Yield Line Analysis of a Web Connection in Direct Tension

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YIELD LINE ANALYSIS is a tool for the design of plates, whether they be concrete slabs,[1] steel grids,[2] box columns[3] or the webs of rolled members.[4] This paper is prepared to help fill a void in connection design involving the transfer of tension to the web of a beam or column. The connections discussed here are: a welded plate (Fig. 1a), a bolted Tee connection (Fig. 1b), and a bolted Tee connection with reinforcing (Fig. 1c). One conservative analysis is made by Blodgett[3] for the Fig. 1a connection.

These three connection types are used for light bracing. The usual design procedure is to size the connection and assume the tension transfer. This paper will show, however, that the member web may not be adequate to carry the tension. Checking the web should be a standard design procedure. If a web is inadequate or cannot be economically reinforced, some other connection type should be selected by the designer.

NOMENCLATURE

EW = external work
F_y = yield stress of steel
IW = internal work
L = distance between inner yield lines, along the member
M_p = plastic moment
P_a = total allowable tension
P_u = total ultimate tension
b = distance from inner yield line to outer yield line, across the member
c = distance between inner yield lines, across the member
d = depth of member
e = distance from inner yield line to outer yield line, along the member
t = thickness of plate in welded plate connection
t_f = thickness of member flange
t_w = thickness of web or doubler plate
Δ = virtual deflection
ϕ, ϕ_1, ϕ_2, ϕ_3, ϕ_4, ϕ_5 = virtual rotation
ψ = $F_y t_w^2$

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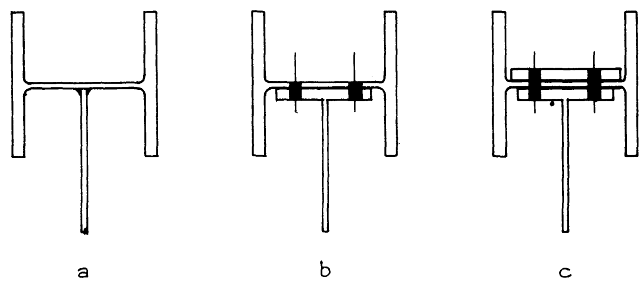

Fig. 1. Connection types

ANALYSIS

The model for analysis is shown in Fig. 2 with the yield line geometry and connection dimensions shown. The analysis assumes the distance Δ is small, so that $\tan \phi = \phi$, and that the mechanism shown has enough plate above and below so that it can actually form. On this basis, the angles ϕ_1, ϕ_2 and ϕ_3 are as follows:

$$\phi_1 = \Delta/e$$
$$\phi_2 = \Delta/b$$
$$\phi_3 = \phi_4 + \phi_5 = \Delta\sqrt{e^2 + b^2}/eb$$

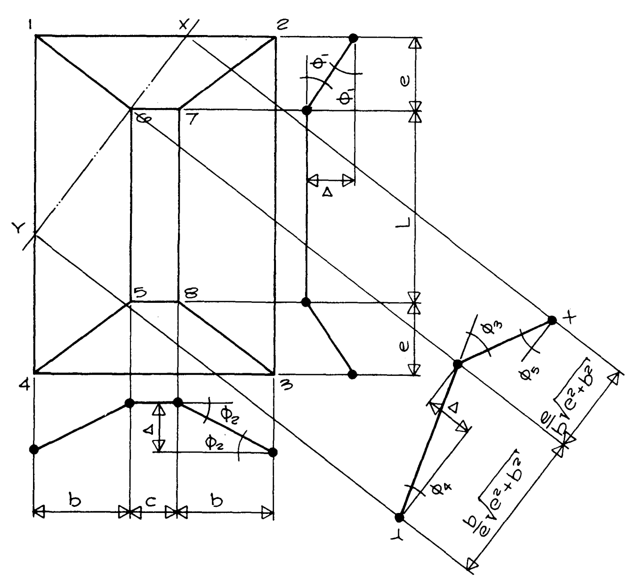

Fig. 2. Model geometry

For a plate element fixed at the flanges, the internal work is:

$$IW = M_p[\phi_1(2)(2b + c) + \phi_1(2c) + \phi_2(2L) + \phi_2(2)(L + 2e) + \phi_3(4)(\sqrt{e^2 + b^2})]$$
$$= (4M_p\Delta/eb)(2b^2 + 2e^2 + cb + Le)$$

$$M_p = F_y t_w^2/4 \quad (1)$$

$$\psi = F_y t_w^2 \quad (2)$$

The external work is $EW = P_u\Delta$. Equating external work and internal work, dividing through by Δ, and substituting Eqs. (1) and (2) results in Eq. (3):

$$P_u = (\psi/eb)\{2b^2 + 2e^2 + cb + Le\} \quad (3)$$

For any given problem, ψ, b, c, and L are known and P_u depends directly on e. The value of P_u is minimized by taking the first derivative of Eq. (3) with respect to e and solving for e.

