aim is to obtain the complete horizontal load-lateral deflection curve for each story of the frame.

This method has been thoroughly described in Ref. 1, and Ref. 2 is an abstract of the method. Its bases appear in Ref. 3, and the charts required have been published in Ref. 4.

The purpose of this paper is to present a modification to the original method, which allows us to obtain the horizontal load-sway deflection curve ($Q\Delta$ curve) for each story of the frame without employing the charts of Ref. 4 and reducing, at the same time, the numerical work required to solve the problem. The final results are very close to those obtained by application of the original method, and the small differences between both methods are not significant in most practical cases.

The basic assumptions relative to the behavior of the structure are maintained (see Ref. 1, p. 4), but the condition of uniformly distributed gravity loads on the girders is deleted, because solutions can be obtained for any type of vertical loads.

As in the original method, it is necessary to make firstly a preliminary design and to obtain afterwards the structure's $Q\Delta$ curve; the members selected from the preliminary design will be adequate if the load-deformation behavior of each story is satisfactory.

Oscar de Buen is Professor of Civil Engineering, National University of Mexico, and a Consulting Engineer, Mexico City. He is a Professional Member of AISC.

SUBASSEMBLAGES

The sway subassemblages used in the modified method are those employed in the original method (Ref. 1, p. 6 and Ref. 2, p. 142).

BEHAVIOR OF THE COLUMNS IN THE SWAY SUBASSEMBLAGES

As a first step toward the solution of the problem it is necessary to study the behavior of a restrained column under a constant vertical load and a gradually increasing lateral load, including the effect of the $P\Delta$ moment due to the vertical load applied to the deflected column (see Fig. 1).

The characteristics of the restraining member (shown as a spring in Fig. 1) are assumed to be known, and for the time being are supposed to be constant, without changing under the applied horizontal load. (This assumption is not true, since the restraining members are the girders, and their characteristics are affected by the successive formation of plastic hinges under increasing lateral load, therefore not remaining constant; later in the paper a more detailed discussion will be presented of the restraining characteristics of girders, as well as the

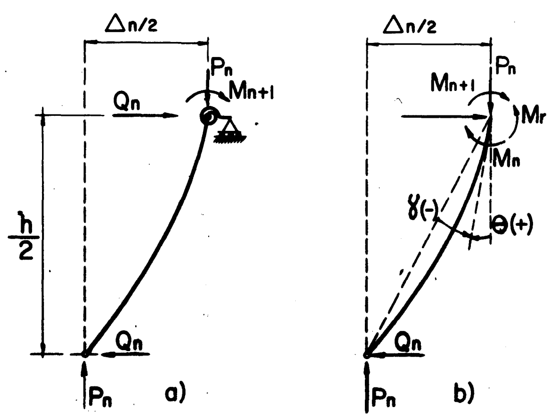

Figure 1

way they can be evaluated.) The column can then be analyzed to obtain its lateral load-sway deflection relationship, including the $P\Delta$ effect.

Figure 1 shows the forces acting on the restrained column and the resulting deformation configuration; also shown is the sign convention adopted for moments and rotations: they are positive when clockwise (this convention refers to moments acting on the joint).

The moment at the upper end of the column is given by

$$M_n = Q_n \frac{h}{2} + P_n \frac{\Delta_n}{2} \tag{1}$$

The total axial force applied to the column, P_n, is equal to the force received by the column from the upper story column, plus the forces transmitted to it by the girders in the level under consideration.

Equilibrium of moments at the upper joint, Fig. 1, requires that:

$$M_{n+1} + M_n - M_r = 0 \tag{2}$$

M_r is the restraining moment furnished by the girder or girders adjacent to the column.

Assuming that M_{n+1} is equal to M_n (this is a conservative supposition, but it is sufficiently accurate for practical purposes; see Ref. 1, p. 11), Eq. (2) reduces to:

$$M_r = 2M_n \tag{3}$$

Using the slope-deflection equations it can be shown that (see Fig. 1):

$$M_n = -3EK\left(\theta - \frac{\Delta_n}{h}\right) = -\frac{6EI}{h}\left(\theta - \frac{\Delta_n}{h}\right)$$

From the above equation:

$$\frac{\Delta_n}{h} = \frac{M_n h}{6EI} + \theta$$

But, $M_r = 2M_n$; then:

$$\frac{\Delta_n}{h} = \frac{M_r h}{12EI} + \theta \tag{4}$$

Equation (4) has been derived assuming: 1) the column behaves elastically and 2) the stiffness of the column does not depend on the magnitude of the axial force acting on it. Neither assumption is strictly true, but studying the M-θ curves of Ref. 5, Part III, it can be seen that the column behavior is practically linearly elastic until $M = M_{pc}$ if $P/P_y \leq 0.6$ and $h/r \leq 60$; besides, the column has an adequate rotation capacity. If $h/r \leq 40$, those conditions are true for $P/P_y \leq 0.8$ (it must be kept in mind that the h of the curves in Ref. 5 is equal to one-half the real column height). Furthermore, the effect of the axial force on the column stiffness is not important if $h/r \leq 40$ (Ref. 1).

