Beam Design in PR Braced Steel Frames

RICCARDO ZANDONINI and PAOLO ZANON

INTRODUCTION

Recent American and European Codes allow the actual behavior of joints to be taken into account in frame design. The AISC-LRFD specifications[1] introduce the concept of partially restrained construction and state the basic design requirements. Eurocode 3^2 permits "semi-continuous" frames and specifies basic criteria for the analysis of this type of structure as well as for the classification and modeling of the moment-rotation of beam-to-column joints (Fig. 1).

This consistent development in Code recommendations recognizes, on the one hand, the significant influence joint response has on the overall frame performance; on the other hand, it reflects a state of knowledge which is now sufficient to enable practicing engineers to undertake the design of partially restrained frames at the required level of reliability. Studies of the stability of this form of framework were recently carried out, the main problems related to the analysis investigated, and design methods established.[3-10]

The most important aspect, as to practical purposes, is the capability of approximating the moment-rotation ($M - \phi$) curve of the joint. Satisfactory prediction models were proposed in the past few years, covering the most popular connection forms.[11-15] Research work is currently underway to set up more refined models as well as to extend their scope.

As far as braced frames are concerned, it was pointed out that weight savings, with respect to simple framing, may be achieved by recognizing the stiffness and strength characteristics of the joints. In many instances the same connection forms used in simple frames can be retained, with limited (or even no) increase in cost of details.[16,17] The more favorable distribution of moments permits lighter beams to be selected, while the end restraint provided to the columns, even by flexible connections, usually seems sufficient to balance the effect of the moment transmitted from the beams to the columns via the connection.[18] Similarly, semi-rigid joint action is of significant benefit for secondary beams as well as for roof purlins.

The traditional assumptions which braced frame design is based on hold true also when the actual joint response is incorporated into the analysis: i.e., (1) the horizontal forces are resisted by the bracing system alone; (2) the frame can be designed by a component analysis.

According to this hypothesis, a beam in PR frame can be modeled as a flexural member subject to vertical loads and rotationally restrained at the ends (Fig. 2); the $M - \phi$ law of the restraints may incorporate column rotation, besides joint deformation.

Due to the significant nonlinearity exhibited by most connections (Fig. 1), the method of joint-beam system analysis must be capable of dealing with nonlinear responses. Furthermore, this should be kept as simple as possible in order to avoid undue complexity in calculations, thus reducing the benefits achievable through the PR design.

The authors developed a method of analysis of partially restrained beams, permitting straightforward determination

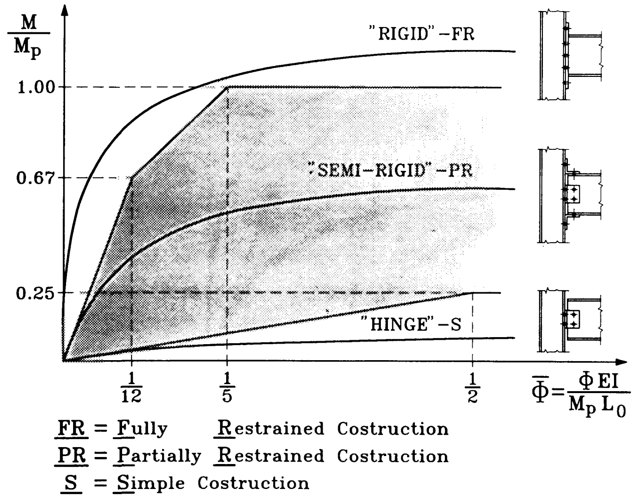

Fig. 1. Classification of joints in non-sway frames after Eurocode 3.

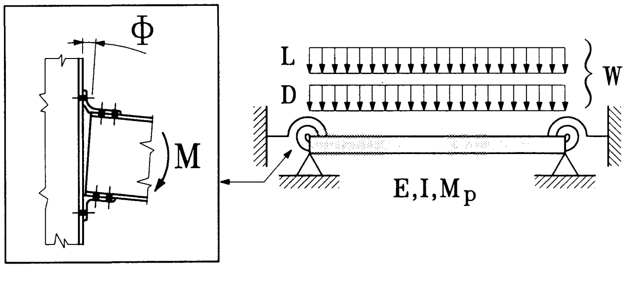

Fig. 2. End-restrained beam.

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Riccardo Zandonini is associate professor of steel structures at the University of Trento, Italy.

Paolo Zanon is professor of theory of structures at the University of Trento, Italy.

of the response as well as a comprehensive checking against the limit state conditions, either elastic or plastic. The approach extends the beam line concept, which most designers are already familiar with; however, the main novelty is that a set of complementary limit state domains is defined and then arranged so that the performance of the joint-beam system can be controlled with simultaneous reference to the different parameters governing the joint-beam system response. The method makes an engineering optimization possible, in the sense that the minimum joint stiffness and strength values required can be determined.

This paper intends to present the main features of the method of analysis, to explain its use in design, and to illustrate through examples its practicality and effectiveness. The results are finally arranged so that the benefits, in terms of weight reduction, attainable by means of PR design are emphasized. This represents only a first and rough indication of the possible savings; the detailing cost may actually increase.

LIMIT STATE ANALYSIS

Assuming that the spreading of plasticity in the cross-section and along the member may be disregarded, i.e., that the post-elastic behavior is fully described by the plastic hinge model, the ultimate limit states of the partially restrained beam under consideration (Fig. 2) are those identified in Fig. 3: the formation either of a plastic hinge at midspan (a) or of plastic hinges at the beam ends (b) defines the elastic ultimate limit state, while the plastic mechanism (c) defines the plastic ultimate limit state. In the presence of rotational end restraints, the beam deformation corresponding to the attainment of the mechanism condition may be unacceptably large; it is hence necessary to associate the plastic collapse mode as well to the attainment of a limit value of the deflection (δ_u in (d)).

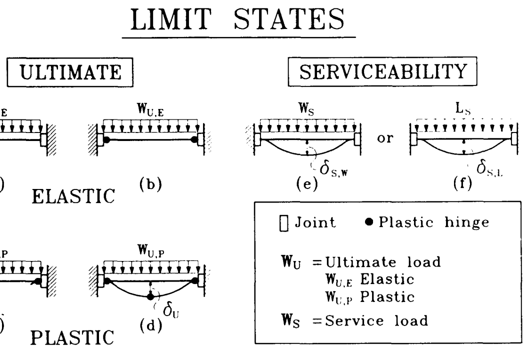

Fig. 3. The beam limit states.

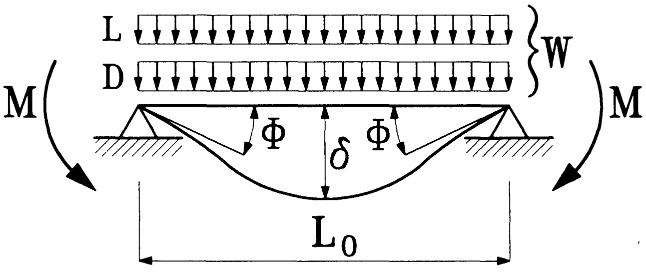

Fig. 4. Assumed parameters.

The check of the performance under normal service conditions (serviceability limit state) relates to a deflection limit δ_s under unfactored design loads; reference can be made[1,2] to the sole live load ($\delta_{s,L}$) or to the total load $W = L + D$ ($\delta_{s,W}$). For simplicity's sake, but without lack of generality, the latter form is adopted in this paper and, according to the Eurocode,[2] a reference value $\delta_s = \delta_{s,W} = L_0/250$ is assumed. However, mention is made of how to deal with a $\delta_{s,L}$ limit. In order to comprehensively describe the response, the following parameters can be chosen related to its strength and deformation (Fig. 4): the applied load W, the end moment M, the end rotation ϕ, and the midspan deflection δ.

