Design Criterion for Vibrations Due to Walking

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ABSTRACT

A design criterion for walking vibrations of broader application than previous criteria is proposed for steel floor or footbridge structures. The criterion is based on the dynamic response of steel structures to walking forces, as well as the sensitivity of occupants to vibration motion. The criterion is applicable to structures with natural frequencies below 9 Hz, where resonance can occur with a harmonic of the step frequency, but is extended beyond 9 Hz where footstep impulse response becomes important.

INTRODUCTION

Walking, good for body and soul, sometimes produces vibrations which are annoying to others. This is not a new problem. Tredgold (1828) wrote that girders over long spans should be "made deep to avoid the inconvenience of not being able to move on the floor without shaking everything in the room." It also became common practice for soldiers to break step when marching across bridges to avoid large and potentially dangerous resonance vibrations. Both stiffness and resonance are therefore important considerations in the design of steel floor structures and footbridges for walking vibrations.

Stiffness has been taken into account for many years in the design of floor structures using criteria dating from Tredgold's time. A traditional stiffness criterion for residential floors is to limit the deflection under 2 kPa (42 psf) to less than span/360. This criterion is restricted to traditional wood floor construction with high transverse stiffness. The American Institute of Steel Construction Allowable Stress Design Specification (AISC, 1989) limits the live load deflection of beams and girders supporting "plastered ceilings" to span/360, a limitation which has also been widely applied to steel floor systems in an attempt to control vibrations. A better stiffness criterion applicable to all floor construction is to limit the deflection due to 1 kN (225 lb.) concentrated load to less than approximately 1 mm (0.04 in.).

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Resonance, however, has been ignored in the design of floors and footbridges for walking vibrations until recently. Approximately 30 years ago, problems arose with walking vibrations for steel-joist floors that satisfied code stiffness criteria. Lenzen (1966) determined that damping and mass, not stiffness, were the most important factors in preventing unacceptable walking vibrations for these floors. To take damping and mass into account, a simple dynamic design criterion based on heel-impact response was developed (Allen and Rainer, 1976) and introduced 18 years ago into an Appendix to the Canadian design standard for steel structures (Canadian Standards Association, 1989), In 1981, Murray recommended a similar dynamic design criterion based on data from 91 floor measurements (Murray, 1981). More recently a design criterion for footbridges has been introduced into British and Canadian bridge standards based on resonance response to a sinusoidal force (BSI, 1978; OHBDC, 1983).

Since these criteria were introduced, more has been learned about the loading function due to walking, in particular that resonance can occur at a harmonic multiple of the step frequency. This has been verified by reviewing past cases of vibration problems with steel joist and beam floors, most of which corresponded to third harmonic resonance of the step frequency (6 Hz floors approximately), but more recently also to second harmonic resonance (4 Hz floors approximately). Also the Canadian CSA criterion has recently been found not to correctly predict the vibration behavior of two-way joist girder systems.

In this paper a simple yet rational design criterion for walking vibration is proposed based on harmonic resonance. The criterion is calibrated to floor experience. It is similar to one recently recommended by Wyatt (1989). The criterion is extended to floor frequencies beyond 9 Hz to control impulse vibration from footsteps.

VIBRATION LIMIT STATE— ACCELERATION LIMITS

International Standards Association (ISO, 1989; ISO, 1992) recommends vibration limits below which the probability of adverse reaction is low. Limits for different occupancies are given in terms of rms acceleration as a multiple of the baseline curve shown in Figure 1. For offices, ISO recommends a multiplier of 4 for continuous or intermittent vibrations and 60 to 128 for transient vibrations. Intermittent vibration is defined as a string of vibration incidents such as those caused by a pile driver, whereas transient vibration is caused rarely, for example by blasting. Walking vibration is intermittent in nature but not as frequent and repetitive as vibration caused by a pile driver. It is therefore estimated that the multiplier for walking vibration in offices is in the range of 5 to 8, which corresponds to an rms acceleration in the range 0.25 to 0.4 percent g for the critical frequency range 4 to 8 Hz shown in Figure 1. Based on an estimated ratio of peak to rms acceleration of approximately 1.7 for typical walking vibration, the annoyance criterion for peak acceleration is estimated to be in the range 0.4 to 0.7 percent g. From experience (Allen and Rainer, 1976), a value of 0.5 percent g is recommended for the frequency range 4–8 Hz. The resulting acceleration limit for offices is shown in Figure 1.

For footbridges, ISO (1992) recommends a multiplier of 60 which, combined with an estimated ratio of peak to rms acceleration of 1.7, results in a criterion of approximately ten times the vibration limit for offices. People in shopping centres will accept something in between, depending on whether they are standing or sitting down. Suggested peak acceleration limits for these occupancies are given in Figure 1.

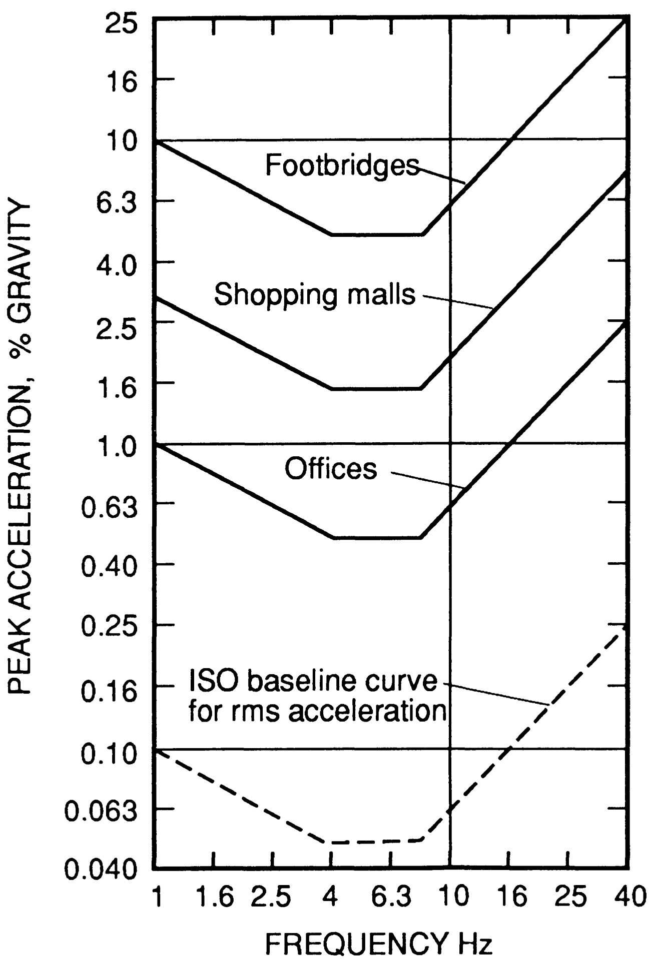

Fig. 1. Recommended acceleration limits for walking vibration (vertical).