From Eq. (1)

$$Q_n = \frac{M_n - (P_n\Delta_n/2)}{h/2}$$

But $M_n = M_r/2$; then:

$$Q_n = \frac{M_r - P_n\Delta_n}{h}$$

$$= \frac{M_r}{h} - P_n \frac{\Delta_n}{h} \tag{5}$$

Equation (5) shows that the column capacity to resist the horizontal load decreases when the sway displacement Δ_n increases, M_r and P remaining constant.

For each pair of values of the restraining moment M_r and the joint rotation θ, Eq. (4) allows us to compute the chord rotation Δ_n/h and Eq. (5) gives the corresponding Q_n value; in that way, a point of the horizontal load-sway displacement curve can be plotted, and the complete curve can be plotted if every value of the joint rotation θ can be computed. (In Ref. 1 the problem is solved by using the charts given in Ref. 4, which permit one to draw the $Q\Delta$ curves for restrained columns for several combinations of the parameters P/P_y and h/r and constant restraining moments; as already mentioned, one of the purposes of this paper is to find a way to obtain those curves without employing the charts.)

RESTRAINING CHARACTERISTICS OF GIRDERS; CONSTRUCTION OF $Q\Delta$ CURVES OF SWAY SUBASSEMBLAGES

The behavior of a restrained column under the combined action of vertical and horizontal loads has already been discussed, assuming that the characteristic of the restraining member does not change with the applied load. However, the assumption is not correct if the column belongs to a subassemblage, because the characteristics of the restraining girders do change when the horizontal loads increase and a succession of plastic hinges are formed in them. To obtain the horizontal load-sway deflection curves of a subassemblage it is then necessary to compute, in every stage of the loading process, the restraining characteristics of the girders adjacent to the column under consideration.

The restraint to which each column in a story is initially subjected is a function of the elastic and inelastic properties of all columns and girders in the story. Under increasing horizontal load this restraint gradually diminishes because of the successive formation of plastic hinges in the columns and girders, and eventually reduces to zero when the story is transformed into a mechanism.

It is obvious that a practical design method cannot consider the influence of all columns and girders in a story on the restraining characteristics of a joint, especially if the frame contains many bays. For simplifying the problem (obtaining, nevertheless, sufficient accuracy for design purposes) it will be assumed that in the first loading stage all the joints in every floor rotate through the same angle. This assumption makes possible the simplified analysis of multi-story frames, with little loss of accuracy for most practical conditions, and is frequently employed in elastic methods of analysis.[6] In Ref. 1 the initial restraining coefficients are computed by means of a more involved assumption, but the small increase in accuracy is more than offset by the considerable amount of additional numerical work involved.

The behavior of the windward subassemblage will now be considered. The subassemblage is composed of a column and a girder, and is acted upon by the constant vertical load and a horizontal load, directed from left to right, which increases from zero to the maximum intensity which can be resisted by the subassemblage (this is the horizontal load which corresponds to the formation of a mechanism). The behavior of the interior subassemblages is similar to this, but slightly more involved because both girders contribute to the restraining moment, and it is therefore necessary to consider the formation of plastic hinges in both girders.

Figure 2 shows the windward subassemblage under vertical loading only: P, which arises from the upper levels, and the load applied on the girder due to the slab which lies directly on it. This load will be considered as uniformly distributed, but the same procedure applies to any other type of load. Figure 2a shows the subassemblage and the column and girder which connect to the joint **B**, because it is necessary to check the moment at the top of that column in order to see if a plastic hinge appears there during the loading process.

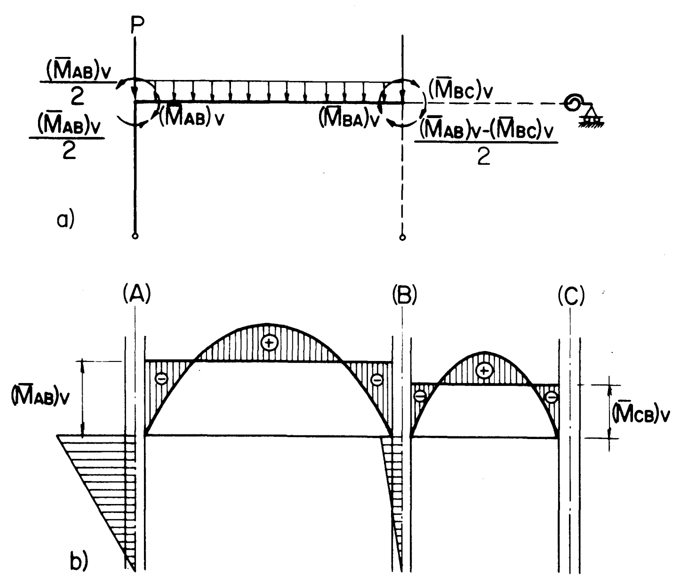

Figure 2

The unbalanced girder moments due to gravity loads which act on every joint are divided in equal parts between the two columns meeting at the joint (the upper column is not shown in the figure); $(\bar{M}_{AB})_V$ and $(\bar{M}_{BC})_V$ are the fixed-end moments and are computed using the actual beam length as given by the clear distance between the column faces.

