LRFD Analysis for Semi-Rigid Frame Design

WON-SUN KING AND WAI-FAH CHEN

ABSTRACT

A practical LRFD-based analysis method for the design of semi-rigid frames is proposed. The proposed method uses first-order elastic analysis with a notional lateral load for the second-order effects. In the proposed method, a simplified three-parameter model describing the tangent rotational stiffness of semi-rigid connections is used.

1. INTRODUCTION

Although partially restrained (PR) construction is permitted by the AISC *Specification for Structural Steel Buildings—Load and Resistance Factor Design*, no specific analysis or design guidance is given in the current LRFD and ASD specifications for these partially restrained frames.

Recently, a simplified procedure for the analysis and design of semi-rigid frames was proposed by Barakat and Chen,[1] using the B_1 and B_2 amplification factors together with the beam-line concept. However, the beam-line method can not adequately predict the drift of unbraced frames and the calculation of effective length factor is cumbersome and time-consuming.

A simplified procedure to improve these drawbacks is introduced in this paper. Here, as in the Barakat method, the proposed method is based on first-order linear elastic analysis, but the second-order effect will be included with the use of notional lateral loads.

2. MODELING OF SEMI-RIGID CONNECTIONS

2.1 Connection Models

Most existing connection models express the moment in terms of rotation from which the tangent stiffness can be derived. This paper proposes a direct tangent-stiffness expression for flexible connections. This proposed tangent-stiffness model is based on the concept that connection stiffness degrades gradually from an initial stiffness, K_i, to zero following a nonlinear relationship of the simple form:

Won-Sun King is associate professor, Department of Civil Engineering, Chung Cheng Institute of Technology, Ta-Hsi, Tao-Yuan, Taiwan.

Wai-Fah Chen is professor and head of Structural Engineering, School of Civil Engineering, Purdue University.

$$\frac{dM}{d\theta_r} = K_t = K_i\left[1 - \left(\frac{M}{M_u}\right)^c\right] \tag{1}$$

where

K_t = tangent stiffness
K_i = initial connection stiffness
M_u = ultimate bending moment capacity
M = connection moment
C = shape factor account for decay rate of K_t, $C > 0$

The moment-rotation ($M - \theta_r$) behavior of bolted extended end-plate beam-to-column connections tested by Yee and Melchers[2] is compared with the proposed model in Figure 1 and a good agreement is observed with $C = 1.6$. In Figure 1, the initial stiffness, K_i = 546,666 in-kip/rad, is the tangent to the starting point of the curve. The ultimate moment capacity, M_u = 3,539 in-kip, is determined by test. The value C is used to control the shape of a convex curve. If C is equal to 1, K_t decreases linearly. When C is less than 1, K_t decreases more rapidly. If C is greater than 1, K_t decreases much slower. This is illustrated in Figure 1 with C = 1.0, 1.6, and 2.2 respectively.

In the following, the proposed tangent stiffness connection model will be applied to several types of connections, including the extended end-plate, top and seat angle with double web angles, framing angles, and single-plate connections.

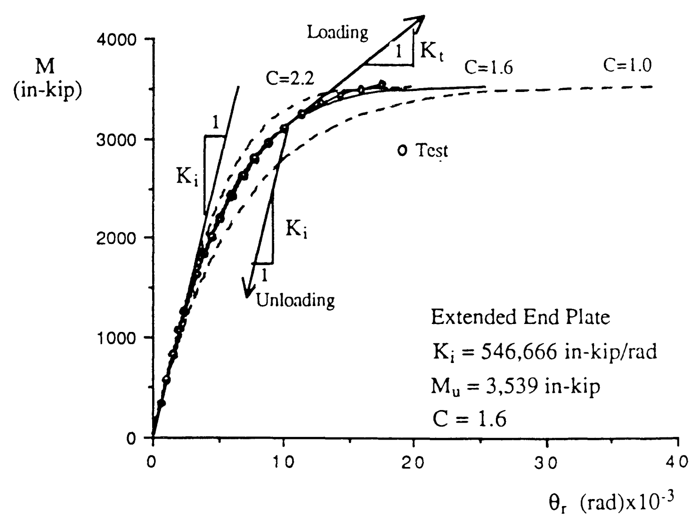

Fig. 1. Moment-rotation curves of Yee connection (1986).

The connections ranged from very stiff to rather soft connections. The moment-rotation curve is obtained by numerical integration of tangent-stiffness Equation 1.

(a) Jenkins Bolted Extended End Plate

A comparison of the proposed model with one of the Jenkins, Tong, and Prescott extended end-plate connection test[3] is shown in Figure 2. In this test, the beam is 305×165 UB54, the column is 254×254 UC132 (stiffened), bolts are M20 grade 8.8, thickness of end plate is 20 mm and K_i = 786,732 in.-kip/rad and M_u = 1,989 in.-kip. Good agreement is observed with C = 0.555.

(b) Bolted Top and Bottom Angles with Web Angles

Azizinamini, Bradburn, and Radziminski[4] reported test results on bolted semi-rigid steel beam-to-column connections. These connections are comprised of top and bottom angles connected to the flanges along with web angles. ASTM A36 steel was used for the members and the connection elements. Eighteen specimens were tested. The beam tested was a W14×38, the bolt diameter is 22.2 mm, and the web angles are 2L4×3½×¼. The thickness of flange angles is 15.9 mm, and the length of the test beam is 203.2 mm. The test number 14S8 with K_i = 677,025 in.-kip/rad and M_u = 1,707 in.-kip compares well with that of the proposed model in Figure 3 with C = 0.34.