Limit state domains can be defined for the beam, the boundaries of which are determined as the loci of the points identified by the values of the four characteristic parameters corresponding to the different limit state conditions: i.e.,

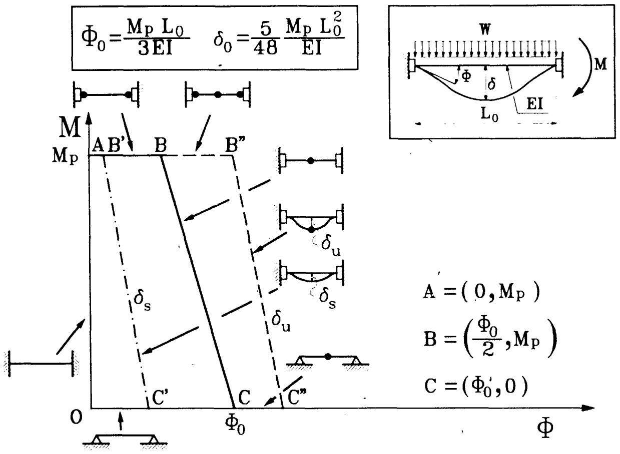

Fig. 5. M − φ domains.

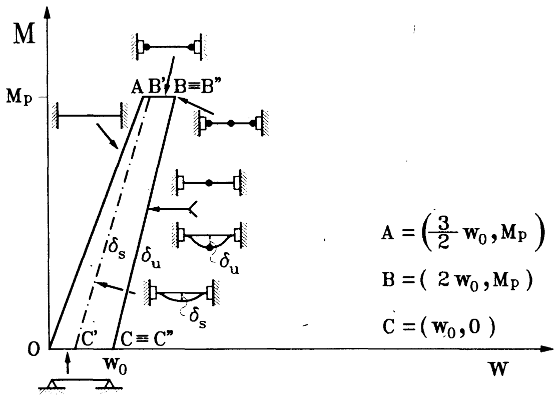

Fig. 6. M − W domains.

ultimate elastic, ultimate plastic, and serviceability.

A representation of these domains may be obtained, which brings significant advantages in terms of practical effectiveness and simplicity of use.

Let us consider the four parameters by pairs: e.g., M and ϕ, M and W, W and δ, δ and ϕ. Each pair defines a system of Cartesian coordinates, referring to which the equations of the boundaries of the limit state domains can be written on the basis of equilibrium and compatibility.[19] They are shown in Figs. 5 to 8, where the relationship between limit state and boundary is also shown.

Lines OABC and BB″C″C bound the ultimate limit state domains, elastic and plastic respectively, while line OAB′C′ bounds the serviceability domain. The elastic ultimate domains are fully defined and are easily drawn, the span and section of the beam being known, as becomes apparent from the coordinates of the vertices A, B, and C also given in the figures. If the resistance of the cross-section is associated to first yield rather than to plastic strength, the elastic limit state domain is obtained simply by substituting the first yield moment M_e to M_p.

The serviceability domain and partly the plastic domain, depend on the deflection limits (δ_s and δ_u) assumed. It may be further noted that:

1. The elastic boundary lines in the $M - \phi$ plane represent the "yield beam line" for the attainment of the cross sectional strength first introduced by Kennedy.[20]
2. In the $M - W$ plane, the elastic and plastic domains coincide: when a plastic hinge is formed at midspan, the beam becomes statically determinate, and points associated to the same end moment must be associated to the same value of W as well, i.e., line BC lies on line B″C″.
3. In the $\delta - \phi$ plane, lines AB and BB″ are parallel to lines OC and CC″ respectively. Each of these lines is related to a constant value of end moment M (lines AB and BB″ to $M = M_p$, lines OC and CC″ to $M = 0$), hence they represent the boundaries of two families of parallel constant end moment lines. It may be easily shown (see point 2, above) that in the plastic domain they are constant load lines as well. The same considerations apply to plane $W - \delta$. The limit state domains of beams with a reduced flexural resistance at the ends (e.g., due to weakening details) maintain the same shape and are straightforwardly obtained.

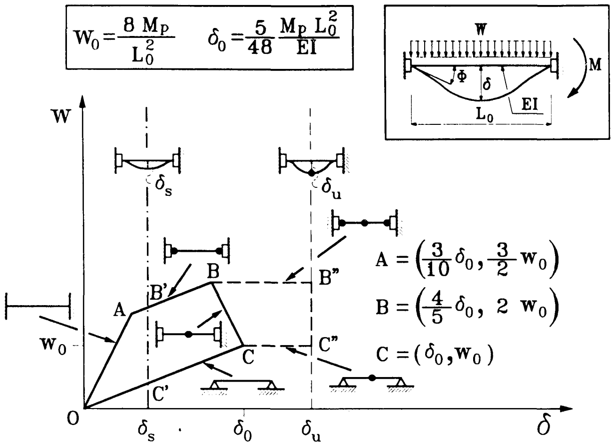

Fig. 7. $W - \delta$ domains.

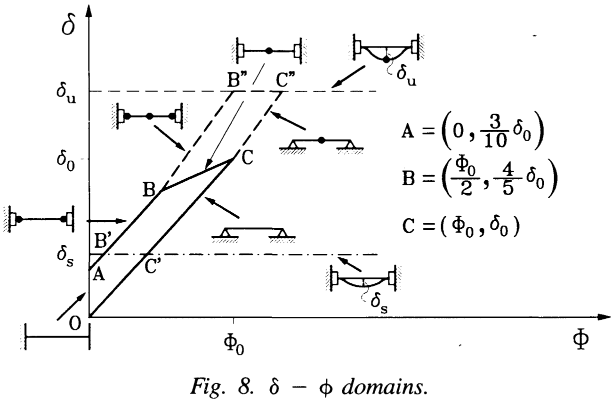

Fig. 8. $\delta - \phi$ domains.

The four systems of coordinates can be arranged as the quadrants of a Cartesian plane, as is illustrated in Fig. 9. In this multi-coordinate representation, the elastic and plastic range of the beam response is defined by elastic and plastic multi-domains.

As a result, the correlation between the selected characteristic parameters can be "shown" immediately and effectively for design practice: e.g., the boundary lines associated to deflection conditions, either δ_s or δ_u, as well as the constant load lines can be drawn by means of the simple graphical constructions illustrated in Figs. 10 and 11 respectively. In effect, any relation between the characteristic parameters can be graphically established.

If the attention is now focused on the system consisting of the beam and the joints, the multi-domains allow its limit state analysis to be conducted, the moment-rotation law of the joint being known: equilibrium and compatibility at beam ends require that the moment-rotation curve of the joints also represents the response curve of the beam in the $M - \phi$ quadrant. The same equilibrium and compatibility equations define the system response in terms of the other parameters. Therefore, superimposing these response curves and the beam multi-domains permits a comprehensive determination of the limit state conditions in terms of end moment, applied load, end rotation, and midspan deflection (intersection points in Fig. 12). The system performance as to stiffness, strength, and ductility can thus be assessed.

Finally, the problem can be conveniently normalized,[19] in order to eliminate dependence on the beam stiffness and strength; to this aim, reference may be made to the values each characteristic parameter assumes at plastic collapse of a simply supported beam, i.e.,

$$W_0 = 8M_p/L_0^2;\ \delta_0 = 5M_pL_0^2/48EI;\ \phi_0 = M_pL_0/3EI$$

where M_p is the beam plastic moment, L_0 its span, and I its moment of inertia. The normalized parameters are, hence,

$$\overline{M} = M/M_p;\ \overline{W} = W/W_0;\ \overline{\delta} = \delta/\delta_0;\ \overline{\phi} = \phi/\phi_0$$

CHECKING THE JOINT-BEAM SYSTEM

The procedure explained in the previous section permits determination of the values of the loads associated to the different limit state conditions (Fig. 12): i.e., W_S to serviceability, $W_{U,E}$ to the elastic ultimate, $W_{U,P}$ to the plastic ultimate, or to the excessive deformation limit.