Figure 2b shows the corresponding bending moment diagram.

Upon application of the horizontal load Q, the subassemblage displaces laterally (Fig. 3a), joints **A**, **B**, and **C** rotate through angles θ and, as a consequence of this rotation, moments appear at the girder ends which act on the columns and restrain their rotations; those moments can be easily computed, through the assumption of equality of joint rotations.

Figure 3a shows the deflections of beams and columns due only to the sway displacement and the bending moments which appear at the ends of every beam and column as a consequence of that displacement. Figure 3b shows the corresponding bending moment diagram, and Fig. 3c shows the total bending moment diagram, which

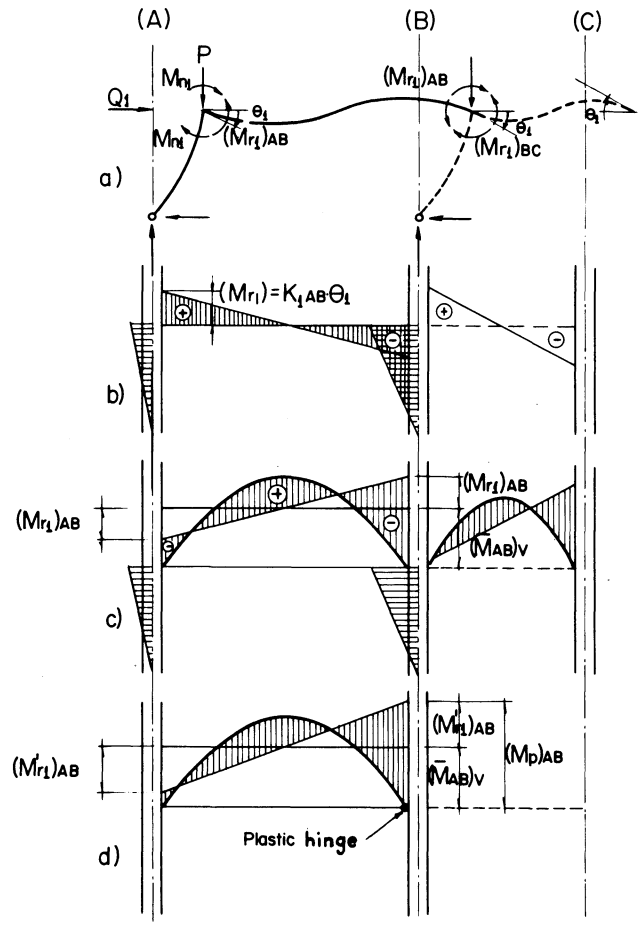

Figure 3

was obtained from the diagrams in Figs. 2b (vertical load) and 3b (sway displacement).

The moment at the left end of beam **AB**, which acts as a restraining moment at the column upper end, is given by

$$(M_{r1})_{AB} = K_{1AB} \cdot \theta \tag{6}$$

where K_{1AB} is the beam stiffness factor which, in this first stage of the loading process, is equal to $6EI_{AB}/L_{AB}$, since both girder ends rotate the same angle, and L_{AB} is the girder clear span length.

The first stage ends with the formation of the first plastic hinge in the girder, when the bending moment in any section reaches the plastic moment value of that section. The first plastic hinge usually forms at the leeward end of the girder, but the upper end of column **B** must be checked because the plastic hinge can form there, and this possibility must be taken into account during the successive stages of the analysis. The columns must be checked using the bending moment at the center line, so it is necessary to modify the bending moments computed until now, as they correspond to the column faces.

Figure 3d allows us to compute the maximum additional bending moment which appears at the windward end of beam **AB**, $(M'_{r1})_{AB}$, which corresponds to the formation of a plastic hinge at the leeward end:

$$(M'_{r1})_{AB} = (M_p)_{AB} - (\overline{M}_{AB})_V$$

$(M_p)_{AB}$ is the beam plastic moment.

(When a real problem is to be solved, the bending moment diagrams will be drawn to scale, and $(M'_{r1})_{AB}$ will be directly measured on them.)