LOADING FUNCTION

Walking across a floor or footbridge produces a moving repetitive force. Figure 2 shows the dynamic reaction at mid-support of a footbridge due to a person walking across it: the Fourier spectrum of the reaction clearly indicates the presence of sinusoidal loading components at the first, second, and third harmonic multiples of the step frequency. The force, F, can therefore be represented in time by a Fourier series

$$F = P\,(1 + \Sigma\alpha_i \cos 2\pi i f t) \tag{1}$$

where P is the person's weight, taken as 0.7 kN (160 lbs) for design, f the step frequency, i the harmonic multiple, α_i is a dynamic coefficient for the harmonic, and t is time. Table 1 recommends design values for these parameters based on test information on dynamic coefficient (Rainer, et al, 1988) and

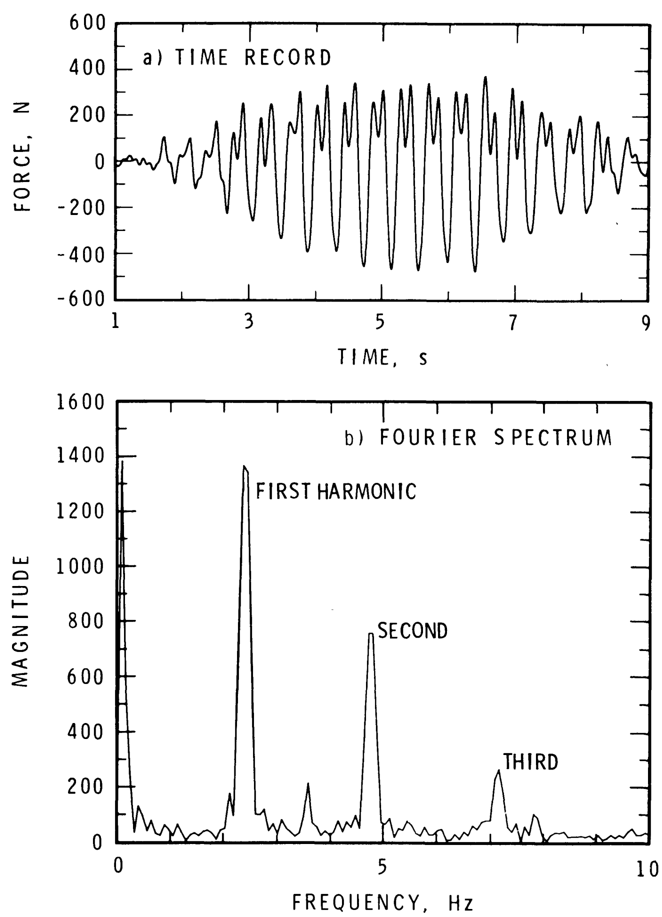

Fig. 2. Center support reaction produced by walking along a footbridge on three supports (Rainer, et al, 1988).

observations of step frequencies which are in the range 1.9 ± 0.3 Hz for offices.

Jogging, or more than one person walking in step, is a more severe dynamic loading, but only for the first two harmonics. Generally such cases are rare enough not to be a problem in practice. Similarly a large group of people walking in an area produces a greater dynamic loading at the step frequency (2 Hz approximately), but lack of coherence at the higher harmonics plus the damping effect of people has meant that, except for footbridges close to entertainment events (Bachmann, 1992) such loading has not been a problem in practice.

RESPONSE

Walking across a footbridge or floor causes a complex dynamic response, involving different natural modes of vibration, as well as motion due to time variation of static deflection. The problem can be simplified by considering a person stepping up and down at mid-span of a simply supported beam which has only one mode of vibration—the fundamental mode. Maximum dynamic response will occur when the natural frequency corresponds to one of the harmonic forcing frequencies. The steady-state acceleration, a, due to harmonic resonance is given by (Rainer, et al, 1988),

$$\frac{a}{g} = \frac{\alpha_i P}{0.5W} \times \frac{R}{2\beta} \times \cos 2\pi i f t = \frac{R\alpha_i P}{\beta W} \times \cos 2\pi i f t \qquad (2)$$

where W is the weight of the beam, β is the damping ratio, g is the acceleration due to gravity, and R is a reduction factor discussed later. The factor $1/(2\beta)$ is the familiar dynamic amplification factor for steady-state resonance and $0.5W/g$ is the mass of an SDOF oscillator which is dynamically equivalent to the simply supported beam of weight W vibrating in its fundamental mode. The other harmonics will also produce steady-state vibrations at their forcing frequencies, but the level of vibration is generally much smaller. For floor structures, an exception occurs when there is resonance of two modes of vibration at two multiples of the step frequency; floor experience indicates, however, that only one resonant mode whose frequency is near to the fundamental frequency need be considered for design.

The reduction factor R is introduced into Equation 2 to take into account (a) that full steady-state resonance is not achieved when someone steps along the beam instead of up and down at mid-span and (b) that the walker and the person annoyed are not simultaneously at the location of maximum modal displacement. Figure 3 shows test results for a person walking across two simply supported footbridges which verify the harmonic resonance response model, Equation 2. The value R = 0.56 in Figure 3a was determined by dynamic analysis of a person walking across the footbridge (Rainer, et al, 1988). It is recommended that for design R be taken as 0.7 or footbridges and 0.5 for floor structures having two-way nodal configurations.

Table 1.
Loading Function for Walking (See Equation 1)

Harmonic i	Frequency Range $i \times f$	Dynamic Load Factor α_i
1	1.6 to 2.2	0.5
2	3.2 to 4.4	0.2
3	4.8 to 6.6	0.1
4	6.4 to 8.8	0.05

PROPOSED DESIGN CRITERIA

Equation 2 predicts peak acceleration due to harmonic resonance, $R\alpha_i P / \beta W$, which can be compared to the acceleration limit, a_o / g shown in Figure 1. It is useful to express this in terms of a minimum value of damping ratio times equivalent mass weight (βW):

$$\beta W \geq \frac{R\alpha_i P}{a_o / g} \qquad (3)$$

Table 2 contains specific minimum values of βW for the values of dynamic loading ($\alpha_i P$) from Table 1, acceleration limit (a_o / g) from Figure 1 and reduction factor (R) recommended above.

As shown in Figure 4 the results of Table 2 can be approximated by the following criterion for walking vibrations:

$$\beta W \geq K \exp(-0.35 f_o) \qquad (4a)$$

where f_o is the fundamental natural frequency (Hz) and K is a constant given in Table 3 which depends on the acceleration limit for the occupancy. Equation 4a can be inverted to express the criterion for walking vibrations in terms of minimum fundamental natural frequency:

$$f_o \geq 2.86 \ln\left[\frac{K}{\beta W}\right] \qquad (4b)$$

The following section provides guidance for estimating the required floor properties for application of Equations 4.