The knowledge of these load values enables designers to check whether the stiffness and strength requirements of the joint-beam system are fulfilled. If D and L denote the design nominal values of the dead and live loads, and $\gamma_{U,D}$ and $\gamma_{U,L}$ the values of the respective load factors for resistance checks, the factored design load is $W_{f,U} = \gamma_{U,D}D + \gamma_{U,L}L$. Therefore, checking consists in ensuring that

$$W_{f,U} \leq W_U \tag{1}$$

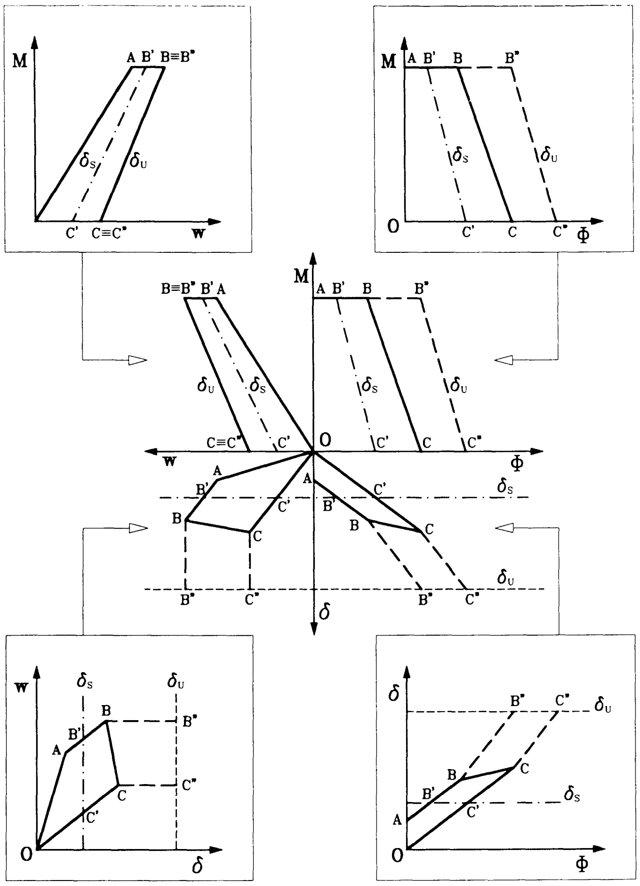

Fig. 9. The limit state multi-domains.

For the beam considered, which is subject solely to vertical loads, it is possible to define one load factor, $\gamma_{U,W'}$, to be applied to the total load W:

$$\gamma_{U,W} = W_{f,U} / W_{unf} \tag{2}$$

where $W_{unf} = D + L$ is the unfactored design load.

An alternative procedure is feasible which

- defines the value of the unfactored total load W_d corresponding to the attainment of a given ultimate limit

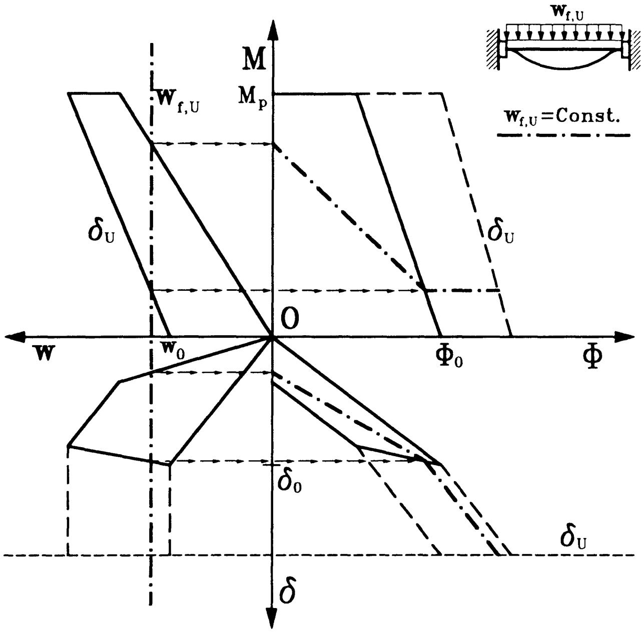

Fig. 10. Graphical determination of the constant load line.

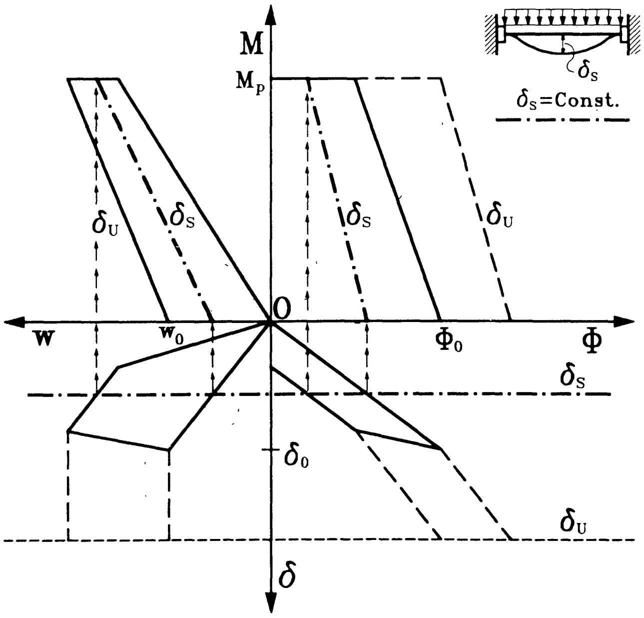

Fig. 11. Graphical determination of the constant deflection line.

state by dividing (Fig. 13) the associated limit state load by $\gamma_{U,W}$ (e.g., $W_{d,E} = W_{U,E} / \gamma_{U,W}$). W_d is the maximum unfactored load W which the joint-beam system is able to resist, while fulfilling a particular ultimate limit state.

- checks that

$$W_{unf} \leq W_{d,P} \tag{3}$$

The advantage of this approach lies in the consistence, in terms of loads, with the serviceability check, the load factors for verification of deflections in service conditions being generally $\gamma_{S,D} = \gamma_{S,L} = \gamma_{S,W} = \gamma_S = 1$.

When the deflection limit is related to the total load ($\delta_{S,W}$), the unfactored load, besides inequality (3), should fulfill also the condition

$$W_{unf} \leq W_{d,S} \tag{4}$$

where $W_{d,S} = W_S / \gamma_S = W_S$.

Depending upon which load is lowest between $W_{d,P}$ and $W_{d,S}$, limit state checking equals to verifying only one inequality (either the (3) or the (4)).

Deflection under sole live loads is often limited ($\delta_{S,L}$); under the conservative assumption that the ultimate limit relationship (3) is fulfilled as an equality ($W_{unf} = W_{d,P}$), the live load $L_{d,P}$ corresponding to the deflection $\delta_{S,L}$ can be easily determined as illustrated in Fig. 14. Serviceability hence requires, besides inequality (3), that

$$L \leq L_{d,P} \tag{5}$$

Relationship (5) may be rather conservative when the design dead load D is significantly lower than L. In this case, it is convenient to determine the actual value of δ_L, which determination is straightforward in the adopted representation of the joint-beam response.