Once $(M'_{r1})_{AB}$ is known, the rotation angle θ'_1, which corresponds to it can be computed; θ'_1 is the maximum rotation in joints **A** and **B** during the first stage.

$$(M'_{r1})_{AB} = \frac{6EI_{AB}}{L_{AB}}\,\theta'_1 \therefore \theta'_1 = (M'_{r1})_{AB}\,\frac{L_{AB}}{6EI_{AB}} \tag{7}$$

After obtaining the rotation angle θ'_1, the corresponding restraining moment at the column center line must be computed; following Ref. 1, p. 26, this restraining moment can be closely approximated by using the center-to-center girder span L'_{AB}, instead of the clear girder span L_{AB}.*

* *This simplification apparently implies an underestimation of the subassemblage sway resistance in a percentage which can be an important part of its total resistance, because the restraining moments computed at the column center line, using the center-to-center girder span, are smaller than those computed at the column faces, whereas the correct situation is generally the reverse. (The application of Eq. (8) to the subassemblage* AC, *Ref. 1, gives a restraining moment $M'_{r2} = 986$ kip-ft, but if that moment is computed adding to the moments at the column faces the additional restraining moments produced by the girder shears at those faces, its value increases to 1204 kip-ft, 22 percent greater.)*

Then,

$$M'_{r1} = \frac{6EI_{AB}}{L'_{AB}} \cdot \theta_1 \tag{8}$$

The restraining moment at the column upper end corresponding to the formation of the first plastic hinge having been computed, Eqs. (4) and (5) allow us to obtain the corresponding sway displacement and horizontal load:

$$\frac{\Delta'_1}{h} = \frac{M'_{r1}h}{12EI_{AB}} + \theta'_1 \tag{4a}$$

$$Q'_1 = \frac{M'_{r1}}{h} - P\frac{\Delta'_1}{h} \tag{5a}$$

(The "n" indexes have been suppressed for the sake of simplicity.)

The pair of Δ and Q values just computed can be represented as a point in a system of coordinate axis $Q\Delta$, which is a point of the subassemblage's horizontal load-sway deflection curve, and the straight line from the origin to that point is a good representation of the first part of the curve, corresponding to the first stage of the loading process, from the beginning to the formation of the first plastic hinge (Fig. 4).

The formation of the first plastic hinge does not mean that the subassemblage's maximum load capacity has been attained, but it does correspond to a change in the beam's properties; from this moment on, through the second part of the loading process, the behavior of the beam will be that of one having a true hinge at its right end, and its stiffness will be reduced to $3EI/L$.

Figure 5a shows the subassemblage during the second stage (column **B** and beam **BC** are not shown in the figure, since new increments in the magnitude of the load Q do not change the bending moment distribution which exists in them, because of the angular discontinuity created by the plastic hinge) and Fig. 5b shows the

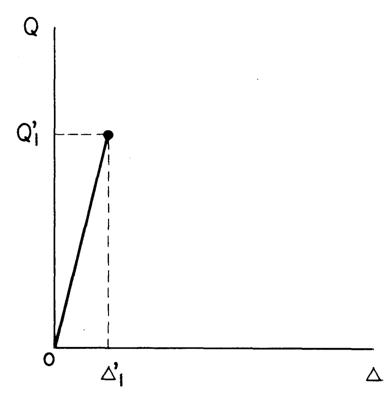

Figure 4

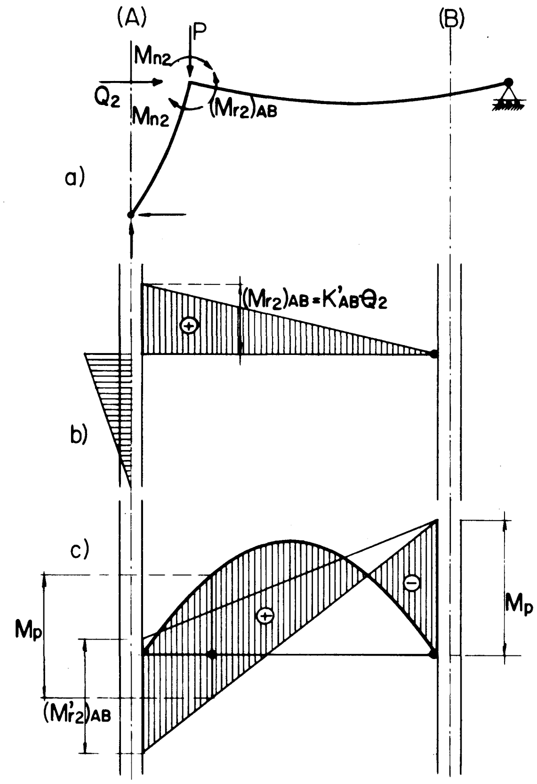

Figure 5

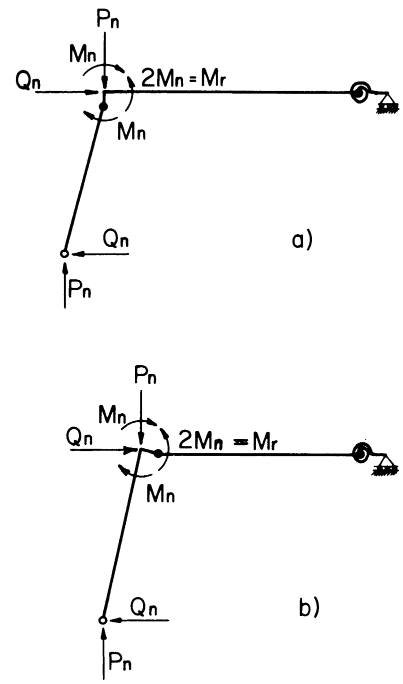

Figure 6

bending moment diagram corresponding to the additional horizontal load, Q; the total bending moment diagram, obtained by superposition of the diagrams in Figs. 3d and 5b, is shown in Fig. 5c.