$$df/de = \psi(2e^2 - 2b^2 - cb)/e^2b = 0$$

$$e = b\sqrt{1 + (c/2b)} \quad (4)$$

Substituting Eq. (4) into Eq. (3) and solving for P_u yields the lowest tension force which produces the mechanism for the connection.

$$P_u = \psi L/b + 4\psi\sqrt{1 + (c/2b)} \quad (5)$$

For a plate element supported, but not fixed, at the flanges, the internal work is reduced because there is no moment capacity along lines 1–4 and 2–3 in Fig. 2.

$$IW = M_p[\phi_1(2)(2b + c) + \phi_1(2c) + \phi_2(2L) + \phi_3(4)\sqrt{e^2 + b^2}]$$

Substituting for the angles, equating external work and internal work, dividing through by Δ, and substituting Eqs. (1) and (2) results in Eq. (6):

$$P_u = (\psi/eb)[2b^2 + e^2 + cb + Le/2] \quad (6)$$

Minimizing and solving for e yields Eq. (7):

$$e = b\sqrt{2 + (c/b)} \quad (7)$$

Substituting Eq. (7) into Eq. (6) and solving for P_u:

$$P_u = \psi L/2b + 2\psi\sqrt{2 + (c/b)} \quad (8)$$

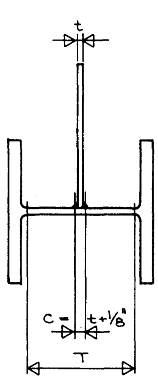

Fig. 3. Welded plate connection

DESIGN

Equations (5) and (8) are to be used to analyze a given connection to decide if it is adequate for a given load. To design a connection (i.e., to calculate the length L, as ψ, b, c, and P_u are now known) requires that Eqs. (3) and (4) and Eqs. (6) and (7) must be solved for L.

For a plate element fixed at the flanges, the length L is:

$$L = P_u b/\psi - 4b\sqrt{1 + c/2b} \quad (9)$$

For a plate element supported at the flanges, the length L is:

$$L = 2P_u b/\psi - 4b\sqrt{2 + c/b} \quad (10)$$

In the design of the connection, e must also be calculated from Eqs. (4) and (7) to determine the length of the member affected by the tension transfer or to determine the total length of reinforcing plate $(L + 2e)$.

APPLICATIONS

Depending on the design assumptions, the engineer can use Eqs. (5) and (8) in a variety of ways. If the engineer is checking the web of a beam or column, he can assume that the flanges give complete fixity or no fixity (or he can derive the equations for intermediate restraint). The engineer should also determine the dimension to use for $(2b + c)$. He can use the T distance of the member, the distance $(d - 2t_f)$, or some intermediate distance. For a welded plate connection, the distance L would be the length of the plate and distance c could be assumed to be the thickness of the plate plus $1/16$-in. on each side for the weld ($c = t + 1/8$, see Fig. 3), or c could be assumed equal to zero. For a bolted connection, L and c should be used as the sides of the rectangle through the bolt centers (see Fig. 4). For a doubler

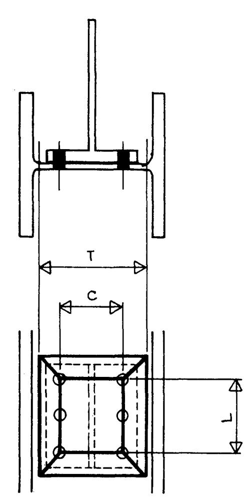

Fig. 4. Bolted Tee connection

plate, the author uses Eq. (8); however, with appropriately thick flanges and a full penetration weld to those flanges, Eq. (5) could be used. The web and doubler plate do not work together. The amount of load each can support must be calculated separately.

Equations (5) and (8) give the ultimate load for the chosen mechanism. To check the allowable load, the appropriate safety factor must be applied. AISC allows a stress of $0.75F_y$ for rectangular sections bent about the weak axis.[5] This results in a safety factor of 1.33. Engineering judgment may dictate a higher factor of safety.