(c) Bolted Framing Angles

Bolted double-web angles were tested by Lewitt, Chesson, and Munse at the University of Illinois. In 1987, Richard, et al[5] proposed a four-parameter formula to describe these full-scale tests. Figure 4 compares the results of the proposed model with one of these tests using a five-bolt design with rivets in the angle-to-beam web connection with K_i = 206,667 in.-kip/rad and M_u = 761 in.-kip.

(d) Single Plate

A total of seven tests were made by Richard[6] on single-plate connections. The first set of two-, three-, five-, and seven-bolt tests were run with the framing connection plate welded to a flange plate which was in turn bolted to the support column. A second set of tests was run on the two-, three-, and five-bolt connections with the framing connection plate welded to the support column. In these tests, three bolts were used to connect the beam and the single plate. The bolts are A325 ¾-in. diameter, and the plate thickness is ⅜-in. The moment-rotation curve of the proposed model compares well with one of the tests as illustrated in Figure 5 with K_i = 51,000 in.-kip/rad, M_u = 137 in.-kip, and C = 0.22.

2.2 Initial Stiffness

For simplicity, researchers[7,8,9] have been using the initial connection stiffness, K_i, for their semi-rigid frames analysis. The use of initial stiffness throughout the flexible frame analysis results in a frame behavior that is generally too stiff when the frame is subjected to a normal loading condition.

Extensive studies of frames by Ackroyd[10] with nonlinear connections indicate that the secant stiffness of beam-to-column connections near ultimate frame capacity was typically 20 percent of the initial stiffness, K_i, at leeward ends of girders and 80 percent of K_i at the windward ends of girders, when the frame is subjected to combined gravity and wind loading. It seems, therefore, reasonable to use an average connection stiffness of $0.5K_i$ when computing the design moments. This is adopted in the present analysis.

3. DESIGN FORMULA IN AISC-LRFD

The equation for the maximum strength of beam-columns is given by AISC-LRFD as

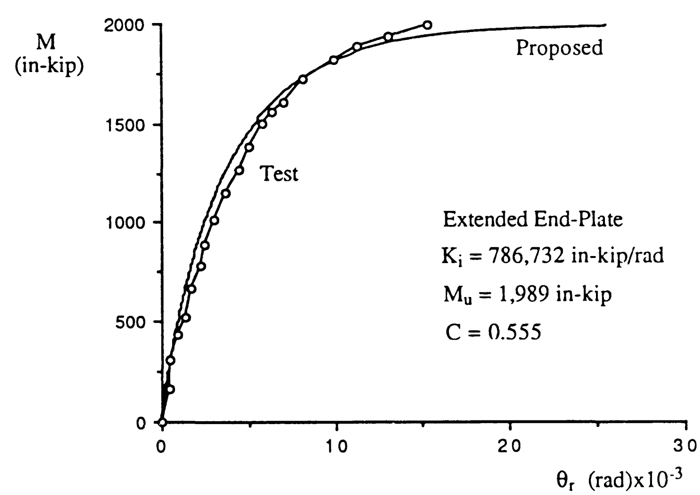

Fig. 2. Moment-rotation curves of Jenkins connection (1986).

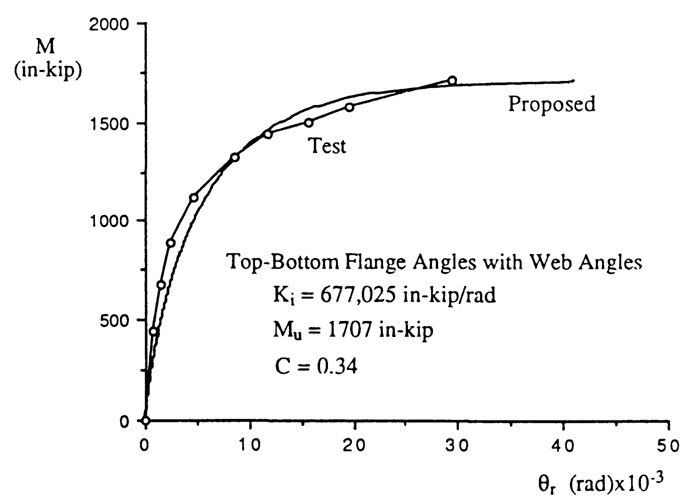

Fig. 3. Moment-rotation curves of Azizinamini connection (1987).

for $\frac{P_u}{\phi_c P_n} \geq 0.2$,

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) \leq 1.0 \quad (2)$$

for $\frac{P_u}{\phi_c P_n} < 0.2$,

$$\frac{P_u}{2\phi_c P_n} + \left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) \leq 1.0 \quad (3)$$

where

P_n = ultimate compression capacity of an axially loaded column

M_{nx}, M_{ny} = ultimate moment-resisting capacity of a laterally unsupported beam about x and y axes, respectively

ϕ_c = column resistance factor (= 0.85)

ϕ_b = beam resistance factor (= 0.9)

P_u = design axial force

M_{ux}, M_{uy} = member design moment about x and y axes, respectively, with:

$$M_u = B_1 M_{nt} + B_2 M_{lt} \quad (4)$$

M_{nt} = first-order moment in the member assuming no lateral translation in the frame

M_{lt} = first-order moment in the member as a result of lateral translation of the frame

B_1 = P-δ moment amplification factor

$$B_1 = \frac{C_m}{1 - \frac{P_u}{P_e}} \geq 1 \quad (5)$$

B_2 = P-Δ moment amplification factor

$$B_2 = \frac{1}{1 - \frac{\Sigma P_u \Delta_o}{\Sigma HL}} \quad (6)$$

C_m = $0.6 - 0.4M_1/M_2$, where M_1/M_2 is the ratio of the smaller to the larger end moment of a member

P_e = $\pi^2 EI/(KL)^2$

ΣP_u = axial loads on all columns in a story

Δ_o = first-order translational deflection of the story under consideration

ΣH = sum of all story horizontal forces producing Δ_o

L = story height

K = effective length factor determined from the alignment chart

The second-order effects are taken into account approximately by the moment amplification factors B_1 and B_2 on the nonsway and sway moments obtained from first-order elastic analyses, respectively. It usually leads to conservative results.