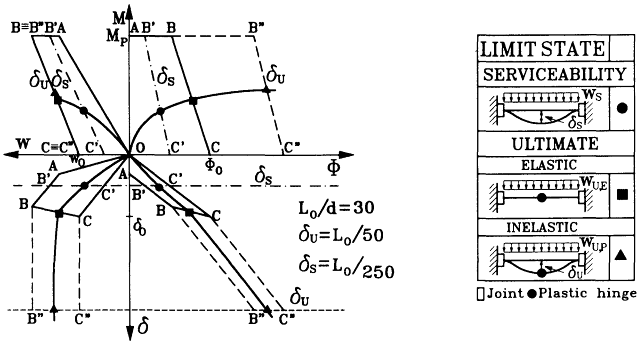

Fig. 12. Analysis of the joint-beam system.

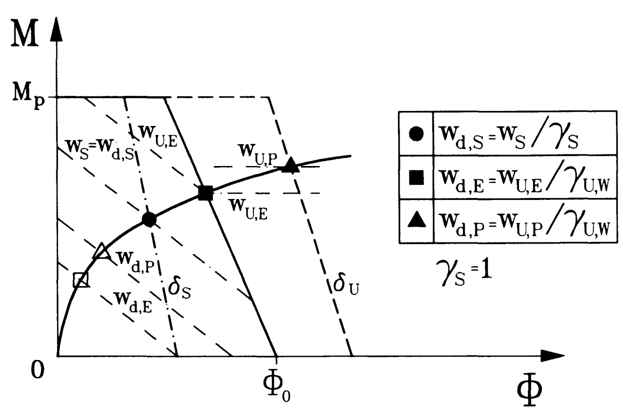

Fig. 13. Definitions of maximum allowable design loads.

DESIGN OF THE JOINT-BEAM SYSTEM

Design Domains of the Joint-Beam System

In design, the nominal loads, the load factors (and then the ultimate design load $W_{f,U}$), the steel grade as well as the deflection limits δ_S and δ_U are given. The beam section is usually sized through preliminary calculations based on

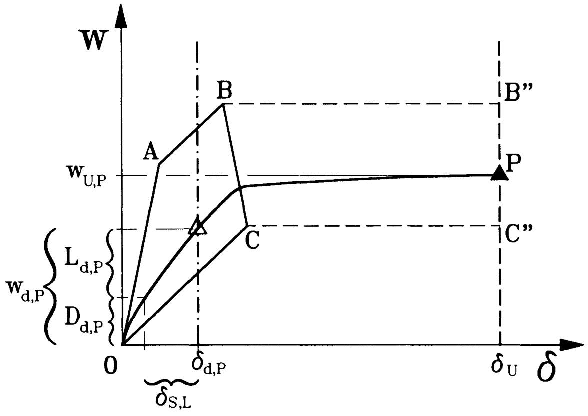

Fig. 14. Design analysis in case of live-load deflection limit.

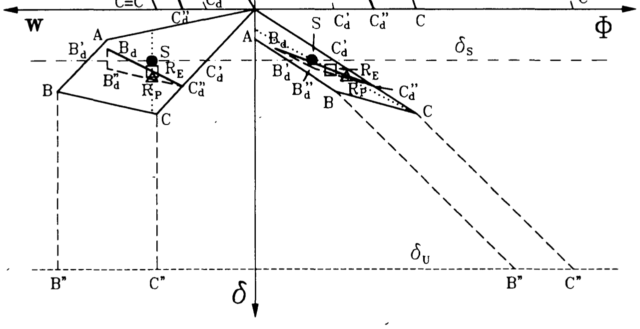

Fig. 15. Joint-beam design domains.

simple models or empirical formulae. The beam multi-domains are then fully defined.

The problem hence is mainly to determine the performance to be required of the joints in order to make the beam to fulfill the design specifications in service and at ultimate conditions (either elastic or plastic). Moreover, because of various fabrication and construction factors, this search is generally conducted with reference to a preferred connection type.

It is advantagous to relate also the ultimate limit state to unfactored loads, thus permitting an immediate comparison between the design requirements in service and at the ultimate conditions.

Let us assume that a mathematical, or mechanical, model of the selected connection form is available to approximate the moment-rotation curves, as a function of the geometrical and mechanical parameters governing the response. For each particular connection of this form, a set of points on the $M - \phi$ curve associated to the maximum allowable unfactored loads W_d (i.e., $W_{d,S}$, $W_{d,E}$, $W_{d,P}$) can be obtained through the procedure discussed in the previous section and illustrated in Fig. 13. While the parameters considered vary with continuity, these points describe a set of lines which bounds the limit state design domains of the joint-beam system.[21]

It should be stressed that these domains depend on the connection form considered. Figure 15 shows typical elastic and plastic design multi-domains; the former are bounded by the B_dC_d'' lines, the latter by $B_dB_d''C_d''$.

If $\gamma_S = 1$, and the deflection limit is referred to the total applied load, the design joint-beam domains and the beam limit state domains related to serviceability coincide ($B_d'C_d' = B'C'$).

The domains are not fully available for a particular design case; their available part is obtained by drawing, in the four quadrants, the constant load lines related to the design unfactored load W_{unf}, as illustrated in Fig. 15. The points of the domain boundaries lying above the intersections between this line and the design domains (i.e., above points S for the serviceability domains, above points R_E and R_P for the elastic and plastic ultimate domains) associate the related limit state condition to unfactored loads higher than those required by the design. In other terms, only if the joint $M - \phi$ curves pass through points R_E, or through R_p, the joint-beam system fails for the attainment of a beam ultimate limit state, elastic or plastic, under the design factored load. Joints characterized by response curves lying above line OR_E, or line OR_P, make the beam to attain its limit state condition for $W \geq W_{f,U}$.

Points S, R_E and R_P hence represent the lower limit of the "effective" part of the domains. For the same connection type, it can be generally assumed that cost increases with stiffness and strength. On this assumption, the "optimal" joint, of a given form, does possess a rotational behavior which makes the joint-beam response to pass through the lower limit points (either S, R_E or R_P) of the effective part of the design domains.

A further check should usually be conducted on the joint rotation capacity. In case the inherent rotation capacity of a connection type is however sufficient to make the beam to achieve the ultimate condition, either elastic or plastic, the sole intersection with the design domains is required.

Design Procedures

Various design approaches may be envisaged; further considerations are useful for a full understanding of the different possibilities.

A more detailed appraisal of the use of the design domains, and of the implications of the design requirements, is first attempted.

Assuming that the connection form was selected and the design domains determined, the unknowns of the design problem are the joint characteristics: i.e., stiffness, strength, and rotation capacity. The solution implies definition of the values of geometrical parameters governing connection response.

If the lines are drawn related to the unfactored and factored design loads, W_{unf} and $W_{f,U}$, the intersection with the beam domains identifies points S, E and P of Fig. 16. Simultaneous consideration of these points, and of points S, R_E and R_P previously defined, allows to determine the range in which the joint-beam response must lie in order to meet design requirements.

1. With reference to serviceability, line OS bounds the part $OSB_d'AO$ (see Fig. 16a) in which the response curve must lie in order for the PR beam to meet the deflection limit in service for $W_{d,S} \geq W_{unf}$. Intersection with line SS′ defines the rotation capacity required to attain of the unfactored design load.
2. With reference to the ultimate limit state in the elastic design (Fig. 16b), the segmental line OR_EE bounds the "effective" part OR_EEBAO, in which the response curve must lie in order to fulfill inequalities related to elastic resistance, $W_{U,E} \geq W_{f,U}$ and $W_{d,E} \geq W_{unf}$. However, intersection with the factored load line E′E permits attainment of the design load $W_{f,d}$ and defines the minimum rotation capacity required. In case the $M - \phi$ curve does not reach line ABE, failure of the joint occurs while the beam still is in the elastic range.
3. With reference to the ultimate limit state in the plastic design (Fig. 16c), the segmental line OR_PP bounds the "effective" part $OR_PPB''AO$ in the plastic beam domains, in which the response curves must lie in order to have a greater plastic resistance than the factored design load, $W_{U,P} \geq W_{f,U}$, and a greater allowable resistance than the unfactored design load, $W_{d,P} \geq W_{unf}$. Furthermore, response curves must intersect the

boundary PB''A in order to possess sufficient rotation capacity to make it possible to attain the deflection limit δ_U. Attainment of the design load $W_{f,U}$, however, implies that joint curves intersect only line E'EP, which defines the minimum rotation capacity required. If intersection occurs with segment E'E, joint failure will occur when the beam is still elastic, while intersection with the segment EP is associated with joint collapse and beam with a midspan plastic hinge.