DAMPING RATIO β

The damping ratio depends primarily on non-structural components and furnishings. The Canadian steel structures specification (CSA, 1989) recommends damping ratios of 0.03 for a bare floor; 0.06 for a finished floor with ceiling, ducts, flooring, and furniture; and 0.12 for a finished floor with partitions. Murray (1991) recommends damping ratios of 0.01 to 0.03 for a bare floor, 0.01 to 0.03 for ceilings, 0.01 to 0.10 for mechanical ducts, and 0.10 to 0.20 for partitions. These damping ratios, however, are based on vibration decay resulting from heel impact and include a component for

Table 2. Minimum Value of βW determined from Equation 3 for Satisfactory Performance			
Floor Frequency f_o (Hz)	Office Floors kN (kips)	Shopping Malls kN (kips)	Footbridges kN (kips)
1.6 to 2.2	28 (6.3)	9.3 (2.10)	4 (0.50)
3.2 to 4.4	14 (3.2)	4.7 (1.05)	2 (0.45)
4.8 to 6.6	7 (1.6)	2.3 (0.52)	1 (0.22)
6.4 to 8.8	3.5 (0.8)	1.1 (0.26)	0.5 (0.11)

geometric dispersion of vibration as well as frictional and material damping. More recent testing of modal damping ratios shows that the frictional and material damping ratios are approximately half of the values determined from heel impact tests. Based on available information (Wyatt, 1989; ISO, 1992), Table 3 recommends damping values for use in the proposed criterion, Equation 4.

NATURAL FREQUENCY, f_o, AND EQUIVALENT MASS WEIGHT, W

In the case of a simply supported panel such as a footbridge, the natural frequency is equal to the fundamental beam frequency of the panel and the equivalent mass weight is equal to the panel weight. Floors of steel construction, however, are two-way systems with many vibration modes having closely spaced frequencies. Natural frequency and equivalent mass weight of a critical mode in resonance with a harmonic of step frequency is therefore difficult to assess. A dynamic modal analysis of the floor structure can be used to determine the critical modal properties, but there are factors that are difficult to incorporate in the structural model. Composite action and discontinuity conditions are two such factors, but more difficult to assess is the effect of partitions and other non-structural components. An unfinished floor with uniform bays can have a variety of modal pattern configurations extending over the whole floor area, but partitions and other non-structural components tend to constrain the modal configurations to local areas in such a way that the floor vibrates locally like a single two-way panel. The following simplified procedure is recommended to estimate the properties of such a panel. Some of the recommendations are based on judgment guided by floor test experience. Further research is needed to obtain better estimates, particularly for W.

The floor is assumed to consist of a concrete slab (or deck) supported on steel joists or beams (open-web or rolled sections) which, in turn are supported on walls or on steel girders between columns. The fundamental natural frequency, f_o, and equivalent mass weight, W, for a critical mode is estimated by first considering a "joist panel" mode and a "girder panel" mode separately and then combining them. If the joist span is less than half the girder span, however, both the joist panel mode and the combined mode should be checked against the criterion, Equations 4.

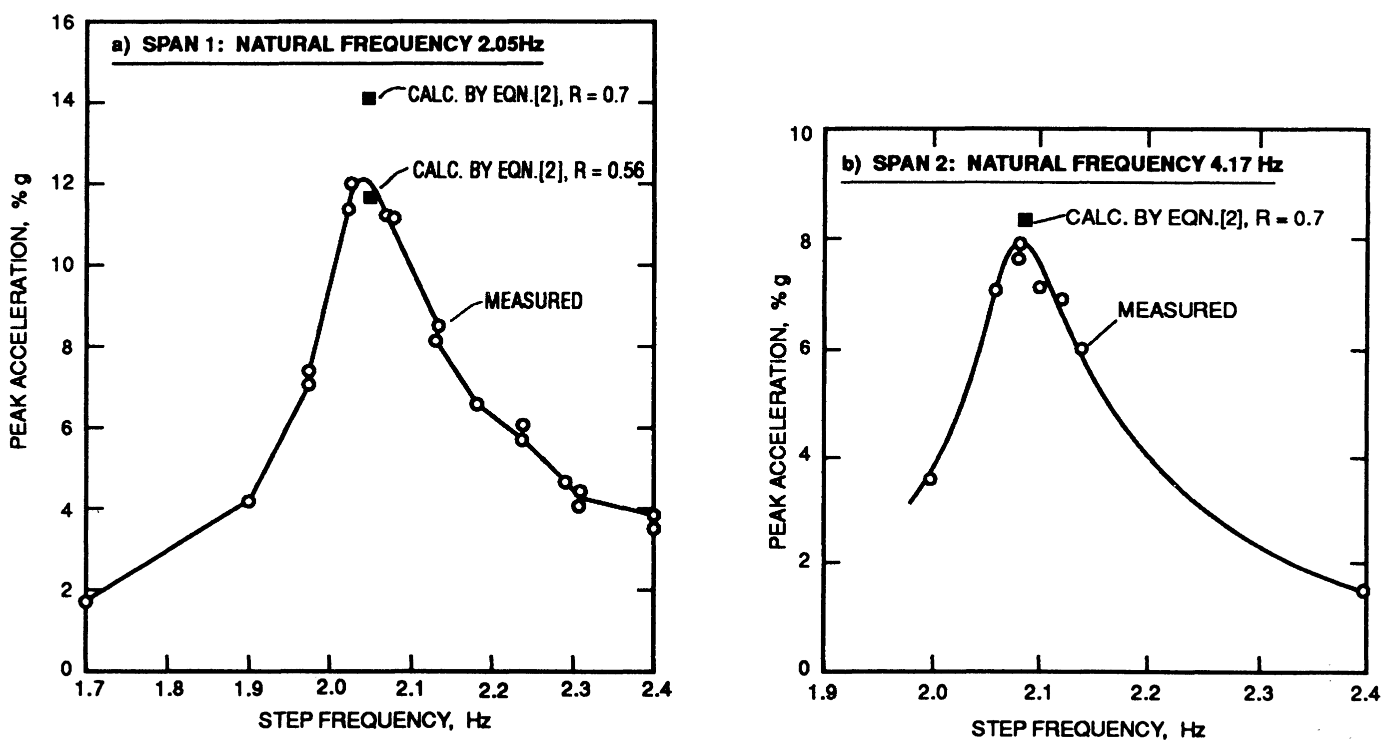

Fig. 3. Peak response of two footbridge spans to a person walking across at different step frequencies (Rainer, et al, 1988).

In the following, the concrete modulus of elasticity is assumed equal to 1.35 times that assumed in current structural standards, the increase being due to the greater stiffness of concrete under dynamic, as compared to static, loading. Also for determining composite moment of inertia, the width of concrete slab is equal to the member spacing but not more than 0.4 times the member span. For edge members, it is half of this value plus the projection of the slab beyond the member center line.

Also the floor weight per unit area, w, should include the sustained component of live load (approximately 0.5 kPa (11 psf) for offices).

JOIST PANEL MODE

The joist panel mode is associated with the natural frequency of the joist or beam alone. The natural frequency of this mode can be estimated from the simple beam formula

$$f_j = 0.18\sqrt{g/\Delta_j} \tag{5}$$

where Δ_j is the deflection of a beam or joist relative to its supports due to the weight supported by the individual beam or joist. Composite action is normally assumed provided the joists are directly connected to the concrete slab by welds to steel deck. Normally the joists or beams are assumed to be simply supported unless dynamic restraint is verified by a dynamic analysis or experiment. For open-web joists, shear deformations should be included in the calculations for Δ_j.