4. BEAM-COLUMN STIFFNESS IN SECOND-ORDER ELASTIC ANALYSIS

For second-order elastic analysis, we use the usual element geometric stiffness matrix combined with the update of the element geometry during the analysis. The first three terms in the Taylor series expansion of the elastic stability functions are retained for the axial compressive force P to increase the accuracy of the element stiffness. The corresponding terms in stiffness matrix were obtained by Goto and Chen[11] as

$$K_{ii} = \frac{4EI}{L} + \frac{2PL}{15} + \frac{44P^2L^3}{25{,}000EI} \quad (7)$$

$$K_{ij} = \frac{2EI}{L} - \frac{PL}{30} - \frac{26P^2L^3}{25{,}000EI} \quad (8)$$

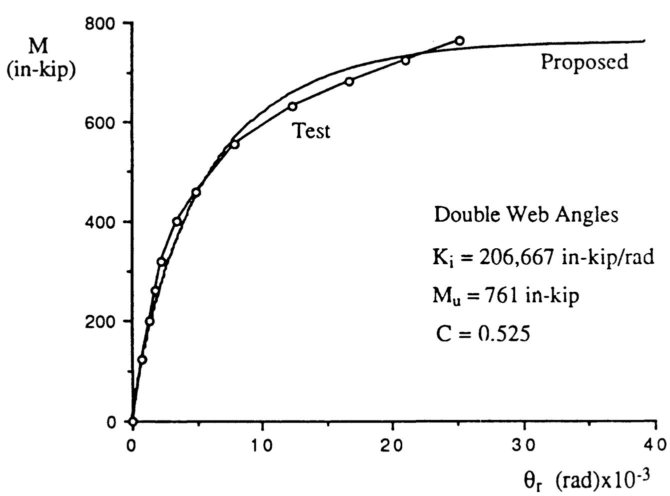

Fig. 4. Moment-rotation curves of Lewitt connection (Richard, 1987).

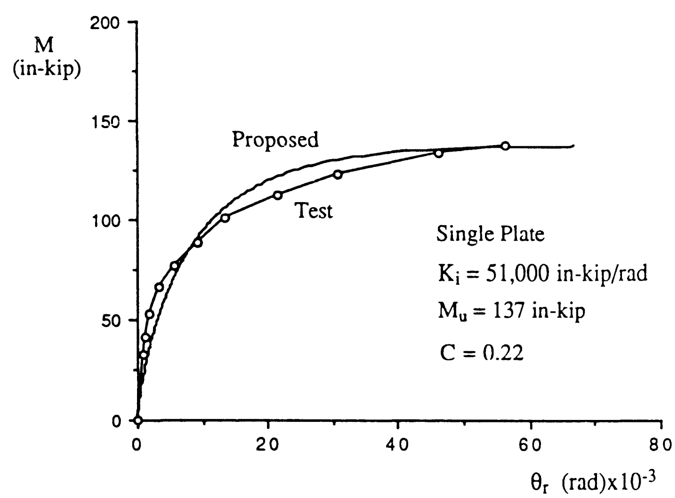

Fig. 5. Moment-rotation curves of Richard connection (1980).

$$K_{ji} = \frac{2EI}{L} - \frac{PL}{30} - \frac{26P^2L^3}{25{,}000EI} \qquad (9)$$

$$K_{jj} = \frac{4EI}{L} + \frac{2PL}{15} + \frac{44P^2L^3}{25{,}000EI} \qquad (10)$$

where

P = axial force in member
A = area of a cross section
r_i = internal reactions at both ends of a member = 1,6
d_i = displacements at both ends of a member = 1,6

The beam stiffness matrix in Equation 11 can be simplified by recognizing that the axial force in beams of rectangular frames is usually negligible. That is, by setting $K_{ii} = K_{jj} = 4EI/L$, $K_{ij} = K_{ji} = 2EI/L$, and $P = 0$.

The stiffness matrix of a beam-column can be modified to include the effect of semi-rigid connections by combining the member stiffness with the connection stiffness using a static condensation. Details of this procedure are given in Chen and Lui,[12] and the resulting member stiffness matrix has the form:

$$\begin{Bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{Bmatrix} = \begin{bmatrix} \frac{AE}{L} & 0 & 0 & \frac{-AE}{L} & 0 & 0 \\ 0 & \frac{(K_{ii}+2K_{ij}+K_{jj})}{L^2}+\frac{P}{L} & \frac{(K_{ii}+K_{ji})}{L} & 0 & \frac{-(K_{ii}+2K_{ij}+K_{jj})}{L^2}-\frac{P}{L} & \frac{(K_{ij}+K_{jj})}{L} \\ 0 & \frac{(K_{ii}+K_{ij})}{L} & K_{ii} & 0 & \frac{-(K_{ii}+K_{ij})}{L} & K_{ij} \\ \frac{-AE}{L} & 0 & 0 & \frac{AE}{L} & 0 & 0 \\ 0 & \frac{-(K_{ii}+2K_{ij}+K_{jj})}{L^2}-\frac{P}{L} & \frac{-(K_{ii}+K_{ji})}{L} & 0 & \frac{(K_{ii}+2K_{ij}+K_{jj})}{L^2}+\frac{P}{L} & \frac{-(K_{ij}+K_{jj})}{L} \\ 0 & \frac{(K_{ji}+K_{jj})}{L} & K_{ji} & 0 & \frac{-(K_{ji}+K_{jj})}{L} & K_{jj} \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \end{Bmatrix} \qquad (11)$$