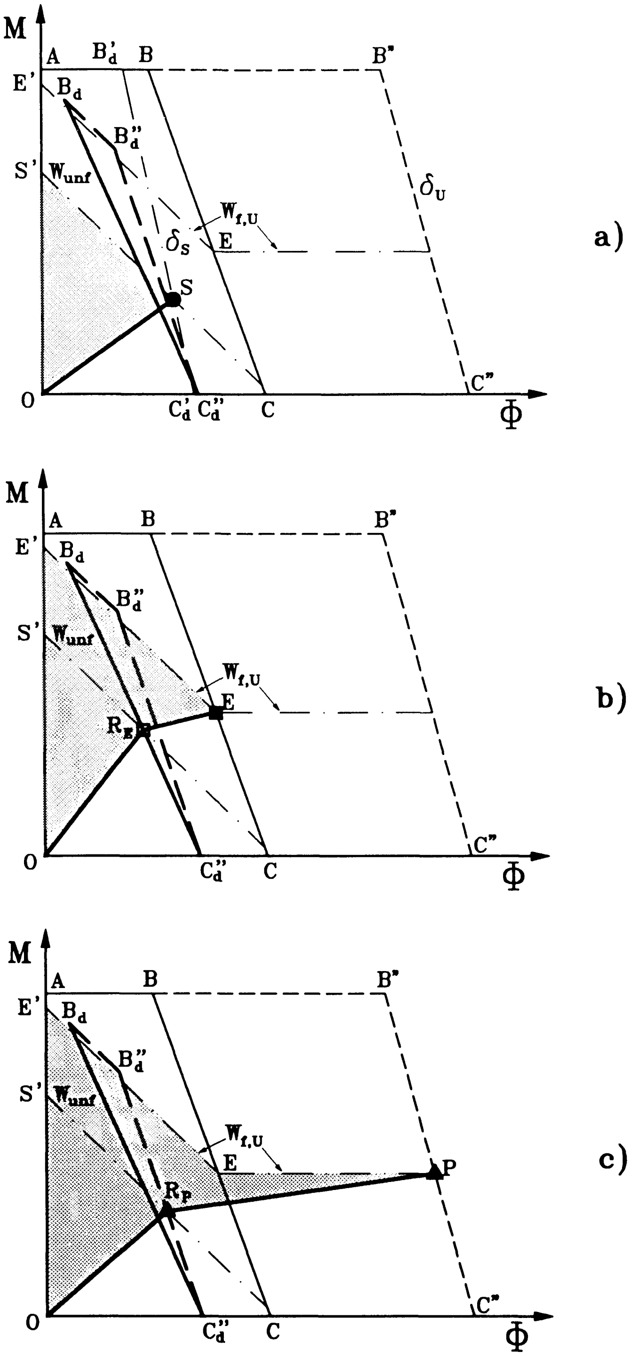

Fig. 16. Effective parts of the beam domains.

A comparative analysis of Figs. 16b and 16c, with reference to Fig. 15 as well, indicates that plastic design is less restrictive than elastic design in terms of stiffness requirements (point R_E is always located above point R_P), while they both require the same joint ultimate strength (although at different values of rotation). Plastic design is, however, significantly stricter as to joint rotation capacity.

Depending on the span-to-depth ratio, the steel grade, and the assumed value for the deflection limit δ_S, serviceability may govern the joint stiffness, i.e., point S may lie above line OR_E in elastic design, or above line OR_P in plastic design, as is shown schematically in Fig. 17a. In this case, determination of the lower boundary OSP of the effective domain is straightforward: S and P are in fact intersections of the beam limit state domains with the constant load lines associated to the design load, unfactored and factored respectively.

On the contrary, when S lies below the R point as in Fig. 17b, the determination of boundary OR_EE (or OR_PP) implies, in principle, definition of the design domains for the connection type considered. However, it may be shown that within the family curves representing the behavior of a given form of connection, no curve intersects the ultimate domains above point E (or P) and the W_{unf} line between S and R. This

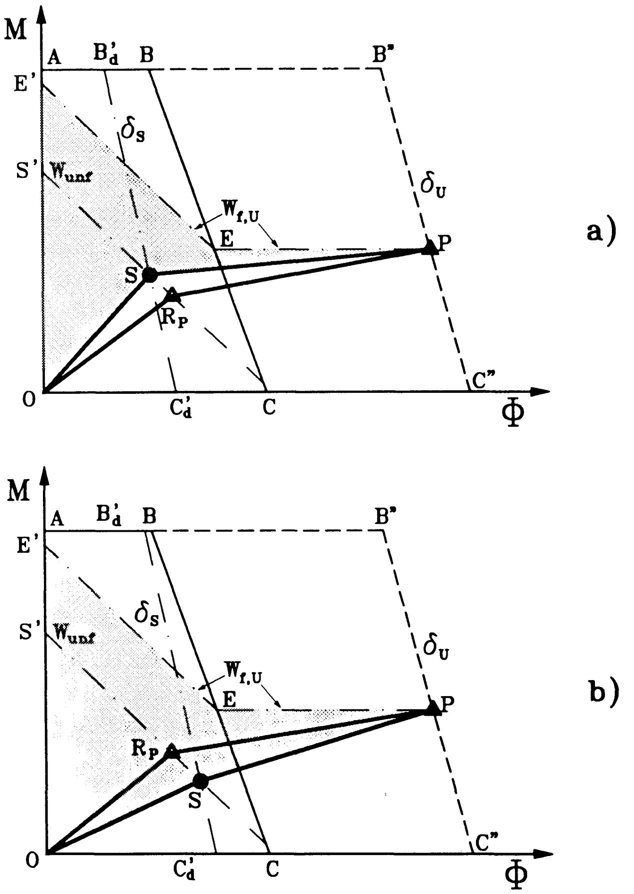

Fig. 17. Limit states governing PR beam design.

implies that boundary OSE (OSP) may be assumed also in this case, thus achieving a substantial simplification.

Furthermore, this result can be generalized: points R depend on the joint-beam design domains, that is, on the joint response, while points S are invariant for a given beam, with respect to the behavior of the connections. The boundary of the effective domains can thus be determined without any preliminary selection of the connection form.

Design Procedure 1

The beam span, the nominal values of the dead and live load, the load factors, and the deflection limits are design data. The steel grade and the beam section are generally selected by simple models in the very preliminary phase.

Unknowns of the problem are the geometrical and mechanical parameters governing joint response.

A design approach, which makes use of the design domains, may consist of the following steps:

1. Selection of the ultimate limit state to be considered: elastic or plastic;
2. Calculation of the factored load $W_{f,U}$ and of the safety factor $\gamma_{U,W}$;
3. Definition of the beam limit state domains;
4. Selection of the connection type and of the model for the approximation of the moment-rotation response;
5. Definition of the design domains;
6. Determination of the load lines associated to the unfactored and factored loads;
7. Determination of the points S, R_E and R_P, by intersecting the domains with the unfactored load line;
8. Determination of point E or point P, intersections between the beam limit state domains and the factored load line;
9. Determination of the joint curve, which intersects the effective part of the design domain and possesses the rotation capacity required by design assumptions.