This new stage of the loading process ends at the formation of a new plastic hinge, which transforms the subassemblage into a mechanism. The cross section in which the second plastic hinge gets formed can be the beam's left end, a point in the beam's central zone, or the column's upper end. (The beam's cross section in which the bending moment is a maximum, equal to M_p, is determined graphically; it must be checked that the bending moment at the column center line is not greater than its reduced plastic moment, M_{pc}.)

The additional rotation belonging to the formation of the second plastic hinge, θ'_2, is computed following the steps employed in the first stage, and the corresponding Δ and Q values, Δ'_2 and Q'_2, are then computed; thus, a second point of the horizontal load-sway deflection curve can be plotted, and a new straight line is drawn from the first to the second point.

After the formation of the second plastic hinge, in the beam or at the column upper end, the subassemblage changes into a mechanism; any additional lateral displacement Δ is then accompanied by a decrease in horizontal load intensity, and the $Q\Delta$ curve changes into a descending straight line which starts at the point—corresponding to the formation of the second plastic hinge.

The failure mechanism can be any of the mechanisms shown in Fig. 6 (if the second plastic hinge forms at the beam's central region, the left end moment can be determined graphically and the situation is similar to that shown in Fig. 6b): if the girder plastic moment M_p is larger than two times the column plastic moment M_{pc}, modified to include the effect of axial compression, the plastic hinge will form at the column end (Fig. 6a); if it is smaller, it will form in the girder (Fig. 6b), and if $M_p = 2M_{pc}$, it will form in any one of them (theoretically, the hinge will form at the intersection of both).

Before the subassemblage has become transformed into a mechanism, the restraining moment M_r afforded by the beam is a function of the joint rotation θ, but it changes into a constant, M'_r, when the second plastic hinge forms; from this stage on, $M_r = M'_r = M_p$ or $M_r = M'_r = 2M_{pc}$, depending on which plastic hinge forms first.

The load-deflection curve corresponding to additional displacements is the $Q\Delta$ curve of the rigid-plastic mechanism end, as mentioned before, is a descending straight line which starts at the point at which the second plastic hinge is formed.

At that moment, Eq. (5) can be written:

$$Q'_2 = \frac{M'_{r2}}{h} - \frac{P\Delta'_2}{h} \qquad \text{(5b)}$$

When the sway displacement Δ increases, the restraining moment M_r keeps its constant value, M'_r, equal to the girder's plastic moment or to twice the column's reduced plastic moment. Since P also is a constant load, Eq. (5b) shows that increasing deflections are only possible when the horizontal load Q decreases.

After formation of the failure mechanism, deflections Δ and horizontal loads Q are related by Eq. (5c):

$$Q = \frac{M'_{r2}}{h} - \frac{P}{h}\Delta \tag{5c}$$

This is the equation of a descending straight line; its slope is $-P/h$, and it passes over point (Δ'_2, Q'_2). The characteristics of this straight line being known, the complete horizontal load-sway deflection curve can be plotted (Fig. 7).

The curves which correspond to the rest of the subassemblages into which the story under consideration has been decomposed can be obtained following the same procedure (the plotting of the interior subassemblage curves involves more work than that required to plot the windward subassemblage curve, because, generally, more plastic hinges are necessary to form a mechanism. However, the leeward subassemblage curve is obtained very easily; obviously, the behavior of the lateral subassemblages is inverted when the direction of the horizontal loads changes).

An interior and an exterior subassemblage are shown in Figs. 8 and 9; a possible sequence of formation of plastic hinges as well as their $Q\Delta$ curves are shown in both cases. Three plastic hinges are required in order to transform the interior subassemblage into a mechanism, but only one is necessary in the exterior subassemblage.

SHEAR RESISTANCE OF A STORY

The total horizontal shear resistance provided by the members in a story is the sum of the shear resistances of the individual sway subassemblages in the story. Consequently, when the $Q\Delta$ curves of every subassemblage are known and plotted, following the procedures developed in the previous section, the load-deflection curve of the story is obtained by graphically combining them.

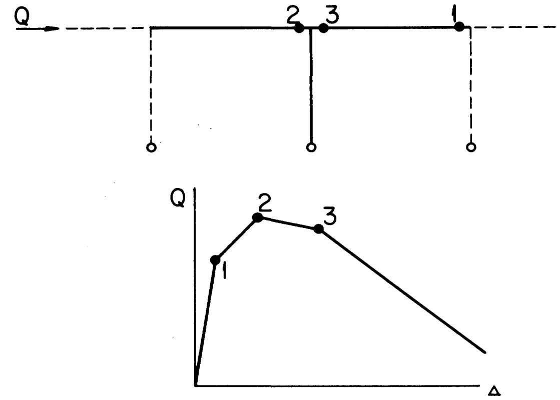

Figure 8

The sequence of formation of the plastic hinges required for transforming the complete story into a mechanism can be found, as the deflection index of the story (Δ/h) at the formation of each plastic hinge in the beams of the subassemblages is known.