$$\begin{Bmatrix} r_1 \\ r_2 \\ r_3 \\ r_4 \\ r_5 \\ r_6 \end{Bmatrix} = \begin{bmatrix} \frac{AE}{L} & 0 & 0 & \frac{-AE}{L} & 0 & 0 \\ 0 & \frac{(K_{ii}'+2K_{ij}'+K_{jj}')}{L^2}+\frac{P}{L} & \frac{(K_{ii}'+K_{ji}')}{L} & 0 & \frac{-(K_{ii}'+2K_{ij}'+K_{jj}')}{L^2}-\frac{P}{L} & \frac{(K_{ij}'+K_{jj}')}{L} \\ 0 & \frac{(K_{ii}'+K_{ij}')}{L} & K_{ii}' & 0 & \frac{-(K_{ii}'+K_{ij}')}{L} & K_{ij}' \\ \frac{-AE}{L} & 0 & 0 & \frac{AE}{L} & 0 & 0 \\ 0 & \frac{-(K_{ii}'+2K_{ij}'+K_{jj}')}{L^2}-\frac{P}{L} & \frac{-(K_{ii}'+K_{ji}')}{L} & 0 & \frac{(K_{ii}'+2K_{ij}'+K_{jj}')}{L^2}+\frac{P}{L} & \frac{-(K_{ij}'+K_{jj}')}{L} \\ 0 & \frac{(K_{ji}'+K_{jj}')}{L} & K_{ji}' & 0 & \frac{-(K_{ji}'+K_{jj}')}{L} & K_{jj}' \end{bmatrix} \begin{Bmatrix} d_1 \\ d_2 \\ d_3 \\ d_4 \\ d_5 \\ d_6 \end{Bmatrix} \qquad (12)$$

where

$$K_{ii}' = \left[S_{ii}' + \frac{S_{ii}'S_{jj}'}{R_j} - \frac{S_{ij}'S_{ij}'}{R_j} \right] \frac{1}{R*} \quad (13)$$

$$K_{jj}' = \left[S_{jj}' + \frac{S_{ii}'S_{jj}'}{R_i} - \frac{S_{ij}'S_{ij}'}{R_i} \right] \frac{1}{R*} \quad (14)$$

$$K_{ij}' = K_{ji}' = \frac{S_{ij}'}{R*} \quad (15)$$

The coefficients R_i and R_j in Equations 13 and 14 are the instantaneous tangent stiffness coefficients of the connections at ends i and j of the member respectively. These coefficients are obtained from Equation 1 when the connection is in the state of loading, and are set equal to K_i when the connection is in the state of unloading. Also, the parameter R^* is given by

$$R* = \left(1 + \frac{S_{ii}'}{R_i}\right)\left(1 + \frac{S_{jj}'}{R_j}\right) - \frac{S_{ij}'S_{ij}'}{R_i R_j} \quad (16)$$

$$S_{ii}' = S_{jj}' = K_{ii} = K_{jj} \quad (17)$$

$$S_{ij}' = S_{ji}' = K_{ij} = K_{ji} \quad (18)$$

in which P is negative for compressive force, and is small or zero for beam elements and can be neglected.

5. THE PROPOSED METHOD

Several simplifications are made in the present formulation. The moments of beam-column joints must be less than the ultimate moment M_u of semi-rigid connections or the plastic moment capacity M_{pc} of beam-columns. The combined axial load and end moments in any member must satisfy the AISC-LRFD bilinear interaction equations.

5.1 Rigid Frame Analysis

1. Perform the first-order elastic rigid frame analysis.
2. Compute notional lateral loads, $\Sigma H'$, using the relationship

$$S_F = \frac{\Sigma H}{\Delta_o} = \frac{\Sigma H + \Sigma P_u \Delta / L}{\Delta} = \frac{\Sigma H'}{\Delta} \quad (19)$$

where Δ_o is the first-order translational deflection of the story, and Δ is the second-order translational deflection of the story under consideration. From Equation 19, we have

$$\Delta = (\Sigma H + \Sigma P_u \Delta / L)\frac{\Delta_o}{\Sigma H} = \left(1 + \frac{\Sigma P_u \Delta}{\Sigma HL}\right)\Delta_o \quad (20)$$

from which we obtain

$$\Delta\left(1 - \frac{\Sigma P_u \Delta_o}{\Sigma HL}\right) = \Delta_o \quad (21)$$

or

$$\Delta = \frac{\Delta_o}{\left(1 - \dfrac{\Sigma P_u \Delta_o}{\Sigma HL}\right)} = B_2 \Delta_o \quad (22)$$

The Δ is the second-order lateral deflection due to P-Δ effect, and the notional lateral load is defined as

$$\Sigma H' = \Sigma H + \Sigma P_u \Delta / L \quad (23)$$

3. Use $\Sigma H'$ and original gravity loads to perform first-order elastic rigid frame analysis. The results of this step include the second-order effect.
4. Calculate B_1 factor with the effective length factor K = 1.0 for each column and multiply the corresponding end moments.
5. Check the AISC-LRFD bilinear interaction equations.