The mass weight of the joist panel mode can be estimated from

$$W_j = wB_jL_j \tag{6}$$

where w is the floor weight per unit area, L_j the joist or beam span, and B_j the effective joist panel width determined from

$$B_j = 2(D_s/D_j)^{1/4}L_j \tag{7}$$

where D_j is the flexural rigidity per unit width in the joist direction and D_s the flexural rigidity per unit width in the slab direction (including a correction for shear in open-web joists) based on the moment of inertia of the uncracked concrete (assume an average thickness t_c for ribbed decks). The form of Equation 7 is based on orthotropic plate action and the factor 2 was determined by calibration to floor data as described later. The effective panel width, B_j, determined by Equation 7 should be assumed to have an upper limit of two-thirds of the total width of the floor perpendicular to the joists or beams.

Where the beams or joists are continuous over their supports (including rolled sections shear connected to girder webs), and an adjacent span is $0.7L_j$ or greater, the effective joist panel weight, W_j, can be increased by 50 percent. The reason for this increase is that continuity over supports engages participation of adjacent floor panels in the fundamental mode of vibration. (Wyatt (1988) recommends an increase of 70 percent where the adjacent span is $0.8L_j$ or greater, 100 percent when it is $1.0L_j$.)

Table 3.
Values of *K* and β for use in Equation (4)

	K kN (kips)	β
Offices, residences, churches	58 (13.0)	0.03*
Shopping Malls	20 (4.5)	0.02
Footbridges	8 (1.8)	0.01

*0.05 for full-height partitions, 0.02 for floors with few non-structural components (ceilings, ducts, partitions, etc.) as can occur in churches

GIRDER PANEL MODE

The girder panel mode is associated with the natural frequency of the girder alone. The natural frequency of this mode can be estimated from

$$f_g = 0.18\sqrt{g/\Delta_g} \tag{8}$$

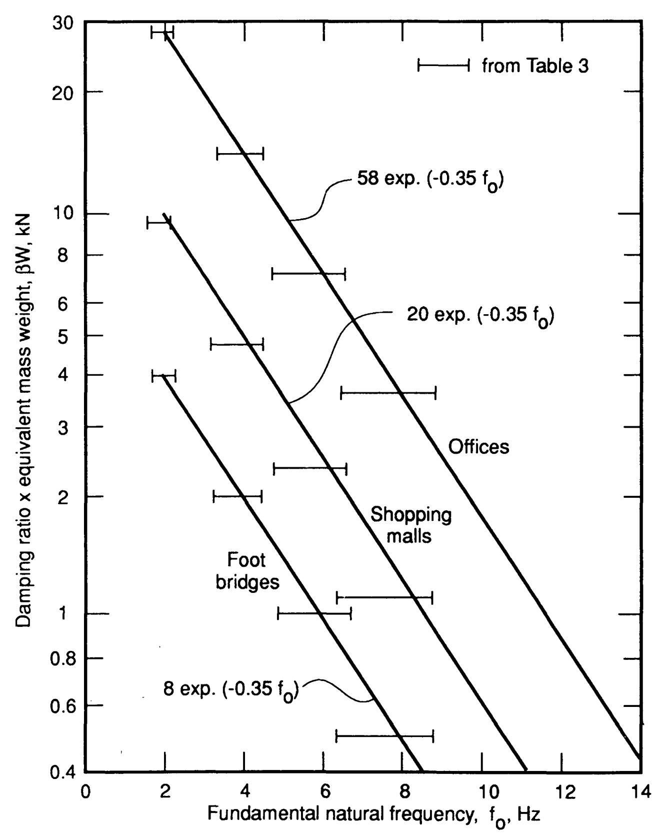

Fig. 4. Proposed criterion for walking vibrations.

where Δ_g is the deflection of individual girders relative to their supports due to the weight supported. Composite action can be assumed when the girders are directly connected to the concrete slab, for example by welds to the steel deck. When the girders are separated from the concrete slab by beams or joist seats (shoes), they act as Vierendeel girders, i.e. partially composite. It is recommended that the moment of inertia of girders supporting joist seats be determined from:

$$I_g = I_{nc} + (I_c - I_{nc}) / 2 \tag{9a}$$

for seat heights 75 mm (3 in.) or less, and

$$I_g = I_{nc} + (I_c - I_{nc}) / 4 \tag{9b}$$

for seat heights 100 mm (4 in.) or more, where I_{nc} and I_c are non-composite and fully composite moments of inertia respectively. (These recommendations are subject to change depending on the results of current research.) Normally the girders are assumed to be simply supported unless dynamic restraint is verified by analysis or experiment.

The mass weight of the girder panel mode can be estimated from

$$W_g = wB_g L_g \tag{10}$$

where L_g is the girder span and B_g is the effective girder panel width determined from

$$B_g = 1.6\,(D_j / D_g)^{1/4} L_g \tag{11}$$

where D_g is the flexural rigidity per unit width in the girder direction and D_j the flexural rigidity per unit width in the joist direction. Equation 11 is the same as Equation 7 except that the factor 2 is reduced to 1.6 to take into account discontinuity of joist systems over supports; if the joists consist of rolled beams shear connected to girder webs the factor 1.6 can be increased to 1.8. B_g determined by Equation 11 should be assumed to have a lower limit equal to the tributary panel width supported by the girder and an upper limit of two-thirds of the total floor width perpendicular to the girders.

Where the girders are continuous over their supports, and an adjacent span is $0.7L_g$ or greater, the mass weight, W_g, can be increased by 50 percent. This is due to participation of adjacent floor panels, as discussed above for the joist panel mode.

COMBINED MODE

Combined flexibilities of the joists and girders reduces the natural frequency and makes the floor more susceptible to noticeable walking vibration. For design purposes this can be taken into account by a "combined" mode whose properties may be estimated using the following interaction equations:

(i) The fundamental natural frequency can be approximated by the Dunkerly relationship:

$$f_o = 0.18 \sqrt{g / (\Delta_j + \Delta_g)} \tag{12}$$

(ii) The equivalent mass weight can be approximated by the interaction formula:

$$W = \frac{\Delta_j}{\Delta_j + \Delta_g} W_j + \frac{\Delta_g}{\Delta_j + \Delta_g} W_g \tag{13}$$

If the girder span, L_g, is less than the joist panel width, B_j, the combined mode is restricted and the system is effectively stiffened. This can be accounted for by reducing the deflection, Δ_g, used in Equations 12 and 13 to

$$\Delta_g' = \frac{L_g}{B_j} (\Delta_g) \tag{14}$$

where

$0.5 \leq L_g / B_j \leq 1.0$

EXAMPLE

Determine if the framing system for the typical interior bay shown in Figure 5 satisfies the proposed criterion for walking vibration. The structural system supports the office floors without full-height partitions. For ease in reading, this example will be carried out using Imperial units.