Design Procedure 2

The beam span, the steel grade, the nominal values of the dead and live load, the load factors, and the deflection limits are design data.

Unknowns of the problem are the beam section and the joint geometrical and mechanical parameters.

Construction of the design domains can be avoided, and a design approach may consist of the following steps:

1. Selection of the ultimate limit state to be considered: elastic or plastic;
2. Calculation of the factored load $W_{f,U}$ and of the safety factor $\gamma_{U,W}$;
3. Selection of the beam on the basis of a predetermined level of ultimate resistance. The load capacity of the fully fixed beam can be advantageously adopted;
4. Definition of the beam limit state domains;
5. Determination of point S, by intersecting the beam serviceability beam domains with the unfactored load line;
6. Determination of point E or point P, intersections between the beam limit state domains, and the factored load line;
7. Determination of the response curve, which lies above line OSE (or above line OSP) and possesses the rotation capacity required by design assumptions.

SOME EXAMPLES

Some examples are presented in this section, aiming at illustrating in detail the procedures for the analysis and design of PR beams.

Reference is made to the case of a beam made of a W section and restrained via semi-rigid bolted connections, as is shown in Fig. 18.

The following design data are assumed:

- deformation limits: $\delta_S = L_0/250$, $\delta_U = L_0/50$;
- load factors, according to AISC-LRFD: $\gamma_{U,D} = 1.2$, $\gamma_{U,L} = 1.6$, $\gamma_S = 1.0$;
- dead load $D = 0.820$ kip/ft, live load $L = 0.655$ kip/ft;
- beam span $L_0 = 30$ ft.

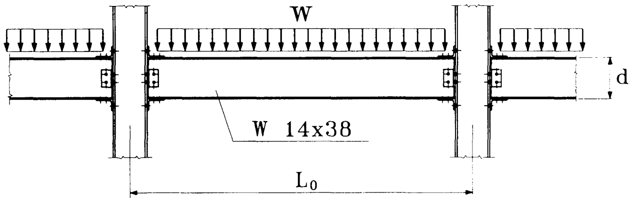

Fig. 18. PR beam considered in the example.

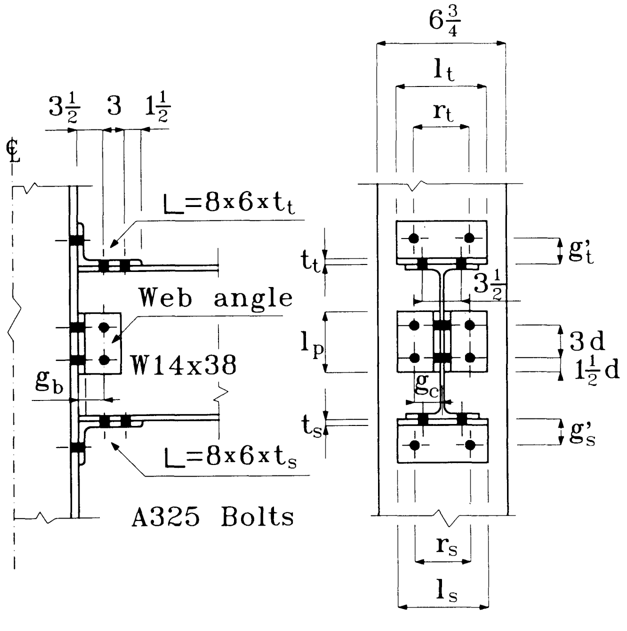

Fig. 19. The connection.

Table 1.

Connection	Bolt Diameter	Top and Seat Angle				Web Angle			Initial Stiffness	Moment at c = 50 mrad	Ultimate Moment
	d (in.)	$t_t = t_s$ (in.)	l_t (in.)	$r_t = r_s$ (in.)	$g_t' = g_s'$ (in.)	Angle	l_p (in.)	$g_b = g_c$ (in.)	$K_{e,t}$ (k-in./rad)	$M_{u,c}$ (k-in.)	M_u (k-in.)
MRC-7/16	3/4	7/16	8	5½	2⅝	2L 4×3½×¼	4½	2⅝	191646	560	618
MRC-½	3/4	½	8	5½	2⅝	2L 4×3½×¼	4½	2⅝	295125	748	816
MRC-⅝	3/4	⅝	6	3½	2⅝	2L 4×3½×¼	4½	2⅝	466024	891	953
MRC-¾	⅞	¾	6	3½	2⅞	2L 4×4×¼	5¼	2⅞	671929	1155	1228
MRC-⅞	1	⅞	6	3½	3	2L 4×4×¼	6	3	1136590	1502	1577

The unfactored design load is hence $W_{unf} = D + L = 1475$ kip/ft, the factored design load $W_{f,U} = 2.032$ kip/ft, and $\gamma_{U,W} = 1.38$.

The rotational response of the bolted connection of Fig. 19, with web and flange angles, is highly nonlinear throughout the range; its resistance may vary significantly, covering practically the whole semi-rigid range of joint behavior shown in Fig. 1. A prediction model was proposed by Kishi et al[12] to approximate the $M - \phi$ curves on the basis of the expression

$$M = K_{e,t}\phi/[1 + (\phi/\phi_{ref})^n]^{1/n} \qquad (6)$$

where $K_{e,t}$ is the initial joint stiffness, $\phi_{ref} = M_U/K_{e,t}$ is the reference value of the plastic rotation, M_U is the joint ultimate capacity, and the system factor n can be assumed as 0.875 for the cases considered.

The joint stiffness $K_{e,t}$ and the moment of resistance M_U can be computed as functions of the steel properties and of the main geometrical variable, i.e., (Fig. 19):

- the thicknesses t_t and t_s of the top and seat angle;
- the thickness t_p of the web angle;
- the angle lengths l_t, l_s and l_p;
- bolt gages r_s, r_t, g_s', g_t' and g_b;
- bolt diameter;
- beam geometrical dimensions.

Calculations are carried out on the assumption that connection rotation capacity is not a critical parameter.

Analysis

A beam section W14×38 was adopted, and spans ranging from 17 ft 7 in. to 41 ft 1 in., in order to have span-to-depth ratios of 15, 25, and 35.

A set of joints was designed so that the parameter governing response was the top-and-seat-angle thickness, which were assumed to be equal. The resulting geometrical dimensions are summarized in Table 1, together with the stiffness and strength values, computed for steel grade ASTM A36. The moment-rotation curves are drawn in Fig. 20. They are asymptotic; however, comparison between the ultimate moment M_u and the moment $M_{U,C}$ at a value of rotation $\phi_c = 50$ mrad indicates that most of the resistance is available for design purposes.

With reference to the criterion of classification given in the Eurocode 3,[2] all of these joints are semi-rigid, with a partial strength.

The procedure of analysis discussed previously was applied (Fig. 21) with the aim of determining the maximum allowable unfactored design load W_d, i.e., the maximum unfactored load permitting to fulfill all design requirements. In the elastic analysis W_d is the lowest between $W_{d,S}$ and $W_{d,E}$; in the plastic analysis W_d is the lowest between $W_{d,S}$ and $W_{d,P}$.

Table 2 presents the results obtained; for comparison, the simply supported and the fully fixed cases are also included.

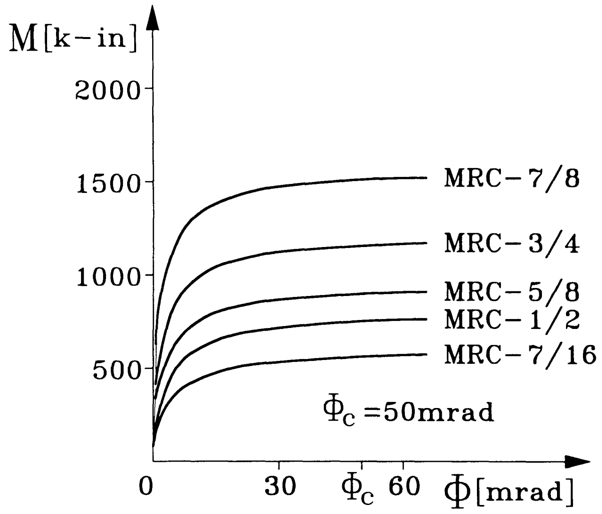

Fig. 20. $M - \phi$ of the connection considered.