5.2 Semi-Rigid Frame Analysis

1. Select connections from the maximum beam-column joint moments in rigid frame analysis.
2. Determine the initial stiffness, K_i, of connections from test results or any other available methods.
3. Substitute $0.5K_i$ of connection stiffness for the semi-rigid joint. The average connection stiffness $0.5K_i$ as suggested by Ackroyd[10] is adopted here.
4. Use $0.5K_i$ for semi-rigid connection stiffness with the notional lateral loads $\Sigma H'$ to carry out the first-order elastic analysis.
5. Calculate B_1 factor with effective length factor K = 1.0 for P_e of each column and multiply the corresponding larger end moments.
6. Check the AISC-LRFD bilinear interaction equations. The effective length factor, K, for the column strength, P_n, has to be modified in the case of semi-rigid frames. For beams connected to columns with semi-rigid connections rotational stiffness, K_i, at both ends, a simple modification of the relative stiffness, G, factors with the modified moment inertia of beam is (Chen and Lui[13]):

$$I' = \frac{I}{1 + \dfrac{2EI}{K_i L}} \quad (24)$$

The I' is used in G factors for the determination of the effective length factor, K, for the value of F_{cr} which is the critical column stress.

6. NUMERICAL EXAMPLES

The proposed method will now be illustrated by numerical examples. Comparisons are made between results using direct second-order elastic analysis, Barakat's method,[1] and the proposed method. The semi-rigid frame examples include single-story and multi-story frames. All examples are analyzed with a personal computer. All beams subjected to uniformly distributed loads are divided into two equal elements.

6.1 Two-Story One-Bay Frame with Concentrated Loads

The two-story one-bay frame as shown in Figure 6 is analyzed with both rigid and semi-rigid connections. Two lateral loads, H, and four constant concentrated gravity loads, P, of 100 kips are applied at the beam-column joints of the frame. The flexible connections used are shown in Figure 2 where M_u is less than the plastic moment M_p of beams and columns. The $0.5K_i$ of Jenkins connection is 393,366 in-kip/rad. The second-order lateral displacement at Joint 5 is

$$\Delta_5' = \frac{\Delta_{o5}}{\left(1 - \dfrac{\Sigma P_u \Delta_{o5}}{\Sigma HL}\right)} = \frac{1.512}{\left(1 - \dfrac{2 \times 100 \times 1.512}{2 \times 10 \times 144}\right)} = 1.69 \text{ in.}$$

The second-order lateral load at Joint 5 is

$$\Sigma H_5' = \Sigma H_5 + \Sigma P_u \Delta_5'/L = 10 + 2 \times 100 \times 1.69/144 = 12.35 \text{ kips}$$

The second-order lateral displacement at Joint 3 is

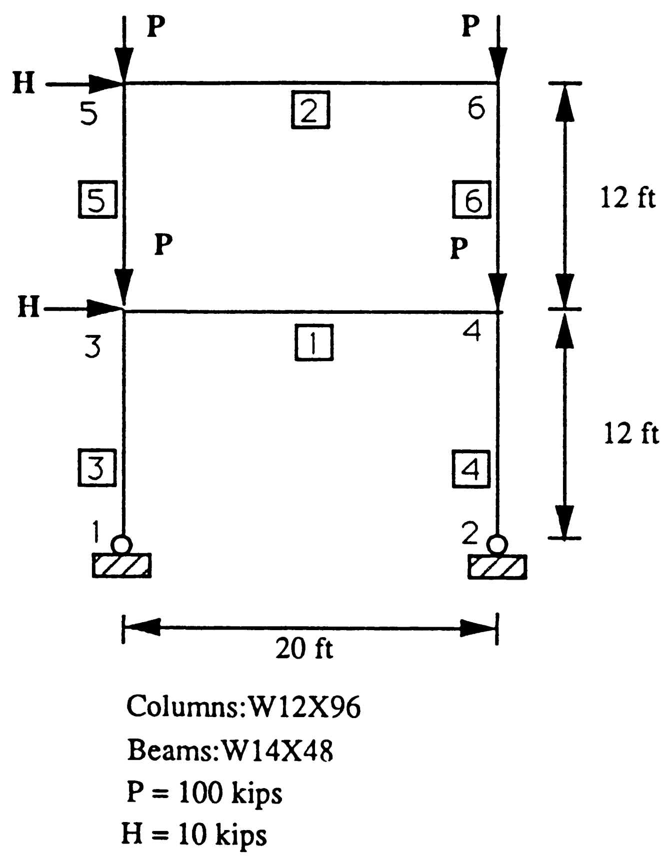

Fig. 6. Two-story one-bay frame with concentrated loads.

Table 1. Maximum Moments in Elastic Rigid Frame Analysis (in.-kips) (Two-story one-bay frame, Figure 6)

Element No.	(1) First-Order (Exact)	(2) Second-Order (Exact)	(3) Proposed	(4) (3) / (2)
1	1449	1649	1839	1.12
2	712	794	894	1.13
3	1443	1670	1847	1.11
4	1437	1664	1839	1.11
5	711	794	893	1.13
6	712	794	894	1.13

$$\Delta_3' = \frac{\Delta_{o3}}{\left(1 - \dfrac{\Sigma P_u \Delta_{o3}}{\Sigma HL}\right)} = \frac{1.01}{\left(1 - \dfrac{4 \times 100 \times 1.01}{2 \times 10 \times 144}\right)} = 1.17 \text{ in.}$$

The notional lateral load at Joint 3 is

$$\Sigma H_3' = \Sigma H_3 + \Sigma P_u \Delta_3'/L = 10 + 4 \times 100 \times 1.17/144 = 13.25 \text{ kips}$$

All moments of beams and columns predicted by the proposed method are normalized with respect to that of second-order elastic analysis and are summarized in Tables 1 and 2. The lateral displacements at windward beam-column joints are shown in Table 3. The mean values are the sum of normalized values of each member divided by the number of total members. All the results predicted by the proposed method are close to the exact solutions. It is found that the maximum moment and lateral displacement can be predicted well by the proposed method.