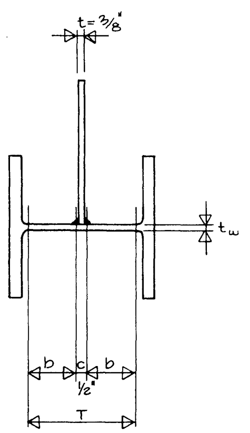

Fig. 5. Example 1

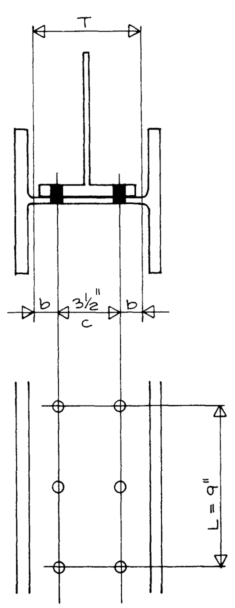

Fig. 6. Example 2

EXAMPLES

Example 1—⅜-in. welded plate connection (Fig. 5).
Column section: W8×31; A36 steel

$L = 12$ in.
$2b + c = T = 6.125$ in.
$c = 0.375 + 0.125 = 0.5$ in.
$b = (6.125 - 0.5)/2 = 2.8125$ in.
$t_w = 0.288$ in.; $\psi = 36 \times (0.288)^2 = 2.986$ kips

Use Eq. (5).
$P_u = 25.20$ kips
$P_a = 0.75P_u = 18.90$ kips

Example 2—Bolted tee connection (Fig. 6).
Column section: W8×31; A36 steel

$L = 9$ in.
$2b + c = T = 6.125$ in.
$c = 3.5$ in.
$b = (6.125 - 3.5)/2 = 1.3125$ in.
$t_w = 0.288$ in.; $\psi = 36 \times (0.288)^2 = 2.986$ kips

Use Eq. (5)
$P_u = 38.72$ kips
$P_a = 0.75P_u = 29.04$ kips

Example 3—Same as example 2, except add ½-in. doubler plate (Fig. 7).

For doubler plate, $\psi = 36 \times (0.5)^2 = 9.00$ kips

Use Eq. (8).
$P_u = 69.74$ kips
$P_a = 0.75P_u = 52.31$ kips

Add P_a to the value of P_a in Example 2.
$P_a = 52.31 + 29.04 = 81.35$ kips

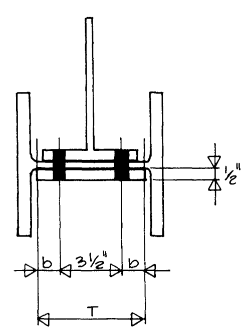

Fig. 7. Example 3

Example 4—Design a 3/8-in. welded plate connection for a W14×68 to carry 35 kips tension (Fig. 8).

$2b + c = T = 11.25$ in.
$c = 0.5$ in.
$b = (11.25 - 0.5)/2 = 5.375$ in.
$t_w = 0.418$ in.; $\psi = 36 \times (0.418)^2 = 6.29$ kips
$P_u = 1.33 \times 35 = 46.55$ kips

Use Eq. (9).
$L = 17.78$ in.; use 18 in.

Determine e, using Eq. (4).
$e = 5.50$ in.

Affected length $= 18 + 2(5.50) = 29.0$ in.

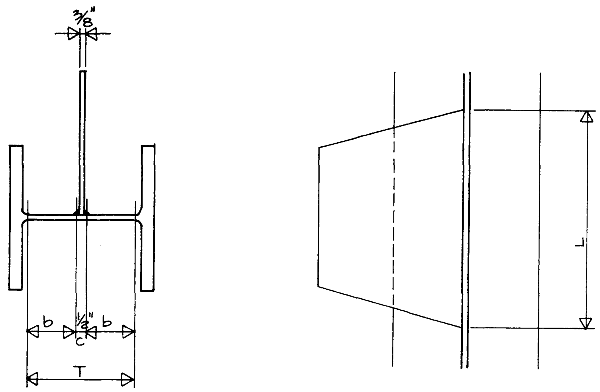

Fig. 8. Example 4

CONCLUSION

A yield line design procedure has been offered to allow the engineer to determine the amount of tension a beam or column web connection can support. The selection of mechanism geometry, design assumptions, and safety factor is left to engineering judgment and the particulars of the design problem. The design procedure (through the examples given) does show that the web may not be able to develop the full connection tension, without some reinforcing. Thus the engineer, as a standard design procedure, should always check the web of a member subjected to a tension force.

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