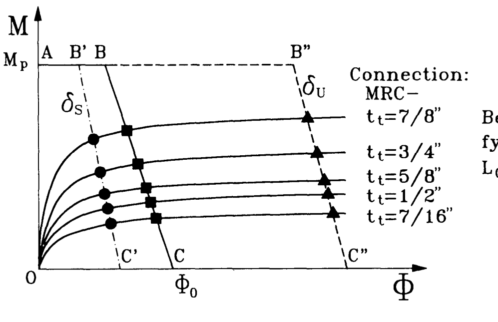

Fig. 21. Response curves of the PR beam.

Table 2.

	Elastic Analysis									Plastic Analysis								
	$L_0/d=15$			$L_0/d=25$			$L_0/d=35$			$L_0/d=15$			$L_0/d=25$			$L_0/d=35$		
Connection	W_d (k/ft)		α	W_d (k/ft)		α	W_d (k/ft)		α	W_d (k/ft)		α	W_d (k/ft)		α	W_d (k/ft)		α
	E	3.09	0.00	S	0.93	0.00	S	0.34	0.00	P	3.09	0.00	S	0.93	0.00	S	0.34	0.00
MRC-7/16	E	3.77	0.44	S	1.34	0.56	S	0.55	0.40	P	3.95	0.28	S	1.34	0.32	S	0.55	0.26
MRC-½	E	4.02	0.60	E	1.48	0.75	S	0.62	0.54	P	4.23	0.37	S	1.49	0.43	S	0.62	0.35
MRC-5/8	E	4.24	0.74	E	1.56	0.86	S	0.68	0.65	P	4.45	0.44	P	1.60	0.52	S	0.68	0.42
MRC-¾	E	4.58	0.95	E	1.69	1.04	S	0.79	0.86	P	4.86	0.57	P	1.75	0.63	S	0.79	0.56
MRC-7/8	E	5.07	1.27	E	1.87	1.29	S	0.92	1.11	P	5.40	0.75	P	1.94	0.78	S	0.92	0.72
	E	4.65	1.00	E	1.66	1.00	E	0.86	1.00	P	6.17	1.00	P	2.22	1.00	P	1.14	1.00

The factor α is an index of the gain, in terms of W_d, made possible by the end restraint; this parameter is calibrated so that $\alpha = 0$ for the simply supported beam and $\alpha = 1.0$ for the fully fixed beam.

The results point out that significant increases of W_d are made possible also by rather flexible connections, in particular when an elastic analysis is conducted. Moreover, it should be noted that, in the case of the elastic analysis, PR beams may resist greater unfactored loads than fully fixed beams.

The value of the span-to-depth ratio L_0/d affects the governing limit state. The load capacity of beams with L_0/d = 15 is always associated to the ultimate limit condition, while, as expected, serviceability becomes increasingly important with the beam span. The presence of semi-rigid connections, however, reduces the dependence of the beam allowable resistance on deflection in service, permitting a better exploitation of the beam section plastic strength.

If a live-load deflection limit is specified, the maximum allowable value L_d of the live load, corresponding to the allowable total load W_d, may be computed according to the approach discussed previously and illustrated in Fig. 14. Tables 3 and 4 report the values of L_d for the cases considered, and a live-load deflection limit $\delta_{S,L} = L_0/360$;[1] the ratio of this load to the total load W_d is also indicated. It ensues that this ratio mainly depends on the value of L_0/d: live load may range from 56% to 79% for $L_0/d = 25$, and from 38% to 48% of the total load for $L_0/d = 35$. For stocky beams ($L_0/d = 15$), deflections are so low that even the total load W_d is associated to deflections lower than the assumed $\delta_{S,L}$: the serviceability check does not imply any limitation to the live load.

Design: Procedure 1

Let us consider the following problem: design of the partially restrained joints of a W14×38 beam, with reference to the connection type of Fig. 19.

The same main data are assumed as were given in the beginning of this example section, the steel grade of both the beam and the connection angles is ASTM A36 with $F_y = 36$ ksi, and joint response model is approximated by expression (6). An appropriate selection of the angle geometry and of the bolt class and diameter restricts the unknowns to the thickness of the top-and-seat-angles.

The available set of data permits determination of the design domains of the joint-beam system shown in Fig. 22 together with the beam limit state domains. Intersection with the unfactored load line identifies points S, R_E and R_P. Point R_E turns out to be the higher, meaning that strength governs the elastic design; on the contrary, the serviceability check is more restrictive in the plastic design, i.e., point S lies above point R_P.

Fulfilment of elastic (plastic) design specifications requires that the connection $M - \phi$ curve passes above point R_E(S) and crosses the factored load line E'E (E'EP).

The lightest connection, among those in Table 1, meeting the elastic design conditions is the MRC½ with angle thickness of ½ in.; the same connection would be selected in plastic design.

An analysis of Fig. 22 points out that this connection permits to actually achieve the beam ultimate limit states. It may hence be concluded that the connection form chosen is satisfactory.

Design: Procedure 2

Elastic Design

Let us consider the following problem: design of a partially

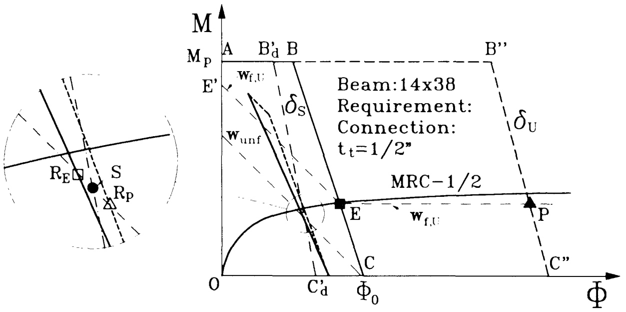

Fig. 22. Design procedure 1.

Table 3.

Connection	Elastic Analysis								
	$L_0/d=15$			$L_0/d=25$			$L_0/d=35$		
	$w_{d,E}$ (k/ft)	$L_{d,E}$ (k/ft)	$\frac{L_{d,E}}{w_{d,E}}$	$w_{d,E}$ (k/ft)	$L_{d,E}$ (k/ft)	$\frac{L_{d,E}}{w_{d,E}}$	$w_{d,E}$ (k/ft)	$L_{d,E}$ (k/ft)	$\frac{L_{d,E}}{w_{d,E}}$
	3.09	3.01	0.97	1.11	0.65	0.58	0.57	0.23	0.40
MRC-7⁄16	3.77	(1)	–	1.39	0.80	0.57	0.71	0.28	0.39
MRC-½	4.02	(1)	–	1.48	0.87	0.59	0.76	0.30	0.39
MRC-⅝	4.24	(1)	–	1.56	0.94	0.60	0.81	0.31	0.38
MRC-¾	4.58	(1)	–	1.69	1.13	0.67	0.88	0.36	0.41
MRC-⅞	5.07	(1)	–	1.87	1.47	0.79	0.97	0.47	0.48
	4.65	(1)	–	1.66	(1)	–	0.86	(1)	–

(1) $L_{d,E}$ is greater than $w_{d,E}$.

Table 4.