6.2 Two-Story One-Bay Frame with Uniformly Distributed Loads

A two-story one-bay frame used by Barakat,[1] et al as shown in Figure 7(a) is employed here for comparison of the maximum moments in members. The semi-rigid connection labeled III-17 is shown in Figure 8 and compared with the proposed connection model. The moments predicted by second-order elastic analysis the Barakat method, and the proposed method are compared in Table 4. The average value of Column 3 in Table 4 is 0.98, while the average value of Column 5 is 0.97. The Barakat method is slightly less conservative in this example.

To verify the validity of the proposed method for soft semi-rigid connections, the bolted framing angles tested by Lewitt[5] are used. The lateral loads and uniformly distributed

Table 2.
Maximum Moments in Elastic Semi-Rigid Frame Analysis (in.-kips)
(Two-story one-bay frame, Figure 6)

Element No.	(1) Second-Order (Exact)	(2) Proposed	(3) (2) / (1)
1	1560	1696	1.09
2	1116	1037	0.93
3	1837	1847	1.01
4	1834	1839	1.00
5	1116	1037	0.93
6	1116	1037	0.93

Table 3.
Lateral Displacements at Windward Beam-Column Joints (in.)
(Two-story one-bay frame, Figure 6)

	Rigid Frame			Semi-Rigid Frame		
	(1)	(2)	(3)	(4)	(5)	(6)
Node No.	Second-Order (Exact)	Proposed	(2) / (1)	Second-Order (Exact)	Proposed	(5) / (4)
3	1.16	1.21	1.04	2.02	1.85	0.92
5	1.73	1.82	1.05	3.26	2.98	0.91

loads are reduced as shown in Figure 7(b), so the maximum moments in members of the two-story one-bay frame are less than the ultimate moment, M_u, semi-rigid connections. The results predicted by the proposed method are compared with that of second-order elastic analysis in Table 4(b) and the lateral displacements are shown in Table 4(c). It can be concluded that the proposed method is valid for soft connections, although the second-order lateral loads in the proposed method are determined from a rigid frame.

6.3 Three-Story One-Bay Frame with Uniformly Distributed Loads

The three-story one-bay frame shown in Figure 9 is analyzed with semi-rigid connections labeled III-17. Three beam-column joints are subjected to concentrated lateral loads. All the beams are subjected to uniformly distributed gravity loads. The results by the Barakat method[1] are compared with those results of the proposed method (Table 5). The average value of Column 3 in Table 5 is 1.00, while the average value of

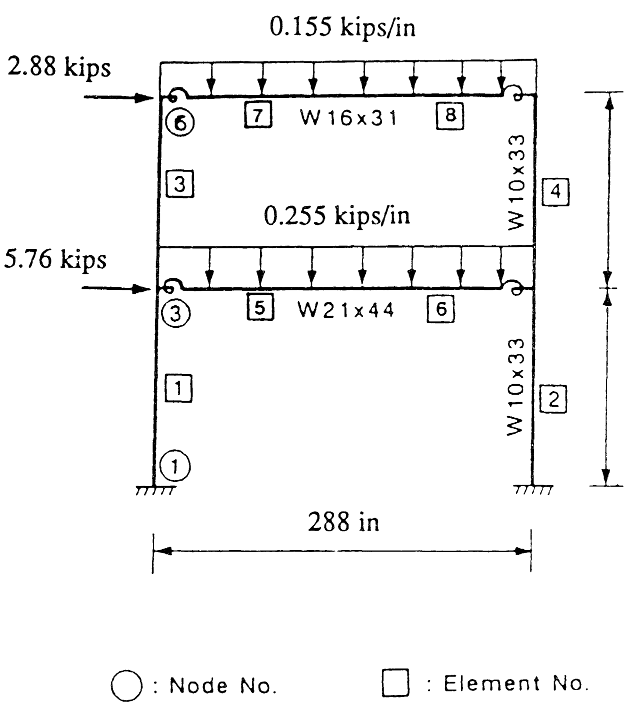

Fig. 7(a). Two-story one-bay frame with uniformly distributed loads.

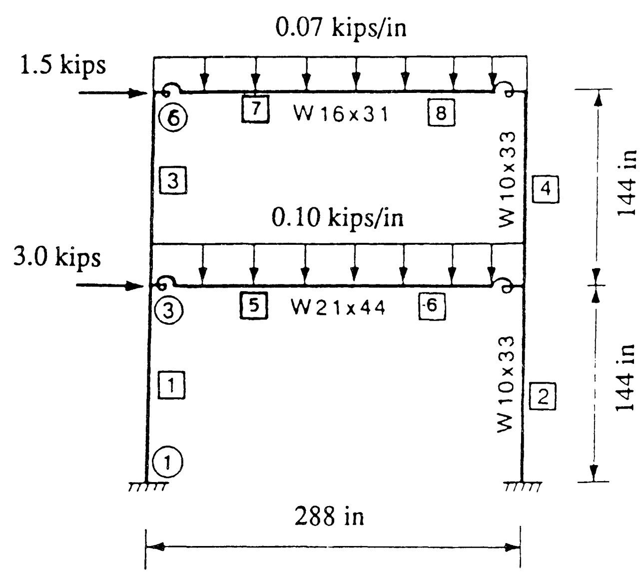

Fig. 7(b). Two-story one-bay frame with uniformly distributed loads.