Concrete: 110 pcf, $f_c' = 4{,}000$ psi; $n = E_s / 1.35\, E_c = 9.3$
Deck thickness = 3.25 in. + 2 in. ribs = 5.25 in.
Deck weight = 42 psf

Beam Mode Properties

With an effective concrete slab width of 120 in. < $0.4\, L_j = 0.4 \times 35 \times 12 = 168$ in., and considering only the concrete above the steel form deck, the transformed moment of inertia $I_j = 2{,}105$ in.^{4} For each beam

$w_j = 10(11 + 42 + 4 + 40 / 10) = 610$ plf

which includes 11 psf live load and 4 psf for mechanical/ceiling, and

$$\Delta_j = \frac{5w_j L_j^4}{384EI_j} = \frac{5 \times 610 \times 35^4 \times 1{,}728}{384 \times 29 \times 10^6 \times 2{,}105} = 0.337 \text{ in.}$$

The beam mode natural frequency from Equation 5 is:

$$f_j = 0.18 \sqrt{\frac{386}{0.337}} = 6.09 \text{ Hz}$$

Using an average concrete thickness, 4.25 in., the transformed moment of inertia per unit width in the slab direction is

$D_s = 12 \times 4.25^3 / 12 \times 9.3 = 8.25$ in.^{4}/ft

The transformed moment of inertia per unit width in the beam direction is (beam spacing is 10 ft)

$D_j = 2{,}105 / 10 = 210.5$ in.^{4}/ft

The effective beam panel width from Equation 7 is:

$B_j = 2(8.25 / 210.5)^{1/4}(35) = 31.3$ ft

Since this is a typical interior bay, the actual floor width is at least $3 \times 30 = 90$ ft, and $\frac{2}{3} \times 90 = 60$ ft > 31.3 ft. Therefore, the effective beam panel width is 31.3 ft.

The mass weight of the beam panel is from Equation 6, adjusted by a factor of 1.5 to account for continuity:

$W_j = 1.5(610/10)(31.3 \times 35) = 100{,}238$ lbs $= 100$ kips

Girder Mode Properties

With an effective slab width of $0.4 \times 30 \times 12 = 144$ in. and considering the concrete in the form of deck ribs, the transformed moment of inertia $I_g = 3{,}279$ in.^{4} For each girder

$w_g = 2.5\ (610 \times 35) / 30 + 50 = 1{,}829$ plf,

$$\Delta_g = \frac{5 \times 1{,}829 \times 30^4 \times 1{,}728}{384 \times 29 \times 10^6 \times 3{,}279} = 0.350 \text{ in.}$$

and

$$f_g = 0.18\sqrt{\frac{386}{0.350}} = 5.98 \text{ Hz}$$

With $D_j = 210.5$ in.^{4}/ft and $D_g = 3{,}279 / 35 = 93.7$ in.^{4}/ft, Equation 11 gives

$B_g = 1.8\ (210.5 / 93.7)^{1/4}(30) = 66.1$ ft

which is less than $\frac{2}{3}\ (3 \times 35) = 70$ ft. From Equation 10

$W_g = (1829 / 35)(66.1 \times 30) = 103{,}626$ lb $= 104$ kip

Combined Mode Properties

In this case the girder span (30 ft) is less than the beam panel width (31.3 ft) and the girder deflection, Δ_g, is therefore reduced according to $0.350 \times 30 / 31.3 = 0.334$ in. From Equation 12,

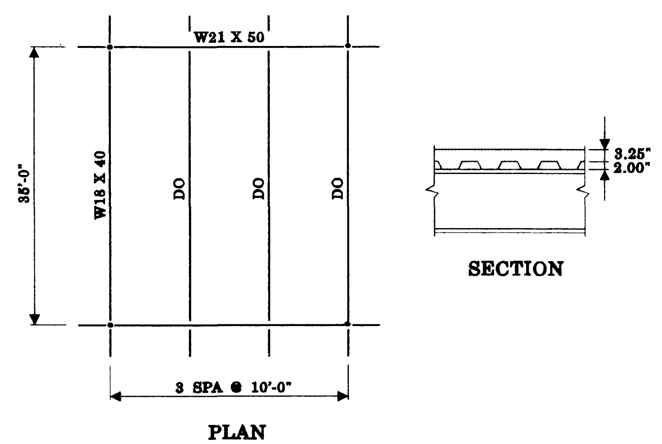

Fig. 5 Floor framing system—typical interior bay.

$f_o = 0.18\ \sqrt{386 / (0.337 + 0.334)} = 4.32$ Hz

and from Equation 13

$$W = \frac{0.337}{0.337 + 0.334}(100) + \frac{0.334}{0.337 + 0.334}(104) = 102 \text{ kips}$$

For office occupancy without full-height partitions, $\beta = 0.03$ from Table 3, thus

$\beta W = 0.03 \times 102 = 3.06$ kips

Evaluation

Application of Equations 4 for offices (see Table 3) results in

$\beta W = 3.06$ kips $> 13 \exp\ (-0.35 \times 4.32) = 2.87$ kips

or

$f_o = 4.32$ Hz $\geq 2.86 \ln\ (13 / 3.06) = 4.14$ Hz

The floor is therefore judged satisfactory.

EDGE PANEL MODE

Unsupported edges of floors can cause a special problem because of low-mass weight and sometimes decreased damping. Normally this is not a problem for exterior floor edges, because of stiffening by exterior cladding or because walkways are not located near exterior walls. Problems have occurred, however, at interior floor edges adjacent to atria. These edge members should often be made stiffer than current practice suggests by use of the following assumptions in the proposal criterion.

Where an interior edge is supported by a joist, the equivalent mass weight of the joist panel can be estimated using Equation 6 by replacing the coefficient 2 with 1 in Equation 7. Where an interior edge is supported by a girder, the equivalent mass weight of the girder panel should be estimated on the basis of the tributary weight supported by the girder. These edge panels are then combined with their orthogonal panels as recommended above.

CALIBRATION OF PROPOSED CRITERION TO EXPERIENCE

The factor 2 in Equation (7) was determined by calibration to data on one-way joist floor systems in Table 1 of Allen and Rainer (1976). The results of applying the proposed criterion, including recommended design parameters, to floors that have been evaluated and tested is given in Tables 4 and 5. Table 4 confirms application of the proposed criterion for one-way systems, two-way systems, and interior edge panels. Application of the CSA criterion (CSA, 1989) to the two-way floor systems in Table 4, on the other hand, predicts that all are satisfactory when in fact floors 12 and 13 are definitely unsatisfactory. Table 5 confirms application of the proposed criterion to two-way systems except for floor 3, a heavy floor (3.6 kPa) with continuity in both directions. Two factors for