Connection	Plastic Analysis								
	$L_0/d=15$			$L_0/d=25$			$L_0/d=35$		
	$w_{d,P}$ (k/ft)	$L_{d,P}$ (k/ft)	$\frac{L_{d,P}}{w_{d,P}}$	$w_{d,P}$ (k/ft)	$L_{d,P}$ (k/ft)	$\frac{L_{d,P}}{w_{d,P}}$	$w_{d,P}$ (k/ft)	$L_{d,P}$ (k/ft)	$\frac{L_{d,P}}{w_{d,P}}$
	3.09	3.01	0.97	1.11	0.65	0.58	0.57	0.23	0.40
MRC-7⁄16	3.95	(1)	–	1.42	0.79	0.56	0.72	0.27	0.37
MRC-½	4.23	(1)	–	1.52	0.85	0.56	0.78	0.29	0.37
MRC-⅝	4.45	(1)	–	1.60	0.91	0.57	0.82	0.31	0.38
MRC-¾	4.86	(1)	–	1.75	1.07	0.61	0.89	0.35	0.39
MRC-⅞	5.40	(1)	–	1.94	1.36	0.70	0.99	0.45	0.45
	6.17	(1)	–	2.22	(1)	–	1.14	(1)	–

(1) $L_{d,P}$ is greater than $w_{d,P}$.

restrained W beam, with reference to the connection type of Fig. 19.

The main data are assumed to be the same, the steel grade of both the beam and the connection angles is ASTM A36 with F_y = 36 ksi, and joint response model is approximated by expression (6). An appropriate selection of the angle geometry and of the bolt class and diameter restricts the connection unknowns to the thickness of the top-and-seat-angles.

A further unknown is the size of the beam section; however, this may be determined on the basis of effective performance requirements. If the PR beam is required to have the same elastic resistance as a fully fixed beam, it follows that the minimum value of the beam section plastic moment is obtained as

$$\phi_b M_p = W_{f,U} L_0^2/12 \qquad (7)$$

where the resistance factor ϕ_b may be assumed as 0.9 according to AISC-LRFD.[1]

In the case considered, and maintaining, for the sake of comparison, the same beam depth as in the example of the section on analysis, the minimum weight W14 shape with a plastic moment greater than the value given by expression (7) is the W14×34.

The beam limit state domains can then be defined; they are shown in Fig. 23. Intersections S and E, with the load lines associated to the unfactored and factored loads respectively, allow to obtain the effective part of the domains through line OSE, as is illustrated in the figure. The joint response must lie in this area and cross the factored load line.

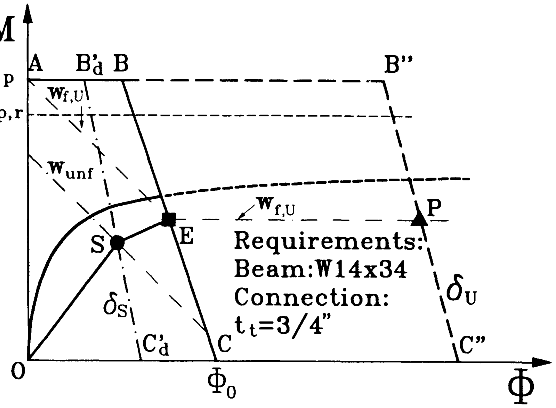

Fig. 23. Design procedure 2: Elastic design.

In order to fulfill all design requirements, the top-and-seat-angles should have a thickness equal to at least ¾ in.

Plastic Design

Let us consider the same problem as given in the section on Design Procedure 1: i.e., design of a partially restrained W beam, with reference to the connection type of Fig. 19. It is assumed that the PR beam is required to have the same plastic resistance as a fully fixed beam; it follows that the minimum value of the beam section plastic moment is obtained from the relationship

$$\phi_b M_p = W_{f,U} L_0^2/16 \qquad (8)$$

where the resistance factor ϕ_b may be assumed as 0.9 according to AISC-LRFD.[1]

If, for the sake of comparison, the same beam depth as in the example of the section on analysis is maintained, the minimum weight W14 shape with a plastic moment greater than the value given by expression (8) is the W14×30.

The beam limit state domains can then be defined; they are shown in Fig. 24.

Intersections S and P, with the load lines associated to the unfactored and factored loads respectively, allow to obtain the effective part of the domains through line OSP, as is illustrated in the figure. The joint response must lie in the effective area and cross the factored load line.

The connection form adopted weakens the end part of the beam, which subsequently has a reduced plastic resistance $M_{p,r}$; a further check of the joint strength required by point P is necessary, which should be lower than $M_{p,r}$.

In order to fulfill all design requirements, the top-and-seat-angles should have a thickness equal to at least ¾ in. A plastic analysis hence permits a reduction in the beam weight, without requiring a heavier connection.

Further Results

Besides the single design cases, it appears of interest to evaluate the combination connection-beam shape fulfilling the assumed design requirements. To this end, the W section of minimum weight was defined for each of the connections in Table 1. An analysis was also conducted for the simply supported and the fully fixed cases. The results are reported in Table 5. They indicate that:

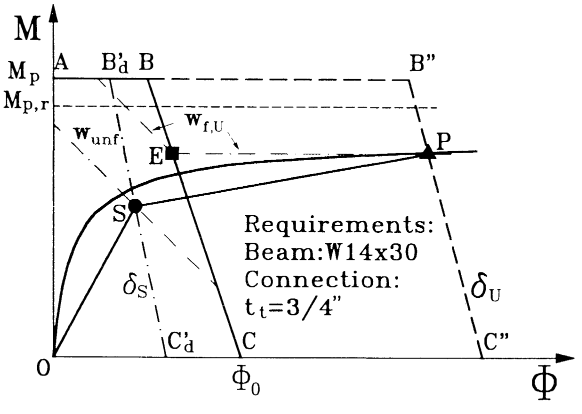

Fig. 24. Design procedure 2: Plastic design.

Table 5.

Connection	Elastic Analysis W Shape	Elastic Analysis % *w*	Plastic Analysis W Shape	Plastic Analysis % *w*
	W18×46	0.00	W18×46	0.00
MRCH-7/16	W18×35	23.9	W18×35	23.9
MRCH-1/2	W18×35	23.9	W18×35	23.9
MRCH-5/8	W18×35	23.9	W16×31	32.6
MRCH-3/4	W16×31	32.6	W16×26	43.5
MRCH-7/8	W16×31	32.6	W16×31	32.6
	W14×34	26.1	W16×26	43.5

- in the elastic analysis, even rather flexible joints permit a remarkable reduction in beam weight as compared with simply supported beams; PR beams may be even lighter than fixed ended beams;
- also in the plastic analysis, partially restrained connections may result in beam shapes of the same size as the fully restrained case;
- plastic design, however, allows a greater material saving (up to 43.5%) than elastic design (32.6%); as expected, differences between the two design approaches increase with the moment capacity of the joint;
- the weakening of the end zone of the beam, resulting from the bolt holes, may govern the beam size when the joint strength is close to the beam plastic moment (see joint MRCH-⅞, plastic analysis).

CONCLUSIONS

A method was presented here to determine the response of a partially restrained beam as well as its limit state performance. This approach permits a comprehensive understanding of the influence of the joint rotational behavior, regardless of its inherent nonlinearity.

The use of the method in limit state designing was then discussed, with reference to both the checking and the sizing phases, with the aid of some examples.

The results showed in particular that:

- possible benefits from design incorporation of joint behavior can be straightforwardly assessed through the analysis procedure;
- "optimal" engineering solutions can be defined by appropriate design procedures, adopting the beam shape, the connection performance, or both as unknown parameters;
- PR connections may permit the use of the same beam shape as FR connections.

Simple mathematical relationships express the beam limit state conditions as well as the joint-beam response curves;[19] moreover, the use of this approach appears to be fully effective graphically. Therefore, it is feasible and convenient for a CAD system, which is currently under development.

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