When the load-deflection curve of the story has been constructed, it is possible to determine the maximum shear resistance of the story, the horizontal shear force, which corresponds to the formation of the collapse mechanism (generally smaller than the maximum), or the shear force pertaining to any value of the deflection Δ. (The shear force which corresponds to a given value of the deflection Δ is important in actual design problems because the relative displacements between consecutive floors of buildings under earthquake or wind loads must be maintained below such limits as permitted by partitions and other nonstructural elements, in order to avoid damages under working seismic or wind disturbances.)

Member sizes selected from the preliminary design, which were the basis for the computations leading to the subassemblages and to the story $Q\Delta$ curves, are adequate if the maximum shear strength of the story is equal to or

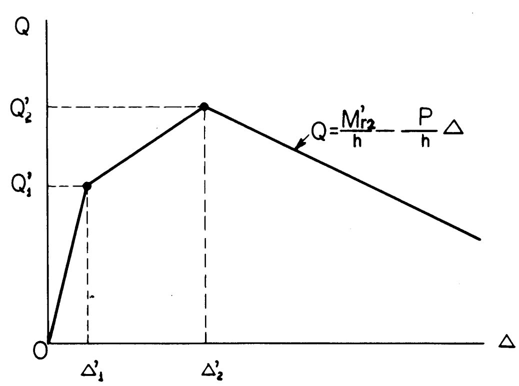

Figure 7

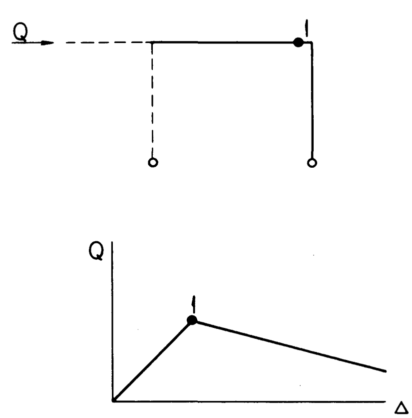

Figure 9

larger than the factored earthquake or wind horizontal load and if, simultaneously, the lateral deflections under working horizontal loads are admissible.

In the frequent cases in which the building floors are made of reinforced concrete slabs resting on the steel girders, their influence on the frame behavior can be considered by increasing the plastic moments in some regions of the girders, when these and the concrete slabs are designed as composite sections, or by only increasing the girder stiffnesses, with the corresponding diminution of lateral displacements and $P\Delta$ effect and increase in the frame's shear strength.

ILLUSTRATIVE EXAMPLE

The method just described will be applied to the analysis of the structure which was used as illustrative example in Ref. 1 (see pp. 31 and 32 of Ref. 1 for a complete description of the problem).

Only the analysis of subassemblage **AC** will be presented, in order to maintain the length of this paper within reasonable limits. Also, the load-deflection curve of the story will be plotted and compared with the curve which was obtained in Ref. 1.

Subassemblage AC

Stage I—During the first stage in the loading process, the bending stiffnesses of the girders are equal to $6EI/L$ (L is the clear girder span), corresponding to beams with the same rotations at both ends.

The first stage ends with the formation of the first plastic hinge in the subassemblage. Obviously, that plastic hinge will be formed in the right-hand end of one of the girders, because the bending moments due to horizontal and vertical loads are both negative at those sections.

A plastic hinge will form at the right-hand end of girder **AB** when the bending moment at that section increases to 600 kip-ft; then, the horizontal load bending moment, M_{BA}, must be equal to:

$$M_{BA} = 600 - 163 = 437 \text{ kip-ft}$$

The value 600 kip-ft is the plastic moment of the shape used in girder **AB**, obtained in a preliminary design, and 163 kip-ft is the fixed-end moment at the face of column **B** (see Fig. 10), due to factored gravity loading (load factor = 1.3).

The joint rotation corresponding to that moment is:

$$\theta = M_{BA}\frac{L_{AB}}{6EI_{AB}} = \frac{437 \times 12 + 223.7}{6 \times 29{,}500 \times 2{,}096} = 0.00316$$

The moment corresponding to the formation of a plastic hinge at the right-hand end of girder **BC** (542.3 kip-ft) and its joint rotation (0.00222) are determined by following the same steps. The first plastic hinge forms at the right-hand end of beam **BC**, for a rotation $\theta'_1 =$