Table 4(a). Maximum Moments in Elastic Semi-Rigid Frame Analysis (in.-kips) (Two-story one-bay frame, Figure 7a)					
	(1)	(2)	(3)	(4)	(5)
Element No.	Second-Order (Exact)	Proposed	(2) / (1)	Barakat	(4) / (1)
1	257	220	0.86	201	0.78
2	547	588	1.07	576	1.05
3	526	552	1.05	557	1.06
4	813	818	1.00	811	0.99
5	1497	1431	0.96	1434	0.96
6	1497	1431	0.96	1434	0.96
7	940	922	0.98	923	0.98
8	940	922	0.98	923	0.98

Table 4(b). Maximum Moments in Elastic Semi-Rigid Frame Analysis (in.-kips) (Two-story one-bay frame, Figure 7b)				
	(1)	(2)	(3)	(4)
Element No.	Linear-Elastic Rigid	Second-Order Elastic Semi-Rigid	Proposed	(3) / (2)
1	88	176	168	0.95
2	324	278	276	0.99
3	283	188	187	0.99
4	400	343	338	0.99
5	562	728	718	0.99
6	673	728	718	0.99
7	384	461	463	1.00
8	400	461	463	1.00

Column 5 is 0.97. It can be seen that the Barakat method is less conservative in this case.

6.4 Four-Story Two-Bay Frame with Uniformly Distributed Loads

A four-story two-bay frame as shown in Figure 10 is investigated here for the maximum column moments in both rigid and semi-rigid frames. The semi-rigid connection of Jenkins is utilized. The average value of Column 3 in Table 6 is 1.06, while the average value of Column 6 in Table 6 is 1.01. The proposed method represents reasonably well the second-order effect for rigid and semi-rigid frames. The lateral displacements at windward beam-column joints are summarized in Table 7. The lateral displacements predicted by the proposed method are less than those of the second-order elastic semi-rigid frame analysis. However, the lateral displacements predicted by the proposed method are larger than that of the second-order elastic rigid frame analysis.

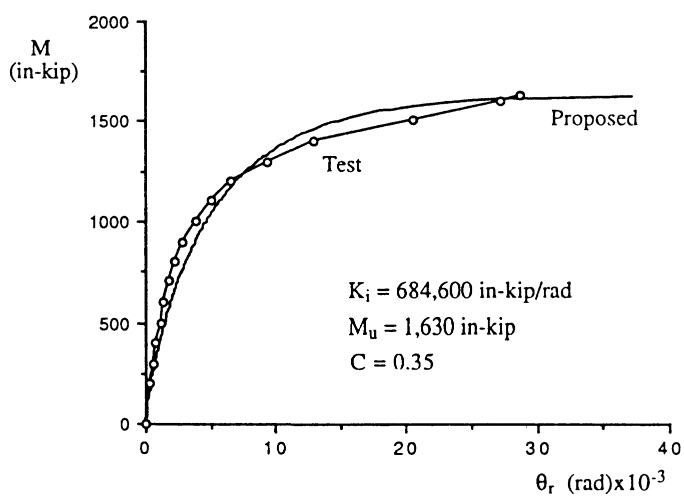

Fig. 8. Experimental III-17 connection curves (Barakat, 1991)

Table 4(c). Lateral Displacements at Windward Beam-Column Joints (in.) (Two-story one-bay frame, Figure 7b)				
	Rigid Frame	Semi-Rigid Frame		
	(1)	(2)	(3)	(4)
Node No.	Linear Elastic (Exact)	Second-Order (Exact)	Proposed	(3) / (2)
3	0.14	0.25	0.24	0.96
6	0.23	0.50	0.47	0.94

7. SUMMARY AND CONCLUSIONS

Several conclusions can be drawn from the present studies:

1. The moment-rotation relationships of semi-rigid connections as represented by a simple tangent stiffness

expression are convenient and can lead to a close moment-rotation curve by numerical integration when compared with test result. Note that only the tangent stiffness is needed in an incremental nonlinear frame analysis.

2. The proposed method gives close results to that of second-order elastic analysis. It can handle both the uniformly distributed gravity loads and concentrated loads, and predicts well the drift of unbraced frames.
3. All mean values of the normalized moment ratios are found close to or slightly greater than one in the proposed method. This shows that the proposed method is more accurate when compared with that of the Barakat method. The proposed method gives a reasonable procedure for estimating the approximate P-Δ column moments for both rigid and semi-rigid frames.
4. The notional lateral loads calculation is relatively simple and straightforward because the tedious determination of the effective length factor, K, can be avoided. It is a simple and practical method for semi-rigid frame design.

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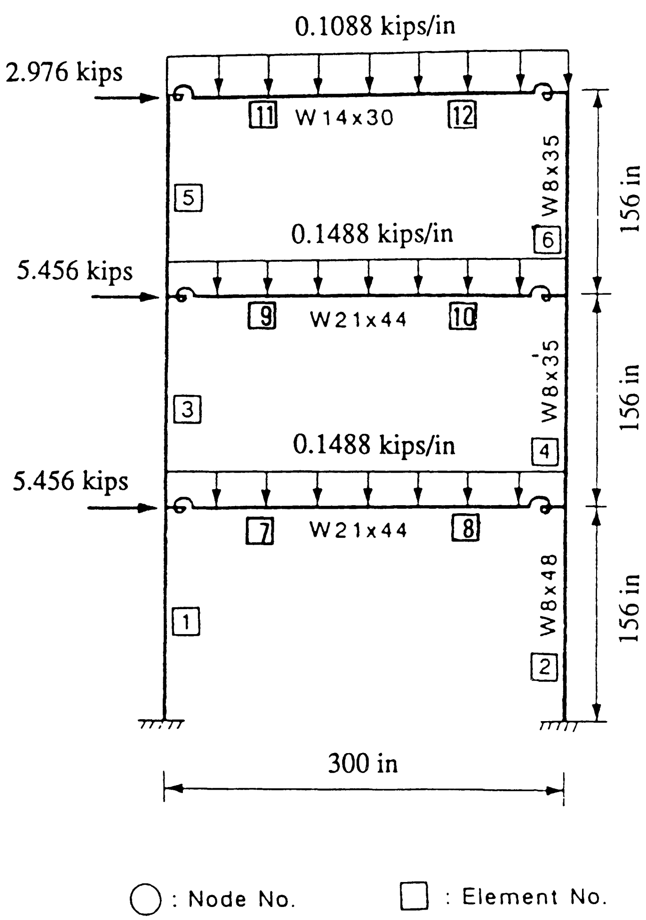

Fig. 9. Three-story one-bay frame with uniformly distributed loads.