Case	Reference or Location	Measured Frequency f_o (Hz)	Span L (m)	Panel Width B (m) Equation 7 & 11	Damping Ratio, β Table 3	βW (kN) Calc.	βW (kN) Criterion Equation 4a	User Rating[2]
	One-Way Joist Systems							
1	Allen and Rainer (1976), #13	4.0	22.2	9.7	0.03	19.3	14.3	S
2	#9	4.5	21.6	11.9	0.03	26.9	12.0	S
3	#24	4.6	16.5	11.2	0.03	16.6	11.6	S
4	#5	5.3	18.3	8.8	0.015	6.0	9.1	U
5	#10	5.3	18.6	7.8	0.015	5.4	9.1	U
6	#2	5.5	14.6	8.6	0.03	9.4	8.5	S
7	#1	6.0	10.7	8.3	0.03	6.6	7.1	U
8	#18	6.0	17.1	9.8	0.015	7.5	7.1	B
9	#22	8.0	10.7	7.1	0.03	5.5	3.5	S
10	#19	8.5	8.9	8.2	0.015	3.3	3.0	B
11	#17	8.8	8.7	7.6	0.015	2.5	2.7	U
	Two-Way Joist—Girder Systems							
12	Quebec City	4.5	(7.6, 7.6)[3]	(9.1, 11.9)[3]	0.03	6.2	12.0	Very U
13	Quebec City	5.4	(7.6, 7.6)	(9.1, 8.6)	0.03	5.4	8.8	U
14	Quebec City	7.2	(7.6, 7.6)	(7.4, 10.7)	0.03	5.2	4.7	S
15	Matthews, et al (1982)	6.2	(9, 12.5)	(9.7, 11.3)	0.03	9.5	6.6	S
16	Pernica and Allen (1982)	5.2	(7.6, 12.2)	(8.1, 15.0)	0.02	11.8	3.2	S
	Interior Edge Panels							
17	Quebec City	5.1	13.7	2.3	0.03	2.4	9.7	Very U
18	Edmonton	5.1	17.5	4.6	0.03	8.4	9.7	U
19	Pernica and Allen (1982)	5.6	12.2	3.3	0.02	2.5	2.8	U

Table 4.
Application of Proposed Design Criterion to Tested Floors

Notes:
[1] $K = 58$ for all cases except #16 and #19, where $K = 20$ applies
[2] U = unsatisfactory, S = satisfactory, B = borderline
[3] The first entry inside the brackets refers to the joist panel, the second refers to the girder panel

unsatisfactory performance of this floor are low damping (criterion just met for $\beta = 0.015$) and vibration transmission due to girder continuity. Floors 7 and 10 are predicted to be marginal.

The proposed criterion can also be compared to existing criteria. Table 6 makes this comparison on the basis of minimum values of βW_j for one-way beam or joist systems. The basis for the values shown in Table 6 is given in Appendix III. For office floors, Table 6 shows that all criteria are similar for resonance with the third harmonic of the step frequency (5 to 7 Hz). This is not surprising because existing design criteria are based to a large extent on experience with floors in the frequency range 5 to 8 Hz.

The criteria, however, differ at other floor frequencies. The CSA criterion is insufficient for frequencies less than 5 Hz and conservative for frequencies beyond 7 Hz. The Murray criterion has tendencies similar to the CSA criterion, but the discrepancy with the proposed criterion is less severe. The Wyatt criterion is close to the proposed criterion within a broad frequency range, 3 to 8 Hz, but is more conservative beyond 8 Hz.

For footbridges the proposed criterion is apparently a little more conservative than the OHBDC (1983) criterion, but this is offset by the difference in recommended values of β (0.01 vs. 0.005 to 0.008 in the OHBDC). Third and fourth harmonic resonance is not adequately considered by the OHBDC but this is not serious in practice because footbridges with these frequencies generally have sufficient mass to satisfy the proposed criterion, Equation 4a.

Information on shopping centers is scarce. Application of Equation 4a for shopping centers to the floor data in Cases 16 and 19 of Table 4, however, indicates agreement with user reaction.

Tables 4–6, as well as Figure 3, therefore confirm the applicability of the proposed criterion for walking vibration to a wide variety of structures and occupancies.

Table 5.
Application of Proposed Design Criterion to Floors Investigated by Murray (1981)[1]

Case	Calculated Frequency f_o (Hz)	Span (m)		Panel Width B(m)		Estimated Damping Ratio, β Table 3	βW (kN)		User Rating
		Beam	Girder	Joist or Beam	Girder		Calculated	Criterion Equation (4a)	
1	7.0	10.5	6.0	6.3	11.1	0.015	2.0	4.9	U
2	7.0	10.5	6.0	6.3	11.1	0.05	6.6	4.9	S
3	4.0	7.3	12.2	9.2[2]	16.9[2]	0.02	18.6	14.1	U
4	7.7	7.0	7.2	7.2	9.0	0.015	2.0	3.9	U
5[3]	5.9	12.2	Wall	7.3	?	0.015	3.5	7.3	U
6	5.9	12.2	Wall	7.3	?	0.05	11.7	9.2	S
7	5.3	13.4	6.4	8.0	19.8	0.03	9.3	9.2	U
8	6.1	9.1	6.1	6.4	11.6	0.03	3.9	6.9	U
9	5.1	5.5	12.5	6.5	9.1	0.03	6.7	9.6	U
10	5.2	11.6	9.8	8.7[2]	19.9	0.02	9.7	9.4	U
11[3]	6.4	12.2	?	7.9	?	0.02	5.1	6.1	U

Notes:
[1]All open web joist on girder systems except #3 and #10 (beams shear connected to girders)
[2]Members continuous over supports (W_j or W_g increased by 1.5)
[3]Joist systems supported on stiff girders, frequency f_o estimated from f_j

NATURAL FREQUENCIES GREATER THAN 9 HZ

When the natural frequency is greater than 9 Hz, harmonic resonance does not occur, but walking vibration can still be a problem. Because the natural frequencies are high compared to the main loading frequencies, the floor response is governed primarily by stiffness relative to a concentrated load. Experience indicates a minimum stiffness of approximately 1 kN per mm (5.7 kips per in.) deflection for office and residential occupancies.

For light floors with natural frequencies in the range 9 to 18 Hz there may also be adverse reaction to floor motion caused by step-impulse forces. Experience indicates that adverse reaction to step impulses depends primarily on mass (initial floor velocity equals impulse divided by mass) and vibration decay time, the shorter the decay time the better. The decay time decreases in proportion to clamping ratio times floor frequency. Wyatt (1989) recommends an impulse criterion beyond 7 Hz floor frequency, but beyond approximately 9 Hz the criterion becomes overly conservative because it ignores the benefits of decreased decay time. Ohlsson (1988) recommends an impulse criterion which takes decay time into account, but the criterion is complex for design. The resonance criterion, Equation 4a, is in a form that correctly reflects impulse discomfort except that the right-hand side has not been correctly determined. If, however, Equation 4a with $K = 58$ for office floors is extended beyond 9 Hz, it decreases rapidly until approximately 18 Hz when the stiffness criterion of 1 kN/mm (5.7 k/in.) starts to control the design of the floor. Application of Equation 4a to the examples in Ohlsson (1988) also indicates that it gives a reasonable evaluation for floors between 9 and 18 Hz.