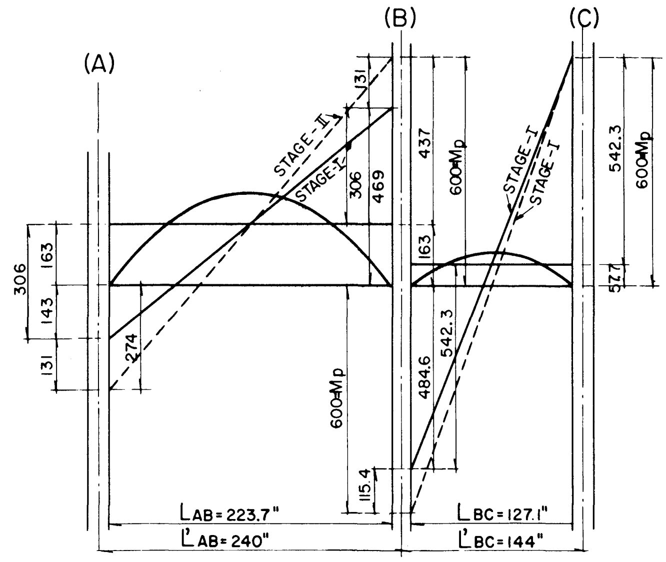

Figure 10

0.00222 (which is smaller than the rotation pertaining to the formation of a plastic hinge in **AB**), when the bending moment at that end of **BC** is equal to:

$$M_{AB} = \frac{6EI_{AB}}{L_{AB}}\theta'_1$$

$$= \frac{6 \times 29{,}500 \times 2{,}096}{223.7 \times 12} \times 0.00222 = 306 \text{ kip-ft}$$

The bending moment diagram pertaining to the end of stage 1 is shown in Fig. 10.

The restraining moment M'_{r1} is now computed employing Eq. (9), taking into account that there are two girders framed to the column's upper end because the subassemblage under discussion is an interior one:

$$M'_{r1} = \left(\frac{6EI_{AB}}{L'_{AB}} + \frac{6EI_{BC}}{L'_{BC}}\right)\theta'_1$$

$$= (128{,}000 + 215{,}000)0.00222 = 762 \text{ kip-ft}$$

The sway displacement Δ (in the form Δ/h) and the horizontal load, Q, which belongs to the ending of stage I, are now computed by means of Equations (4a) and (5a).

$$\frac{\Delta'_1}{h} = \frac{762 \times 142 \times 12}{12 \times 29{,}500 \times 3{,}229} + 0.00222 = 0.00337$$

$$Q'_1 = \frac{762}{12} - 1{,}384 \times 0.00337 = 63.5 - 4.7 = 58.8 \text{ kips}$$

In Ref. 1, Q = 58.6 kips corresponds to Δ/h = 0.00337 (see Table IV, p. 58, Ref. 1); the difference between both results is only 0.3 per cent.

By inspecting the bending moment diagram shown in Fig. 10, it can be seen, without computing the bending moments at the column's axis, that no plastic hinge forms in columns **A** and **B** (in column **A**, $2M_{pc} = 2000$ kip-ft > 143; in column **B**, $2M_{pc} = 1550$ kip-ft $> 469 + 484.6$). To know if a plastic hinge forms in column **C** the bending moment at the left-hand end of girder **CB** has to be computed:

$$M_{CD} = \frac{6EI_{CD}}{L_{CD}} \theta'_1 = 215 \text{ kip-ft}$$

Then, the total bending moment which acts upon column **C** is equal to $600 - 309 + 215 = 506$ kip-ft $< 2M_{pc}$ (772 kip-ft).

No plastic hinge forms in any column.

Stage II—The bending stiffness of girder **AB** is equal to $6EI_{AB}/L_{AB}$, as in stage **I**, but that of girder **BC** reduces to $3EI_{BC}/L_{BC}$, because there is a plastic hinge at the right hand end of the girder.

The additional bending moment which is necessary for the formation of a second plastic hinge at the right-hand end of beam **AB** is equal to $600 - 469 = 131$ kip-ft; this moment is arrived at when the joint rotation increases by 0.00095 rad.

A plastic hinge forms at the left-hand end of beam **BC** when the bending moment in that section increases in $600 - 484.6 = 115.4$ kip-ft, corresponding to an additional joint rotation of 0.00095 rad (computed with the beam's reduced bending stiffness).

Both plastic hinges form, then, at the same time, when the joints have rotated an additional amount $\theta_2 = 0.00095$ rad, the total joint rotation being $\theta'_2 = 0.00222 + 0.00095 = 0.00317$; see the corresponding bending moment diagram in Fig. 10 (with dotted line). In that moment, the subassemblage turns into a mechanism.

Following the procedure used in stage **I**, it can be easily proven that no plastic hinge appears at the columns' upper ends.

Additional restraining moment (which corresponds to stage **II**):

$$M_{r2} = \left(\frac{6EI_{AB}}{L'_{AB}} + \frac{3EI_{BC}}{L'_{BC}}\right) 0.00095 = 224 \text{ kip-ft}$$

Restraining moment corresponding to the termination of stage 2:

$$M'_{r2} = 762 + 224 = 986 \text{ kip-ft}$$

$$\frac{\Delta'_2}{h} = \frac{M'_{r2}h}{12EI_{AB}} + \theta'_2$$

$$= \frac{986 \times 144 \times 12}{12 \times 29{,}500 \times 3{,}229} + 0.00317$$

$$= 0.00148 + 0.00317 = 0.00465$$

$$Q'_2 = \frac{M'_{r2}}{h} - P\frac{\Delta'_2}{h} - \frac{986}{12} = 1{,}384 \times 0.00465$$

$$= 82.2 - 6.5 = 75.7 \text{ kips}$$

The horizontal load corresponding to $\Delta/h = 0.00465$ in Ref. 1 is $Q = 73.4$ kips; the difference between both results is 3.1 per cent.