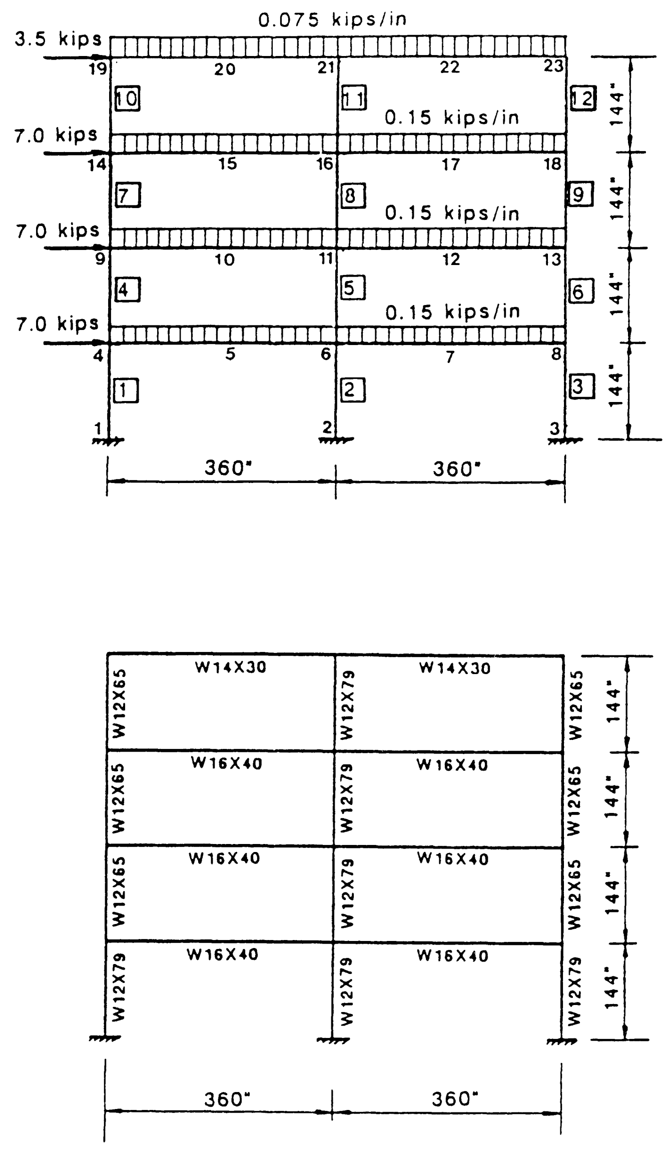

Fig. 10. Four-story two-bay frame.

Table 5.
Maximum Moments in Elastic Semi-Rigid Frame Analysis (in.-kips)
(Three-story one-bay frame, Figure 9)

Element No.	(1) Second-Order (Exact)	(2) Proposed	(3) (2) / (1)	(4) Barakat	(5) (4) / (1)
1	599	575	0.96	533	0.89
2	845	879	1.04	836	0.99
3	152	123	0.81	115	0.75
4	618	669	1.08	659	1.07
5	349	356	1.02	367	1.05
6	659	663	1.01	651	0.99
7	1181	1075	0.91	1076	0.91
8	1212	1386	1.14	1322	1.09
9	1082	1023	0.95	1025	0.95
10	1082	1148	1.06	1101	1.02
11	722	714	0.99	715	0.99
12	722	714	0.99	715	0.99

Table 6.
Maximum Column Moments in Elastic Frame Analysis (in.-kips)
(Four-story two-bay frame, Figure 10)

Element No.	Rigid Frame			Semi-Rigid		
	(1) Second-Order (Exact)	(2) Proposed	(3) (2) / (1)	(4) Second-Order (Exact)	(5) Proposed	(6) (5) / (4)
1	534	632	1.18	843	799	0.95
2	958	1066	1.12	1170	1182	1.01
3	1202	1296	1.08	1397	1398	1.00
4	455	421	0.93	291	322	1.11
5	656	729	1.11	596	698	1.17
6	1101	1142	1.04	1044	1061	1.02
7	615	603	0.98	559	558	0.99
8	473	525	1.11	542	562	1.04
9	1029	1061	1.03	996	1012	1.02
10	702	703	1.00	705	672	0.95
11	200	221	1.11	313	269	0.86
12	818	829	1.01	846	812	0.96

Table 7.
Lateral Displacments at Windward Beam-column Joints (in.)
(Four-story two-bay frame, Figure 10)

Node No.	Rigid Frame			Semi-Rigid Frame		
	(1) Second-Order (Exact)	(2) Proposed	(3) (2) / (1)	(4) Second-Order (Exact)	(5) Proposed	(6) (5) / (4)
4	0.27	0.30	1.11	0.40	0.37	0.93
9	0.66	0.73	1.11	1.07	0.95	0.89
14	0.94	1.04	1.11	1.61	1.40	0.87
19	1.11	1.23	1.11	1.95	1.68	0.86

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