To ensure satisfactory performance of office and residential floors with frequencies greater than 9 Hz it is recommended that Equations 4 be used in conjunction with the stiffness criterion of 1 kN/mm (5.7 k/in.).

CONCLUSION

Walking forces produce motions which are related to resonance, impulse response, and static stiffness. Resonance controls the design of floors and footbridges with natural frequencies less than approximately 9 Hz, static stiffness controls the design of floors with frequencies greater than approximately 18 Hz, and impulse response controls the design of floors with frequencies in between.

A simple criterion for resonance vibration of floor and footbridge structures, Equations 4, is proposed for design, along with a recommended procedure for determining the

Table 6.
Comparison of Various Design Criteria for Walking Vibrations

Natural Frequency f_o (Hz)	Minimum Value of Damping Ratio Times Effective Mass Weight, βW_j (kN)					
	Offices, Residences				Footbridges	
	Equation 4a	CSA (1989)[1]	Murray (1981)[1]	Wyatt (1989)	Equation 4a	OHBDC (1983)
2	28.8	NA	NA	NA	4.0	3
4	14.3	4	5.8–7.6	17.5	2.0	1.8
6	7.1	6	5.8–7.6	8.8	1.0	—
8	3.5	7	5.8–7.6	3.0[1]	0.5	—
10	1.75	8	5.8–7.6	3.0[1]	0.24	—

Note:
[1]Results are given for a standard case of finished floor without full-height partitions (β = 0.03)

required floor properties. The proposed criterion, based on acceptable vibration for human reaction, compares well with existing criteria and is confirmed by experience with tested floors. Recommended values of the criterion parameters, however, are expected to be improved by further experience and research.

Floors of offices and residential occupancies with frequencies greater than 9 Hz should also be checked both for a minimum static stress under concentrated load of 1 kN/mm (5.7 kips/in.) and for impulse response by means of Equations 4.

APPENDIX I: REFERENCES

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APPENDIX II: NOTATION

The following symbols are used in this paper:

a = acceleration
a_o = acceleration limit
B = effective width of a panel
D = flexural rigidity or transformed moment of inertia per unit width of a panel

E_s	= modulus of elasticity for steel
f	= step frequency
f_j	= natural frequency of joist or beam panel
f_g	= natural frequency of girder panel
f_o	= fundamental natural frequency of floor structure
g	= acceleration due to gravity; subscript indicating girder
i	= ith harmonic of step frequency
j	= subscript indicating joint or beam
K	= factor in Equation 4 taking into account occupant sensitivity to vibration
L	= span of joint, beam, or girder (with subscript j or g)
P	= weight of a person (0.7 kN assumed)
R	= reduction factor in Equation 2
W	= effective mass weight of floor vibrating in the fundamental mode
w	= unit weight of floor panel, including acting live load
w_j or w_g	= unit weight of joist or girder per unit length
α_i	= dynamic load factor for ith harmonic of step frequency
β	= damping ratio
Δ	= deflection of member under weight supported

APPENDIX III: BASIS FOR COMPARISON OF VIBRATION CRITERIA

Existing design criteria for walking vibration can be compared with the proposed criterion by considering a standard joist or beam panel on stiff supports. To make a valid comparison, each criterion must be considered as a total package. This requires adjustments to the criteria to take account of differences in the form of the design equations and in the recommended values of design parameters. To make a comparison, all criteria will be transformed to a common measure βW_j as defined for the proposed criterion.

The following frequency relationship for a simply supported joist panel will be used to transform all criteria to the common measure, βW_j:

$$f_o = \frac{\pi}{2}\sqrt{\frac{gD_j}{wL_j^4}} \tag{A1}$$

where w is the unit weight of the panel

Canadian Standards Association (CSA, 1989)

This criterion has been used in Canada since 1975, with minor modifications in 1984. For the standard joist panel, the CSA criterion can be expressed as follows:

$$w(40t_c)L_j\,(\text{kN}) > 0.6f_o\,/\,(a_o\,/\,g) \tag{A2}$$

where t_c is the effective concrete thickness, $40t_c$ is the effective slab width, and $a_o\,/\,g$ is a limiting heel-impact acceleration determined from Figure 6. Equation A2 can be expressed in terms of W_j if a correction is made for the effective panel width. For a typical case of a 5.5 Hz floor, span L_j = 12 m and concrete thickness t_c = 75 mm, application of Equation 7 results in an effective width of 8.3 m or $110t_c$ compared to $40t_c$ in Equation A2. If Equation A2 is multiplied by $110\beta\,/\,40$ it becomes

$$\beta W_j\,(\text{kN}) > \beta 1.65f_o\,/\,(a_o\,/\,g) \tag{A3}$$

Minimum values of βW_j for the CSA criterion in Table 6 were determined from Equation A3 using the criterion for finished floors in Figure 6 and β = 0.03 from Table 3.

Murray (1981)

On the basis of a review of field data from 91 floors, Murray (1981) recommended the following criterion, presently widely used in the U.S.:

$$\beta > 0.35A_of_o + 0.025 \tag{A4}$$

where A_o is the initial amplitude of vibration (inches) due to

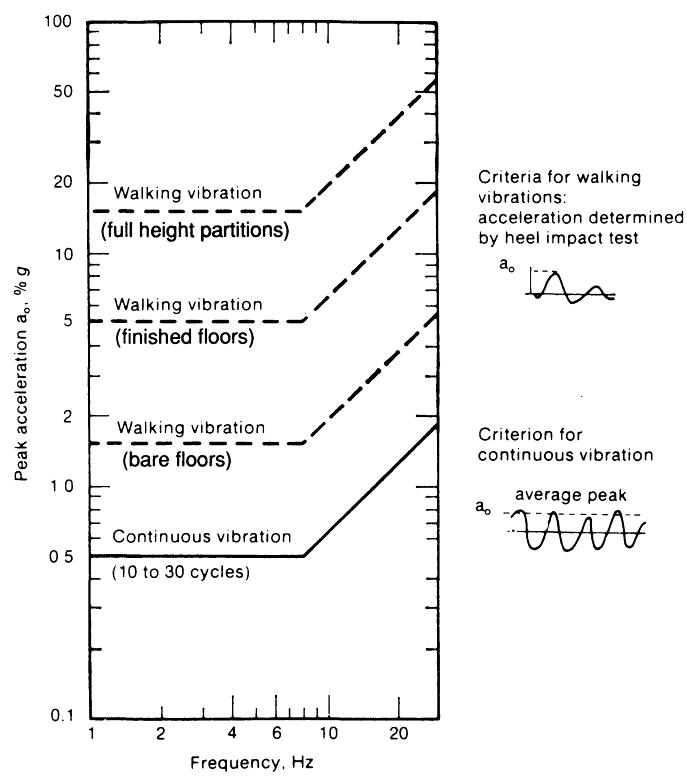

Fig. 6. Annoyance criteria for floor vibrations in residential, school, and office occupancies (CSA, 1989).