Behavior of the subassemblage following mechanism formation—The plastic hinges which form simultaneously at the right-hand end of beam **AB** and at the left-hand end of **BC** when the joint rotation is equal to 0.00317 rad, in combination with the plastic hinge which formed in the first stage, are enough to transform the subassemblage into a mechanism. Then, in order to draw the complete $Q\Delta$ curve it is necessary to determine the straight line corresponding to that mechanism.

The two beams connected to the column develop plastic hinges in their ends near that column.

The maximum restraining moment is $M'_{r2} = 986$ kip-ft (against 993 kip-ft, Ref. 1). This value is maintained during the mechanism lateral displacement.

The behavior of the subassemblage after the formation of the mechanism is depicted by Eq. (5c):

$$Q = \frac{M'_{r2}}{h} - \frac{P}{h}\Delta = -P\frac{\Delta}{h} + \frac{M'_{r2}}{h}$$

$$= -1{,}384\frac{\Delta}{h} + 82.2$$

The corresponding straight line can be easily plotted after determining its intersections with both coordinate axes:

If $\Delta/h = 0$: $Q = 82.2$ kips:

If $Q = 0$: $\Delta/h = 0.059$

Load-deflection curve for the subassemblage—We have already obtained all the information needed to plot the subassemblage's horizontal load-sway deflection curve, shown in Fig. 11. Figure 11 also shows the curve computed in Ref. 1, which was obtained from the data in Table IV, p. 58, Ref. 1. Both curves practically coincide from the origin to the formation of the mechanism; from that point on the agreement continues to be very good.

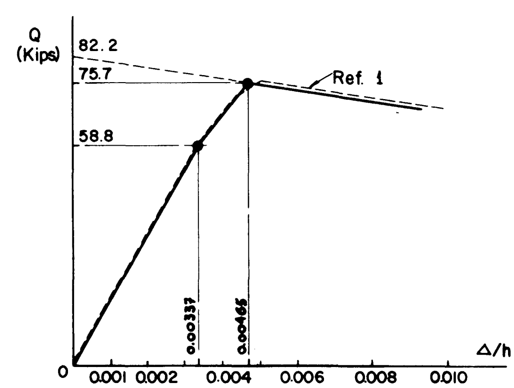

Figure 11

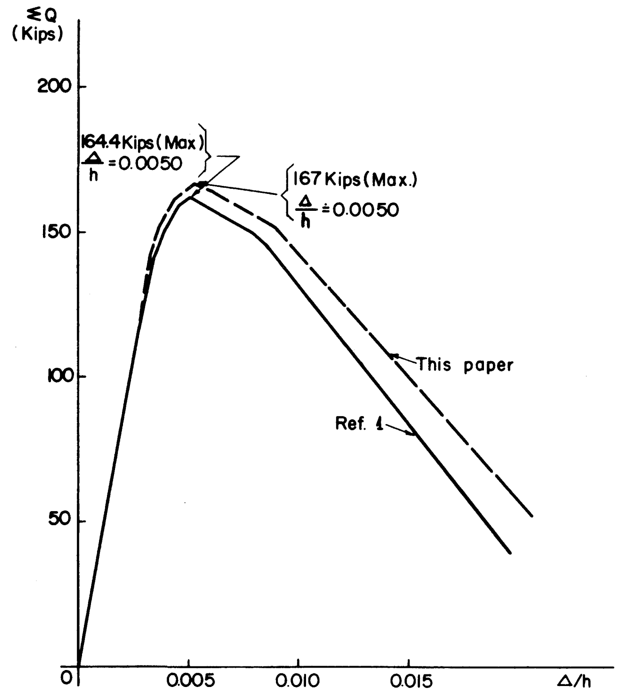

Figure 12

Discussion of results for remaining subassemblages—The results which were obtained for subassemblage **AB** are as accurate as those reported for subassemblage **AC**.

The differences between the results arrived at by employing both methods are more important in the remaining subassemblages, **BD** and **CD**. The discrepancies are probably due to the very high axial loads which act on the columns (P/P_y is equal to 0.85 for column **C** and 0.87 for column **D**, while only 0.43 and 0.53, respectively, for columns **A** and **B**), and to the fact that the restraining moments M'_r approach twice the column plastic moment, M_{pc}, in a range where the $Q\Delta$ curves are no longer straight lines.[4] Nevertheless, the method presented in this paper yields final results which provide very good agreement with those reported in Ref. 1, as can be seen in Fig. 12, where appear the complete story $Q\Delta$ curves obtained following both procedures.

ACKNOWLEDGMENTS

The writer is grateful to Emilio Rosenblueth and Enrique del Valle who read the manuscript and made helpful suggestions.

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