a standard heel impact. Equation A4 is plotted in Figure 7 along with the floor data. To determine A_o Murray provides the following expression for a simply supported one-way floor system:

$$A_o = DLF \times \Delta_s \tag{A5}$$

where Δ_s is the static deflection of the joist panel under a concentrated load of 600 lb. and *DLF* is a dynamic load factor to obtain the maximum amplitude of vibration for a standard heel impact. *DLF* ranges from $0.15f_o$ at f_o = 4 Hz to $0.12f_o$ at f_o = 10 Hz, and can therefore be approximated by $0.14f_o$, its value at f_o = 6 Hz. Thus,

$$A_o = \frac{(1.14f_o)\, 600L_j^3}{48D_j B_M} \tag{A6}$$

where B_M is the effective joist panel width as defined later. Substitution of Equation A6 in Equation A4 after elimination of D_j by means of Equation A1 results in the following criterion:

$$\beta > \frac{584}{wB_M L_j} + 0.025 \tag{A7}$$

For the standard case of finished office floor without full-height partitions, $\beta = 0.03$ according to Table 3 and $\beta = 0.045$ according to Murray. For this case Equation A7 becomes

$$wB_M L_j = 584 / (0.045 - 0.025) = 29{,}200 \tag{A8}$$

Murray (1991) provides expressions for determining B_M in terms of beam or joist spacing times the number of effective joists. Two expressions are used, one for normal hot-rolled beam (spacing more than 30 in.); the other for closely spaced joists (30 in. or less). The expression for narrow spacing is equivalent to

$$B_M = \frac{3\sqrt{2}}{\pi}\left[\frac{D_S}{D_j}\right]^{1/4} L_j = 1.35\left[\frac{D_S}{D_j}\right]^{1/4} L_j = 0.675B_j \tag{A9}$$

where B_j is defined according to Equation 7, and the expression for wide spacing can be approximated by

$$B_m = 1.03\left[\frac{D_S}{D_j}\right]^{1/4} L_j = 0.515\, B_j \tag{A10}$$

Substitution of Equations A9 or A10 in Equation A8 results in minimum values of W_j equal to 43,260 lb for narrow spacing and 56,700 lb for wide spacing. For the standard case, $\beta = 0.03$, the corresponding minimum values of βW_j included in Table 6 are 1,300 lb (5.8 kN) and 1,700 lb (7.6 kN).

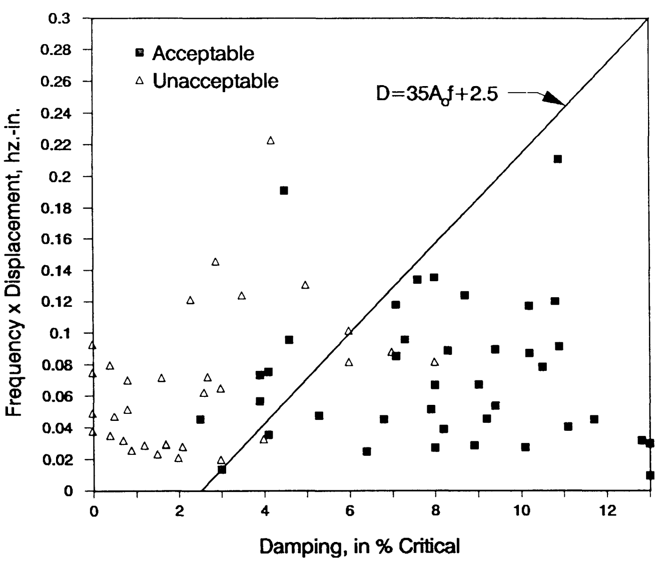

Fig. 7. Murray criterion, Equation (A4), compared to floor data (Murray 1981).

Wyatt (1989)

Wyatt (1989) proposed two design criteria for office floors, one a resonance criterion for floor frequencies up to 7 Hz, the other an impulse response criterion for floor frequencies greater than 7 Hz. For the one-way beam or joist system, the resonance criterion can be expressed (with rearrangement and change of symbols) as

$$\beta(wB_w L_j) > 667C_f / F \tag{A11}$$

where C_f is a loading coefficient (0.4 for second harmonic loading and 0.2 for third harmonic loading), F is a rating factor which depends on the office environment (12 for a busy office, 8 for a general office, and 4 for a special office) and B_w is the joist panel width. For the one-way system Wyatt recommends

$$B_W = 4.5\left[\frac{gD_S}{f_o^2 w}\right]^{1/4} \tag{A12}$$

which can be expressed in the same form as Equation 7 by use of Equation A1. After substitution, B_w in Equation A12 becomes equal to $1.8B_j$, where B_j is defined by Equation 7. Wyatt, however, recommends a concrete modulus elasticity 25 percent higher than recommended for D_s in Equation 7. With this correction B_w, becomes equal to $1.9B_j$. Equation A11 can therefore be expressed as

$$\beta W_j > 351C_f / F \tag{A13}$$

for floor frequencies below 7 Hz. Table 6 contains minimum values of βW_j assuming $F = 8$ for a general office.

For floor frequencies greater than 7 Hz Wyatt recommends the following impulse criterion:

$$wSL_j > 294 / F \tag{A14}$$

where S is the member spacing. Equation A14 may be expressed in terms of βW_j if it is multiplied by $\beta B_j / S$. Based on an assumed beam spacing of 2.5 m used in Wyatt's examples and a typical value $B_j = 6.8$ m for an 8 Hz floor of span 9 m and concrete thickness of 75 mm. Equation A14 can be approximated by

$$\beta W_j > 800\beta / F \tag{A15}$$

Table 6 contains a minimum value of Equation A15 at $f_o = 8$ Hz for a general office floor for which $F = 8$ and $\beta = 0.03$.

Footbridges—Ontario Highway Bridge Design Code (OHBDC, 1983)

The OHBDC (1983) design criterion for footbridges is based on a pedestrian or jogger exerting a dynamic force of $\alpha P \cos 2\pi f t$ where P is 0.7 kN, $\alpha = 0.257$ and f, the step frequency, takes on any value between 1 and 4 Hz. The footbridge is modeled as an SDOF beam which vibrates at the first flexural frequency, f_o. For a simply supported footbridge, the resonance response for flexural frequencies up to 4 Hz can be determined from Equation 2 with a value of R which is determined by the length of the footbridge. If, for a typical case R is assumed equal to 0.7, the maximum acceleration is determined from

$$a_{max} / g = 0.7(0.257)0.7 / \beta W_j = 0.126\, \beta W_j \tag{A17}$$

where W_j is the weight of the footbridge. The OHBDC recommends limiting values of a_{max} / g equal to 0.042 at $f_o = 2$ Hz and 0.072 at $f_o = 4$ Hz. Thus Equation A17 can be inverted to a criterion for minimum value of βW_j equal to 0.126 / 0.042 = 3 kN at $f_o = 2$ Hz and 0.126 / 0.072 = 1.8 kN at $f_o = 4$ Hz.

For a flexural frequency beyond 4 Hz, the OHBDC gives an incorrect assessment because it neglects resonance with the higher harmonics of the walking and jogging forces.